Week 25

AP Physics 1

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

7. Oscillations

Starter Quiz · 课前小测

AP Physics 1 Name: ____________ Score: ______

1. A spring has $k = 50\ \frac{\text{N}}{\text{m}}$. What is the size of the restoring force at a displacement of 0.2 m, in N?

   ____________ N

2. An oscillator has a period of 0.25 s. What is its frequency, in Hz?

   ____________ Hz

3. An oscillator is $x(t) = 0.1\cos(4t)$ (metres). What is its position at $t = 0$, in metres?

   ____________ m

4. A spring with $k = 200\ \frac{\text{N}}{\text{m}}$ oscillates with amplitude 0.1 m. What is the total energy, in joules?

   ____________ J

5. What defines simple harmonic motion?

   ☐ a constant force

   ☐ a restoring force proportional to displacement

   ☐ no force at all

   ☐ a force that grows with speed

6. For SHM, swinging with a larger amplitude makes the period longer.

   ☐ True ☐ False

7. Where is an SHM oscillator moving fastest?

   ☐ at the extremes (turning points)

   ☐ at the equilibrium (middle)

   ☐ everywhere the same

   ☐ it never moves

8. Where in the swing is all the energy kinetic?

   ☐ at the extremes

   ☐ at the middle (equilibrium)

   ☐ nowhere

   ☐ everywhere equally

9. In $F = -kx$, the minus sign means the force points ______ toward equilibrium.

   ____

10. The period of a simple pendulum depends on:

☐ the mass of the bob

☐ its length and g

☐ the amplitude only

☐ the colour of the string

AP Physics 1

1. **10 N**

   $|F| = kx = 50 \times 0.2 = 10$ N, pointing back toward equilibrium.

2. **4 Hz**

   $f = 1/T = 1/0.25 = 4$ Hz.

3. **0.1 m**

   $x(0) = 0.1 \cos 0 = 0.1$ m — the maximum displacement (amplitude).

4. **1 J**

   $E = \frac{1}{2}kA^2 = \frac{1}{2}(200)(0.01) = 1$ J.

5. **a restoring force proportional to displacement**

   SHM needs $F = -kx$: a restoring force proportional to displacement, pointing back to equilibrium.

6. **False**

   The period of SHM is independent of amplitude — a wider swing is also faster, so the time is unchanged.

7. **at the equilibrium (middle)**

   Speed is greatest at the middle, where the acceleration (and force) is zero.

8. **at the middle (equilibrium)**

   At the middle the speed is greatest and $x = 0$, so all the energy is kinetic.

9. **back**

   The force always opposes the displacement — it points **back** toward the middle.

10. **its length and g**

    $T = 2\pi\sqrt{L/g}$ — only length and gravity, not the bob's mass.

7.1 Defining Simple Harmonic Motion (SHM)

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS equilibrium 平衡 • restoring force 回复力 • simple harmonic motion 简谐运动 • period 周期

Objective

Build the skills to answer exam questions on **defining simple harmonic motion (SHM)**.

You must be able to:

- define **simple harmonic motion 简谐运动** as motion where the restoring force is proportional to the displacement
- identify the condition $F = -kx$ that produces SHM
- give examples of SHM, such as a mass on a spring and a small-angle pendulum

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The condition for SHM

The **restoring force 回复力** is proportional to the displacement and points back toward equilibrium: $F = -kx$. The negative sign is what makes the motion oscillate.

■ Examples

A mass on a spring, and a pendulum swinging through a **small** angle, both obey $F = -kx$ (approximately, for the pendulum).

■ Not SHM

A ball bouncing off the floor is periodic but **not** SHM — its force is not proportional to displacement.

2 Practice

2.1 State the condition on the restoring force for simple harmonic motion. [1]

2.2 Give two examples of systems that perform SHM. [2]

2.3 A spring obeys $F = -kx$. State the direction of the force when the mass is displaced to the right of equilibrium. [1]

3 Exam-style questions

3.1 In simple harmonic motion, the restoring force is [1]

- **A** constant
- **B** proportional to the displacement
- **C** proportional to the speed
- **D** always zero

3.2 Which of these is an example of SHM? [1]

- **A** a ball rolling down a hill
- **B** a mass oscillating on a spring
- **C** a car braking to a stop
- **D** a satellite in a circular orbit

3.3 A 0.50 kg mass on a spring of constant $k = 200\ \mathrm{N\ m^{-1}}$ is pulled 0.10 m from equilibrium.

(a) Find the size of the restoring force. [2]

(b) State the direction of this force. [1]

Solutions

2.1 it must be proportional to the displacement and directed back toward equilibrium ($F = -kx$).

2.2 a mass on a spring; a small-angle pendulum.

2.3 to the left, back toward equilibrium.

3.1 **B**.

3.2 **B**.

3.3 (a) $F = kx = 200 \times 0.10 = 20$ N. (b) toward equilibrium (to the left).

AP PHYSICS 1 • EXERCISE SHEET

7.2 Frequency and Period of SHM

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS period 周期 • frequency 频率 • amplitude 振幅 • oscillation 振动

Objective

Build the skills to answer exam questions on **the frequency and period of SHM**.

You must be able to:

- define the **period 周期** T and **frequency 频率** f and relate them by $f = 1/T$
- state that for SHM the period is **independent of amplitude 振幅**
- use $T = 2\pi\sqrt{m/k}$ for a mass-spring system and $T = 2\pi\sqrt{L/g}$ for a simple pendulum

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Period and frequency

T is the time for one full oscillation; $f = \dfrac{1}{T}$ is the number per second (Hz).

■ The two standard formulas

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(mass-spring)}, \qquad T = 2\pi\sqrt{\frac{L}{g}} \quad \text{(pendulum)}.$$

■ Amplitude does not matter

For SHM the period does **not** depend on the amplitude — a wider swing simply moves faster.

2 Practice

2.1 A pendulum has a period of 2.0 s. Find its frequency. [1]

2.2 A 0.20 kg mass oscillates on a spring of constant 80 N m^{-1}. Find the period. [2]

2.3 State how the period of a mass-spring system changes if the amplitude is increased.[1]

3 Exam-style questions

3.1 The period of a simple pendulum depends on [1]

- **A** its mass
- **B** its amplitude
- **C** its length
- **D** the material of the bob

3.2 To double the period of a mass-spring system, the mass must be [1]

- **A** doubled
- **B** quadrupled
- **C** halved
- **D** left unchanged

3.3 A simple pendulum has length 1.0 m (take $g = 9.8$ m s^{-2}).

(a) Find its period. [2]

(b) Find its frequency. [1]

Solutions

2.1 $f = \dfrac{1}{T} = \dfrac{1}{2.0} = 0.50$ Hz.

2.2 $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.20}{80}} = 0.31$ s.

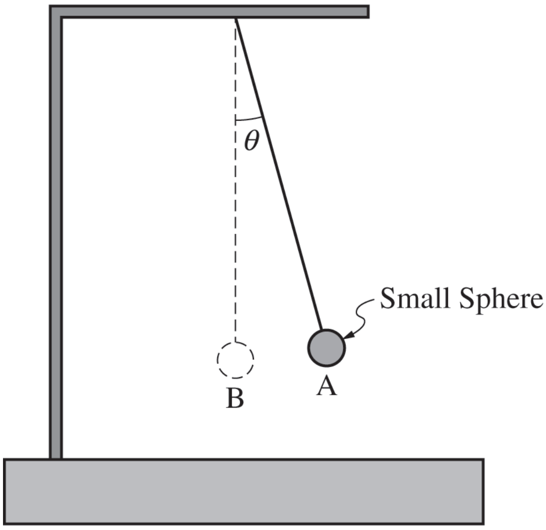
2.3 it does not change — the period is independent of amplitude.

3.1 **C**.

3.2 **B** — $T \propto \sqrt{m}$, so a factor of 2 in T needs a factor of 4 in m.

3.3 (a) $T = 2\pi\sqrt{\dfrac{1.0}{9.8}} = 2.0$ s. (b) $f = \dfrac{1}{2.0} = 0.50$ Hz.

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

A simple pendulum consists of a small sphere that hangs from a string with negligible mass. The top end of the string is fixed. The sphere is pulled to Point A so that the string makes a small angle θ with the vertical, as shown. The sphere is then released from rest and swings through its lowest point at Point B. The work done on the sphere by Earth between points A and B is W_E.

The pendulum is then taken to Planet X. The mass of Planet X is the same as the mass of Earth, but the radius of Planet X is greater than the radius of Earth. The sphere is again brought to Point A (displaced θ from the vertical), released from rest, and swings through its lowest point at Point B. The work done on the sphere by Planet X between points A and B is W_X.

(a) **Justify** why W_X is less than W_E.

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Continue your response to **QUESTION 4** on this page.

A new pendulum is made by hanging the same small sphere from a different string with negligible mass. The new string is slightly elastic, and the length of the string may increase or decrease depending on the tension applied to the string. On Earth, when the sphere is again displaced θ from the vertical and released from rest, the new pendulum oscillates with period T_E.

The new pendulum is then taken to a different planet, Planet Y. The radius of Planet Y is the same as the radius of Earth, but the mass of Planet Y is larger than the mass of Earth. On Planet Y, when the sphere is again displaced from the vertical and released from rest, the new pendulum oscillates with period T_Y.

(b) In a clear, coherent paragraph-length response that may also contain drawings, **explain** how T_Y could be larger than T_E but also could be smaller than T_E.

GO ON TO THE NEXT PAGE.

PHYSICS 1

SECTION II

Time—1 hour and 30 minutes

5 Questions

Directions: Questions 1, 4, and 5 are short free-response questions that require about 13 minutes each to answer and re worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to nswer and are worth 12 points each. Show your work for each part in the space provided after that part.

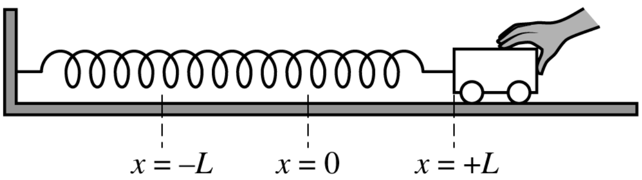

Figure 1

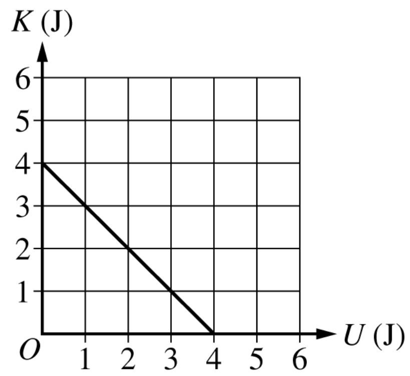

Figure 2

. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

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Continue your response to **QUESTION 1** on this page.

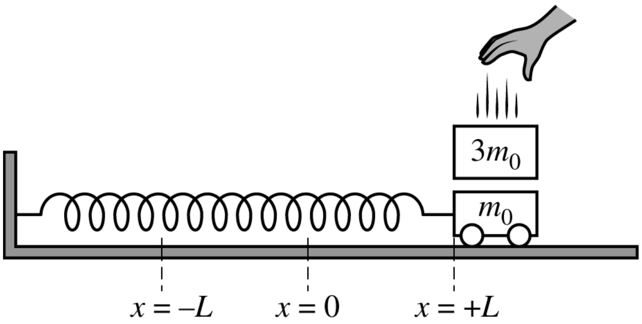

Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\dfrac{f_2}{f_1}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

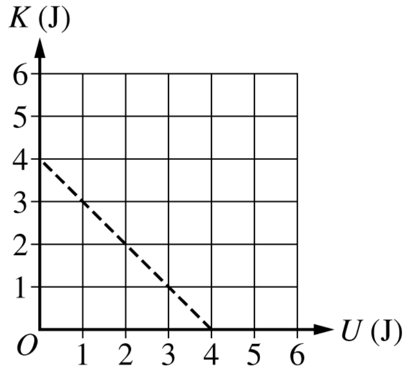

Figure 4

i. Briefly explain why the two graphs must be the same, using physics principles.

ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

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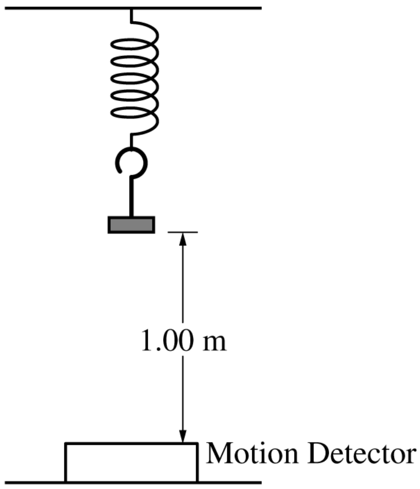

. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

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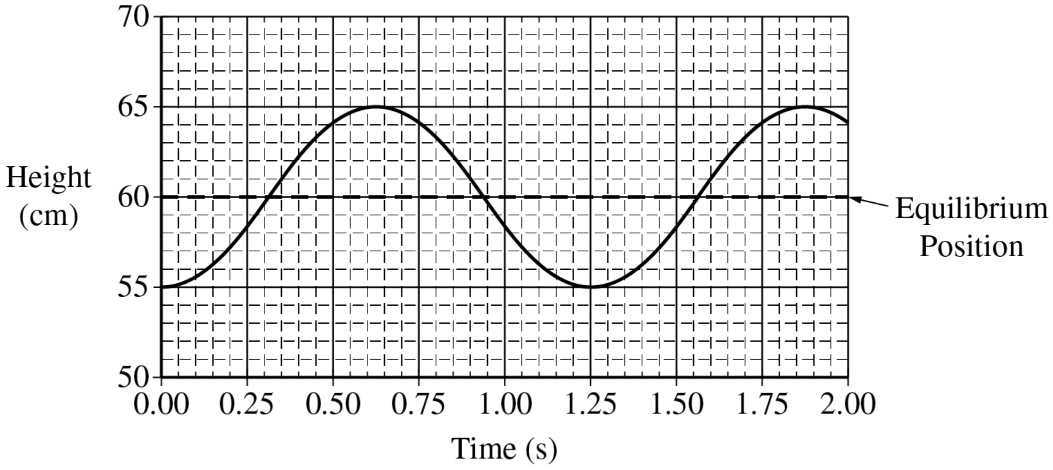
Continue your response to **QUESTION 5** on this page.



(a) Using the information given and information taken from the graph, calculate the spring constant.

(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

ii. Briefly explain why the total potential energy of the system is the same at these two times.

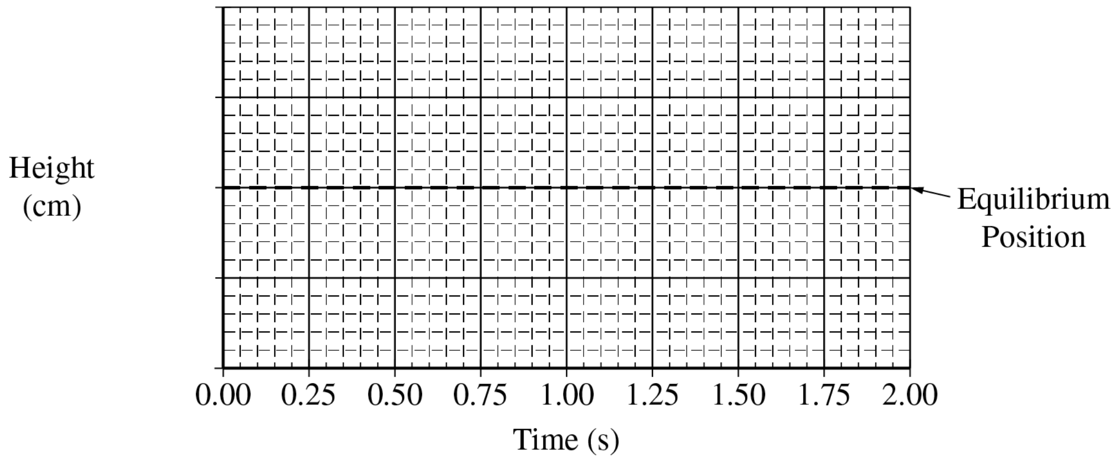
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 5** on this page.

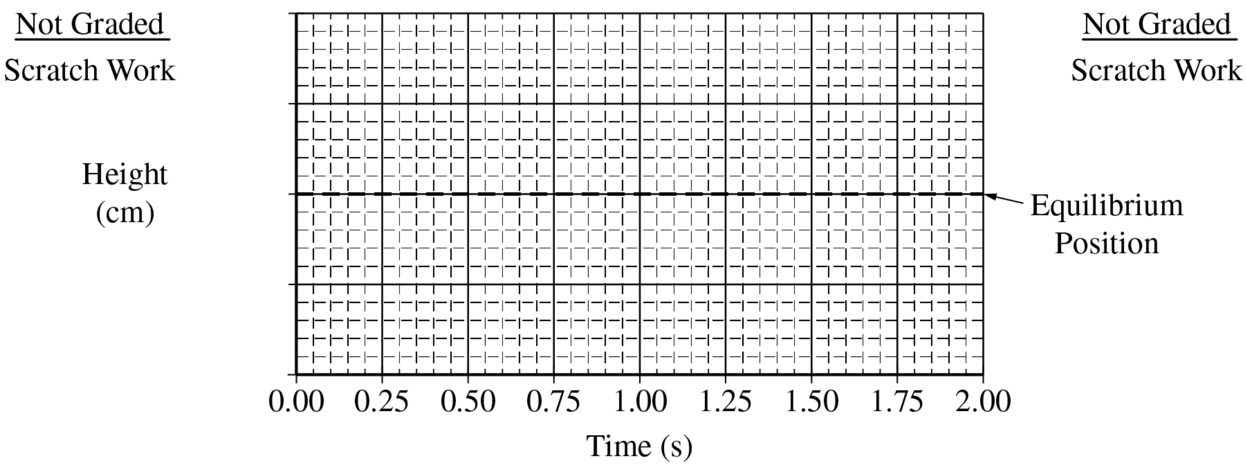
(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The $0.50\ \text{kg}$ object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



The following graph is provided for scratch work only and will <u>not</u> be graded.



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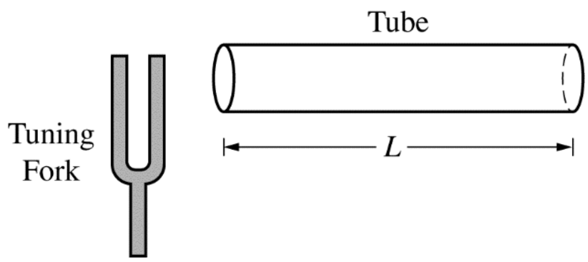
STOP

END OF EXAM

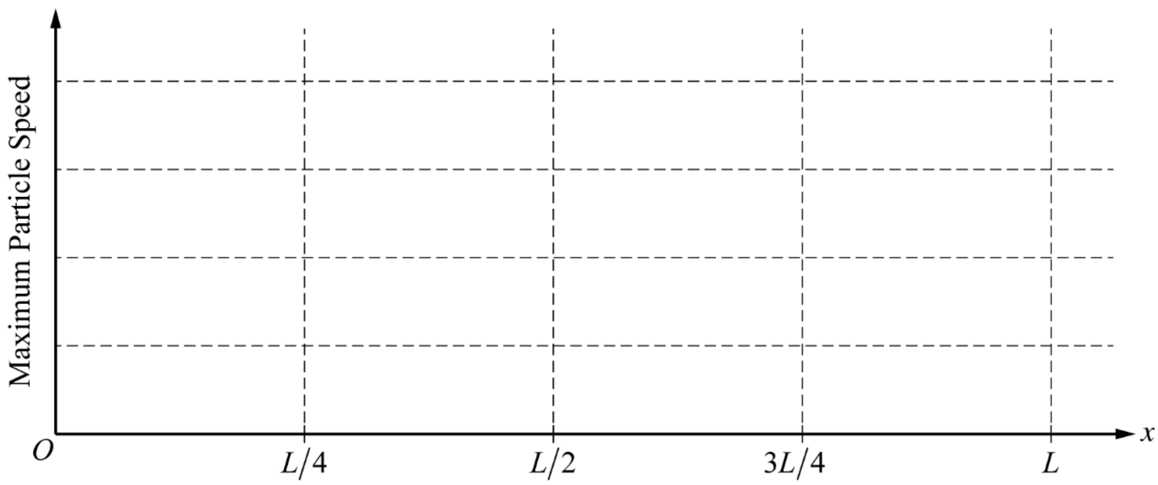


5. (7 points, suggested time 13 minutes)

A tuning fork vibrating at 512 Hz is held near one end of a tube of length L that is open at both ends, as shown above. The column of air in the tube resonates at its fundamental frequency. The speed of sound in air is $340\ \text{m/s}$.

(a) Calculate the length L of the tube.

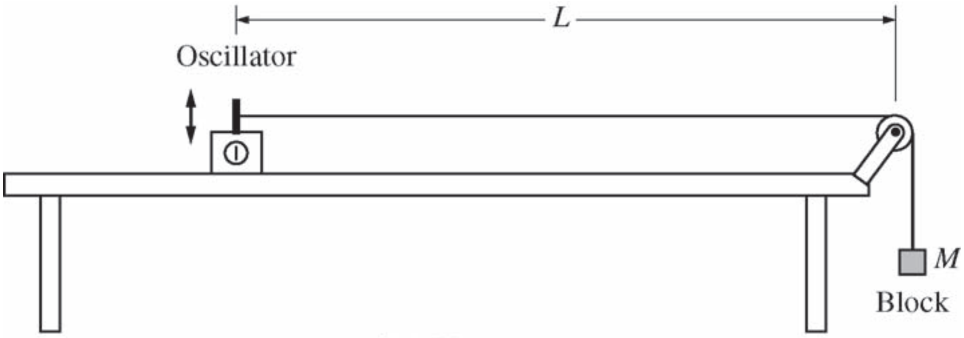
(b) The column of air in the tube is still resonating at its fundamental frequency. On the axes below, sketch a graph of the maximum speed of air molecules as they oscillate in the tube, as a function of position x, from $x = 0$ (left end of tube) to $x = L$ (right end of tube). (Ignore random thermal motion of the air molecules.)



(c) The right end of the tube is now capped shut, and the tube is placed in a chamber that is filled with another gas in which the speed of sound is $1005\ \text{m/s}$. Calculate the new fundamental frequency of the tube.

STOP

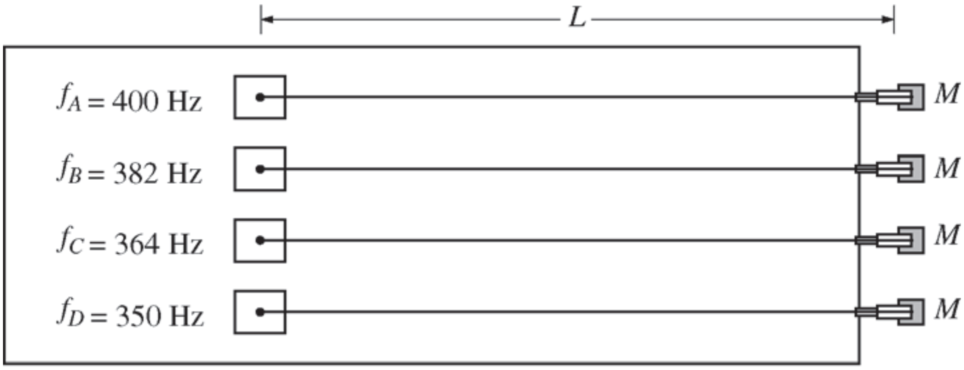
END OF EXAM



Side View

5. (7 points, suggested time 13 minutes)

The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency f. The distance between each oscillator and pulley L is the same, and the mass M of each block is the same. However, the fundamental frequency of each string is different.



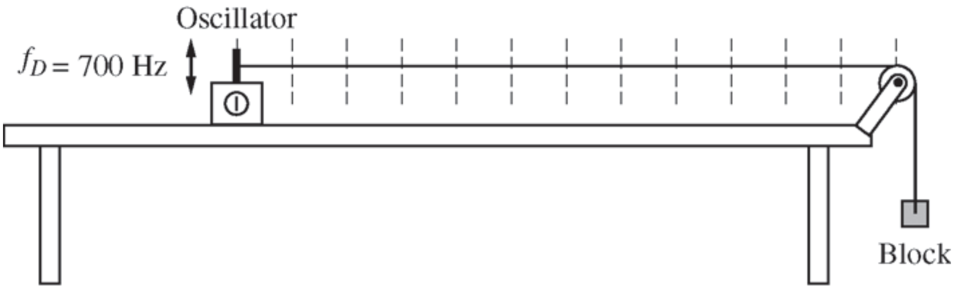
Top View

The equation for the velocity v of a wave on a string is $v = \sqrt{\dfrac{F_T}{m/L}}$, where F_T is the tension of the string and m/L is the mass per unit length (linear mass density) of the string.

(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

(c) The frequency of the oscillator connected to string D is changed so that the string vibrates in its second harmonic. On the side view of string D below, mark and label the points on the string that have the greatest average vertical speed.



Side View

STOP

END OF EXAM

AP PHYSICS 1 • EXERCISE SHEET

7.3 Representing and Analyzing SHM

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

KEY WORDS equilibrium 平衡 • amplitude 振幅 • phase 相位 • oscillation 振动

Objective

Build the skills to answer exam questions on **representing and analyzing SHM**.

You must be able to:

- represent the displacement, velocity, and acceleration of an oscillator as functions of time
- explain that the speed is greatest at the **equilibrium 平衡** position and zero at the **amplitude 振幅**
- relate the **phase 相位** of the motion to graphs of position, velocity, and acceleration

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Where the speed is greatest

The oscillator moves **fastest at equilibrium** ($x = 0$) and is momentarily **at rest at the amplitude** ($x = \pm A$).

■ Where the acceleration is greatest

Because $a = -\frac{k}{m}x$, the acceleration is **largest at the amplitude** and **zero at equilibrium** — the opposite of the speed.

■ Phase

If $x = A\cos(\omega t)$, then velocity and acceleration are shifted in phase — the velocity peaks a quarter-cycle after the displacement passes through zero.

2 Practice

2.1 State where in the oscillation the speed is greatest. [1]

2.2 State where the acceleration is greatest. [1]

2.3 A mass oscillates with amplitude 0.10 m. State its displacement at the instant its speed is maximum. [1]

3 Exam-style questions

3.1 In SHM, the velocity is maximum at the [1]

- **A** amplitude
- **B** equilibrium position
- **C** turning point
- **D** highest point

3.2 In SHM, the acceleration is zero at the [1]

- **A** amplitude
- **B** equilibrium position
- **C** both ends
- **D** every point

3.3 A mass on a spring oscillates between $x = +8.0$ cm and $x = -8.0$ cm.

(a) State the amplitude. [1]

(b) State where the kinetic energy is greatest. [1]

(c) State where the elastic potential energy is greatest. [1]

Solutions

2.1 at the equilibrium position ($x = 0$).

2.2 at the amplitude ($x = \pm A$).

2.3 $x = 0$ (equilibrium).

3.1 **B**.

3.2 **B**.

3.3 (a) 8.0 cm. (b) at equilibrium, $x = 0$. (c) at the amplitude, $x = \pm 8.0$ cm.

Begin your response to **QUESTION 2** on this page.

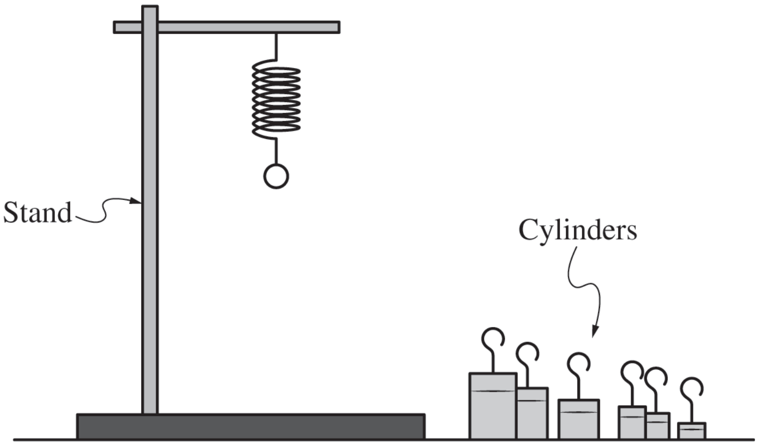


Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(a) Design an experimental procedure the student could use to determine the spring constant k of the spring.

In the following table, list the quantities that would be measured using only the provided equipment in your experiment. Define a symbol to represent each quantity.

In the space below the table, **describe** the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. If needed, you may include a simple diagram of the setup with your procedure.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
		Stopwatch
		Digital scale
Procedure (and diagram, if needed)		

(b)

i. **Indicate** the quantities that could be plotted to produce a linear graph whose slope can be used to determine the spring constant k of the spring.

Vertical axis:____________________ Horizontal axis:____________________

ii. Briefly **describe** how the slope of the graph would be analyzed to determine the spring constant k of the spring.

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Continue your response to **QUESTION 2** on this page.

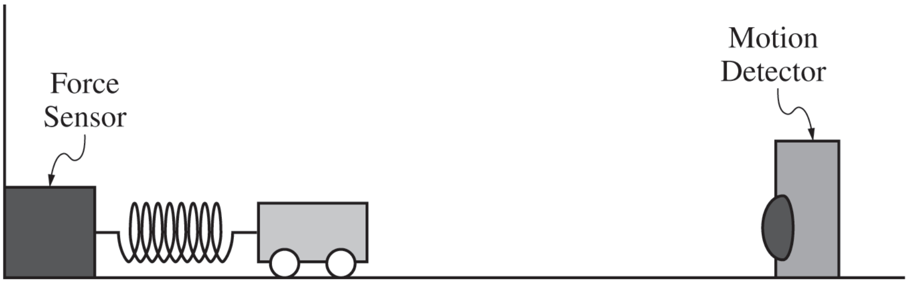
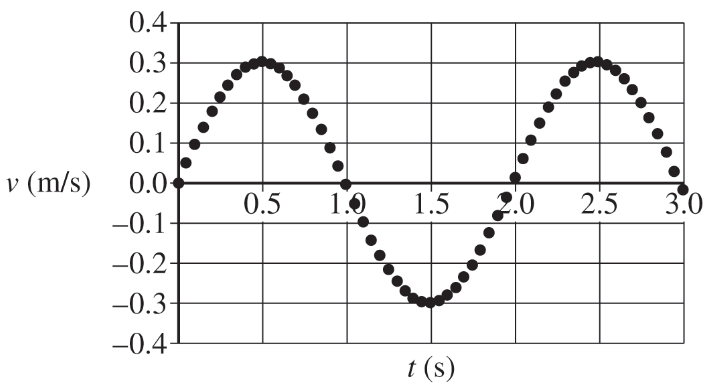
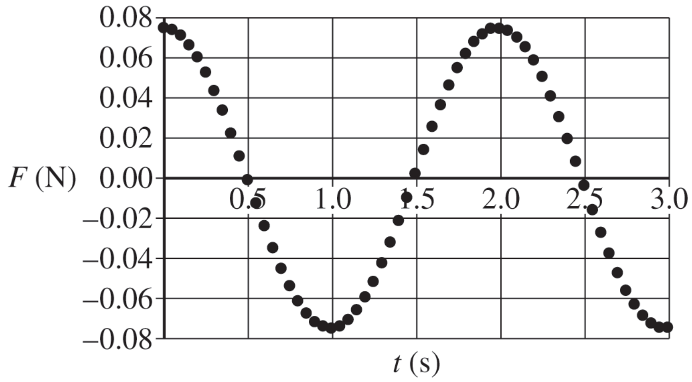


Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t.





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Continue your response to **QUESTION 2** on this page.

(c)

i. Using the data in the velocity-time graph, **calculate** the change in kinetic energy of the cart from $t = 0.5\text{ s}$ to $t = 2.0\text{ s}$. Show your steps and substitutions.

ii. Using the data in the force-time graph, **estimate** the change in momentum of the cart from $t = 0.5\text{ s}$ to $t = 2.5\text{ s}$. Briefly **explain** how you arrived at your estimation.

iii. Do the data from the velocity-time graph confirm your estimation from part (c)(ii) ? Briefly **explain**.

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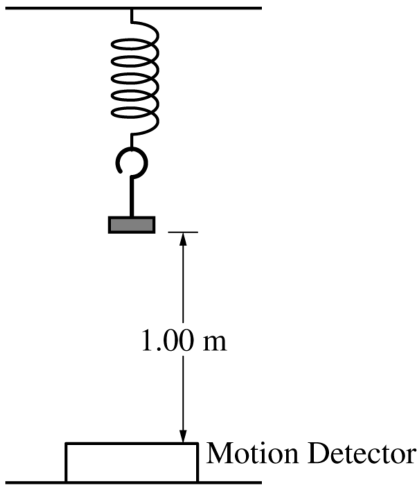

. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

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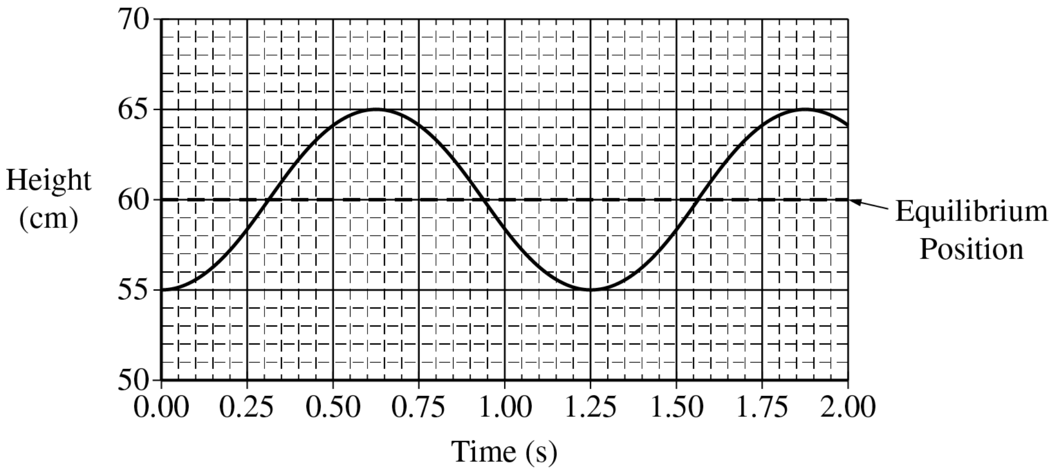
Continue your response to **QUESTION 5** on this page.



(a) Using the information given and information taken from the graph, calculate the spring constant.

(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

ii. Briefly explain why the total potential energy of the system is the same at these two times.

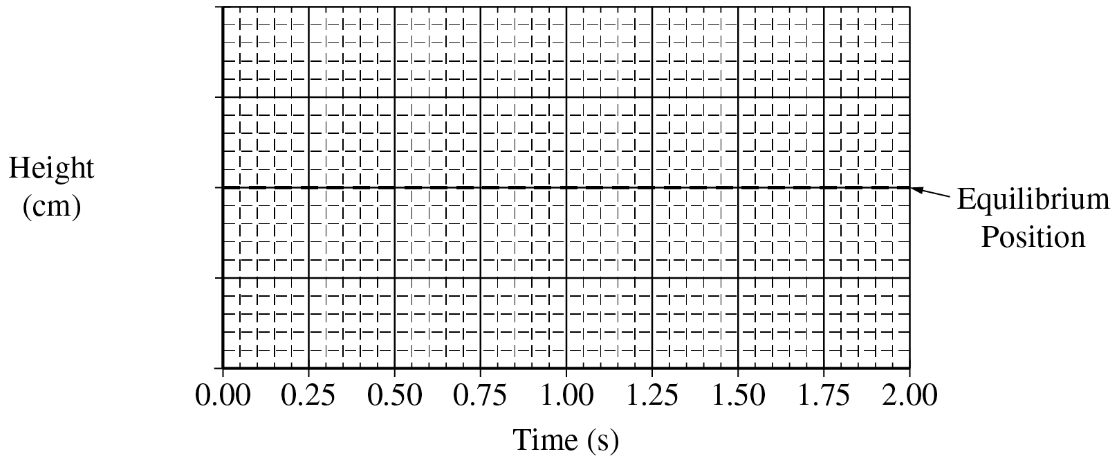
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 5** on this page.

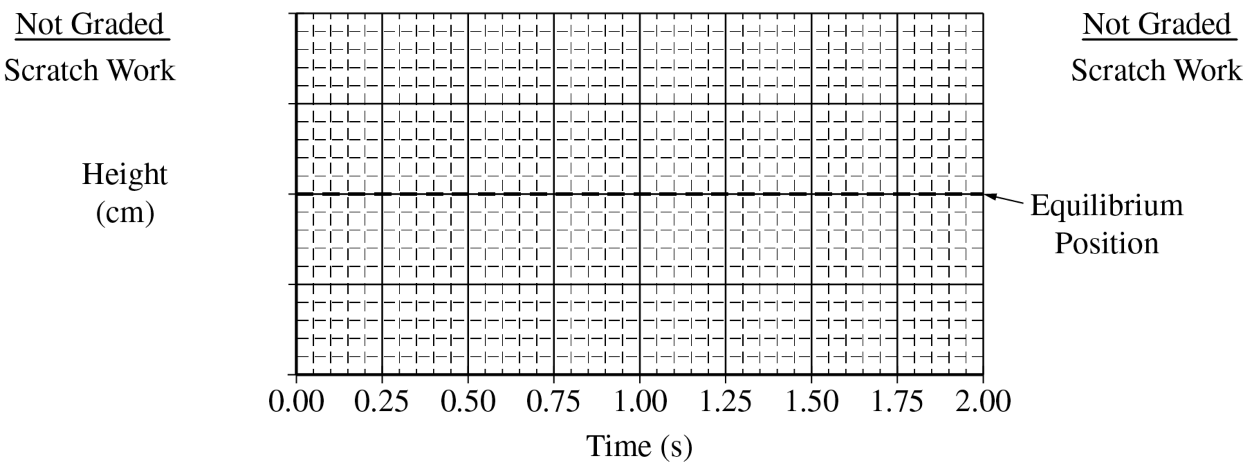
(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



The following graph is provided for scratch work only and will not be graded.



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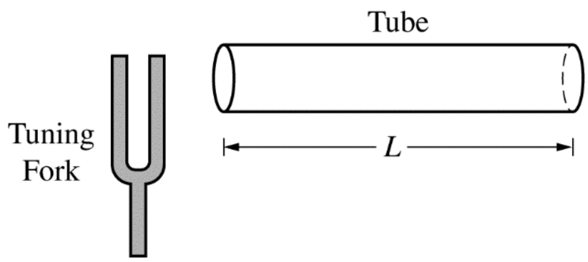
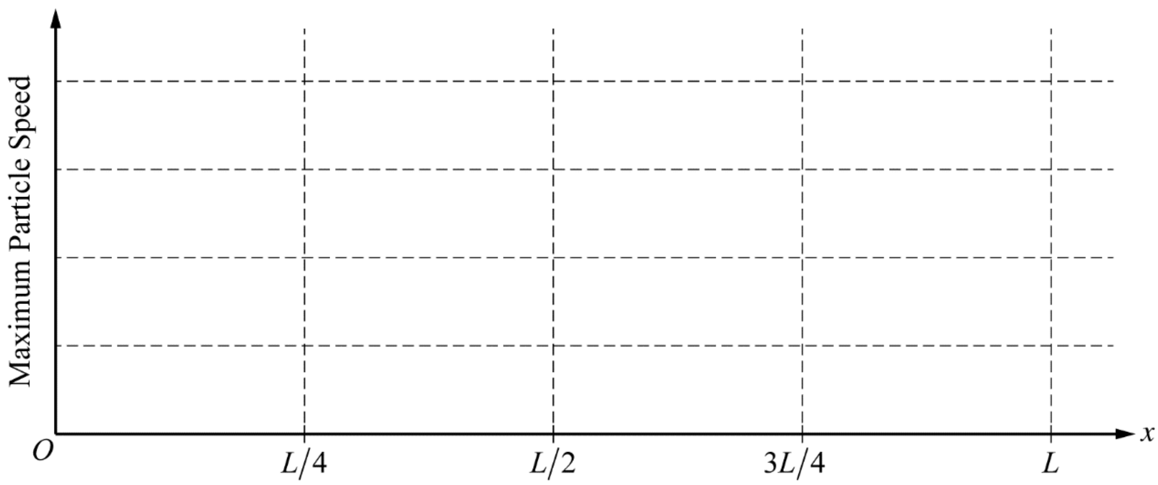


5. (7 points, suggested time 13 minutes)

A tuning fork vibrating at 512 Hz is held near one end of a tube of length L that is open at both ends, as shown above. The column of air in the tube resonates at its fundamental frequency. The speed of sound in air is $340\ \text{m/s}$.

(a) Calculate the length L of the tube.

(b) The column of air in the tube is still resonating at its fundamental frequency. On the axes below, sketch a graph of the maximum speed of air molecules as they oscillate in the tube, as a function of position x, from $x = 0$ (left end of tube) to $x = L$ (right end of tube). (Ignore random thermal motion of the air molecules.)



(c) The right end of the tube is now capped shut, and the tube is placed in a chamber that is filled with another gas in which the speed of sound is $1005\ \text{m/s}$. Calculate the new fundamental frequency of the tube.

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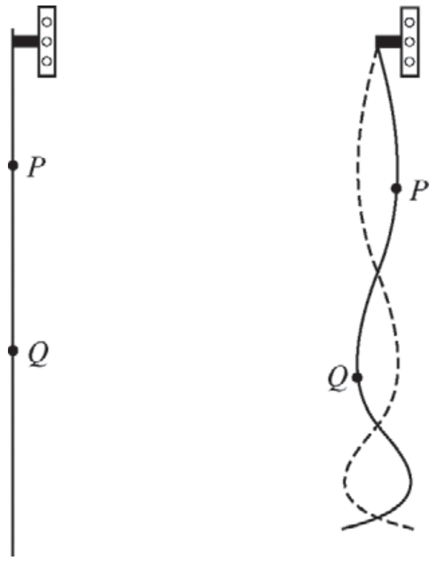
END OF EXAM



5. (7 points, suggested time 13 minutes)

The figure above on the left shows a uniformly thick rope hanging vertically from an oscillator that is turned off. When the oscillator is on and set at a certain frequency, the rope forms the standing wave shown above on the right. *P* and *Q* are two points on the rope.

(a) The tension at point *P* is greater than the tension at point *Q*. Briefly explain why.

(b) A student hypothesizes that increasing the tension in a rope increases the speed at which waves travel along the rope. In a clear, coherent paragraph-length response that may also contain figures and/or equations, explain why the standing wave shown above supports the student's hypothesis.

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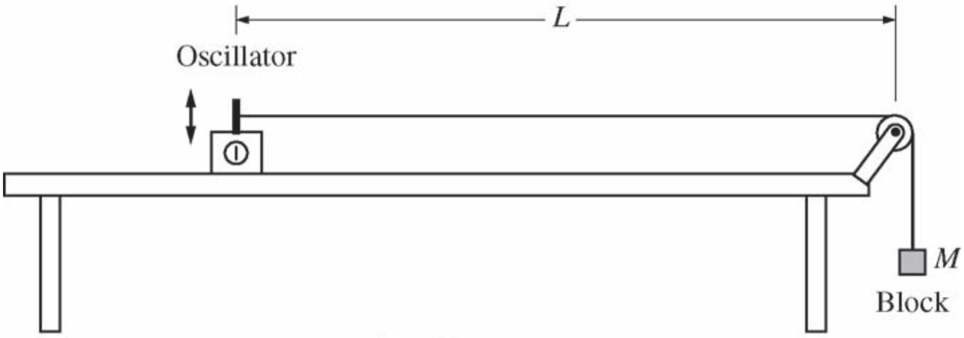
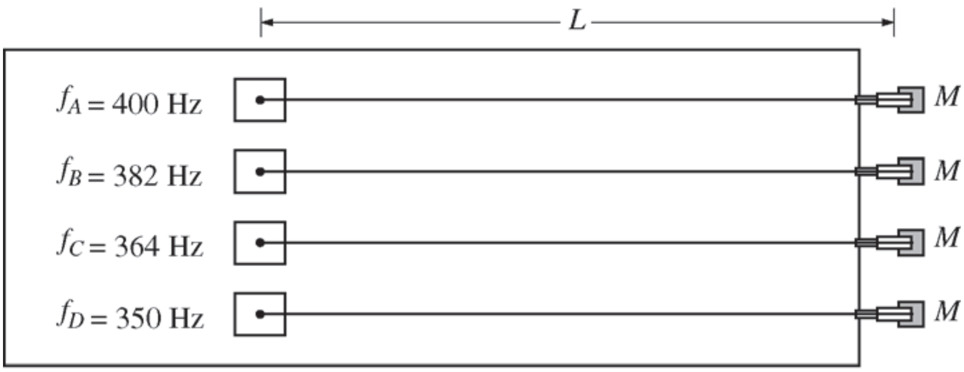


Side View

5. (7 points, suggested time 13 minutes)

The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency f. The distance between each oscillator and pulley L is the same, and the mass M of each block is the same. However, the fundamental frequency of each string is different.



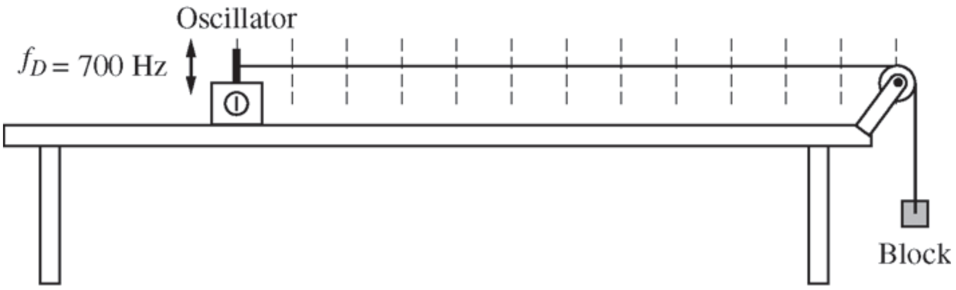
Top View

The equation for the velocity v of a wave on a string is $v = \sqrt{\dfrac{F_T}{m/L}}$, where F_T is the tension of the string and m/L is the mass per unit length (linear mass density) of the string.

(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

(c) The frequency of the oscillator connected to string D is changed so that the string vibrates in its second harmonic. On the side view of string D below, mark and label the points on the string that have the greatest average vertical speed.



Side View

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END OF EXAM

AP PHYSICS 1 • EXERCISE SHEET

7.4 Energy of Simple Harmonic Oscillators

Name: ______ Class: ______ Date: ______ **Total: 10 marks**

KEY WORDS total mechanical energy 总机械能 • equilibrium 平衡 • amplitude 振幅

Objective

Build the skills to answer exam questions on **the energy of simple harmonic oscillators**.

You must be able to:

- describe the continuous exchange between **kinetic energy** and **potential energy** during SHM
- state that the **total mechanical energy 总机械能** of an ideal oscillator is constant
- relate the total energy of a mass-spring oscillator to its amplitude, $E = \frac{1}{2}kA^2$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Energy is traded, not lost

During SHM, kinetic and potential energy convert back and forth while the **total stays constant** (no friction).

Total energy from the amplitude

$$E = \frac{1}{2}kA^2.$$

At the amplitude all the energy is potential; at equilibrium all of it is kinetic (so the speed is greatest there).

Example

A spring with $k = 100$ N m^{-1} and amplitude 0.20 m: $E = \frac{1}{2}(100)(0.20)^2 = 2.0$ J.

2 Practice

2.1 A spring of constant 100 N m^{-1} oscillates with amplitude 0.20 m. Find the total energy. [2]

2.2 State where in the motion the kinetic energy is greatest. [1]

2.3 State the potential energy at the amplitude, in terms of the total energy E. [1]

3 Exam-style questions

3.1 The total energy of a mass-spring oscillator is [1]

- **A** $\frac{1}{2}kA$
- **B** $\frac{1}{2}kA^2$
- **C** kA^2
- **D** $\frac{1}{2}mA^2$

3.2 At the amplitude of SHM, the energy is entirely [1]

- **A** kinetic
- **B** potential
- **C** thermal
- **D** zero

3.3 A 0.50 kg mass on a spring of constant 200 N m^{-1} oscillates with amplitude 0.10 m.

(a) Find the total energy. [2]

(b) Find the maximum speed. [2]

Solutions

2.1 $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.20)^2 = 2.0$ J.

2.2 at the equilibrium position.

2.3 all of it — E.

3.1 **B**.

3.2 **B**.

3.3 (a) $E = \frac{1}{2}(200)(0.10)^2 = 1.0$ J. (b) at equilibrium all energy is kinetic: $\frac{1}{2}mv^2 = 1.0 \Rightarrow v = \sqrt{\frac{2(1.0)}{0.50}} = 2.0$ m s^{-1}.

PHYSICS 1

SECTION II

Time—1 hour and 30 minutes

5 Questions

)irections: Questions 1, 4, and 5 are short free-response questions that require about 13 minutes each to answer and re worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to nswer and are worth 12 points each. Show your work for each part in the space provided after that part.

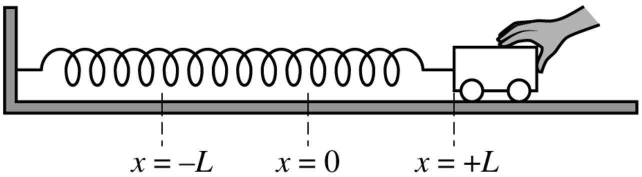

Figure 1

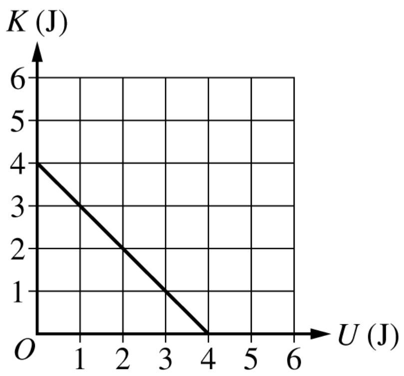

Figure 2

. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

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Continue your response to **QUESTION 1** on this page.

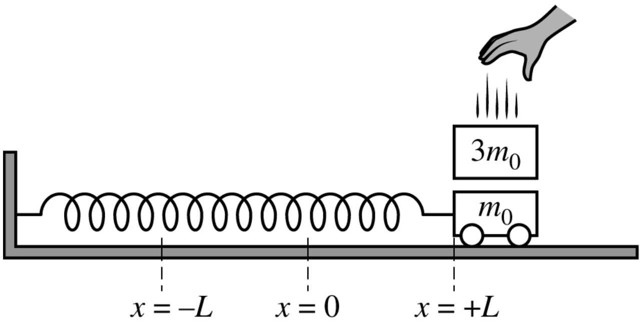

Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

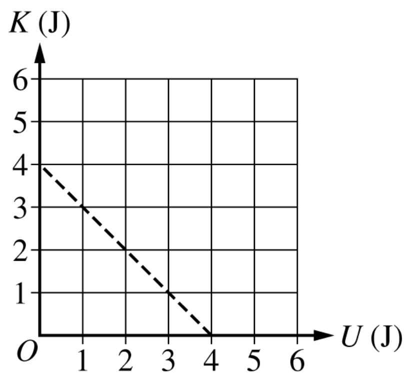

Figure 4

i. Briefly explain why the two graphs must be the same, using physics principles.

ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

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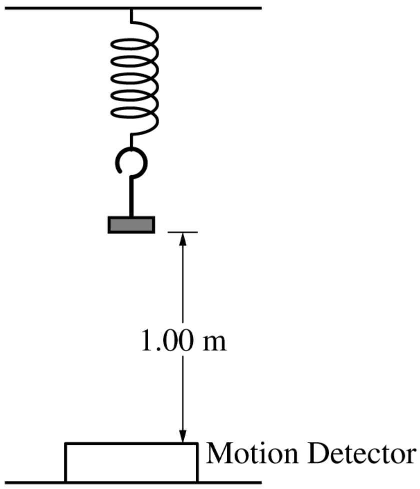

. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

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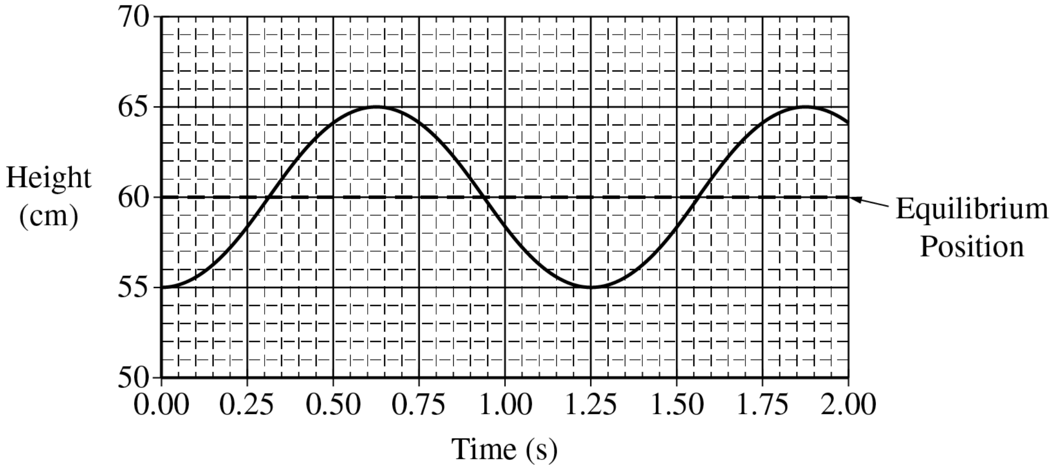
Continue your response to **QUESTION 5** on this page.



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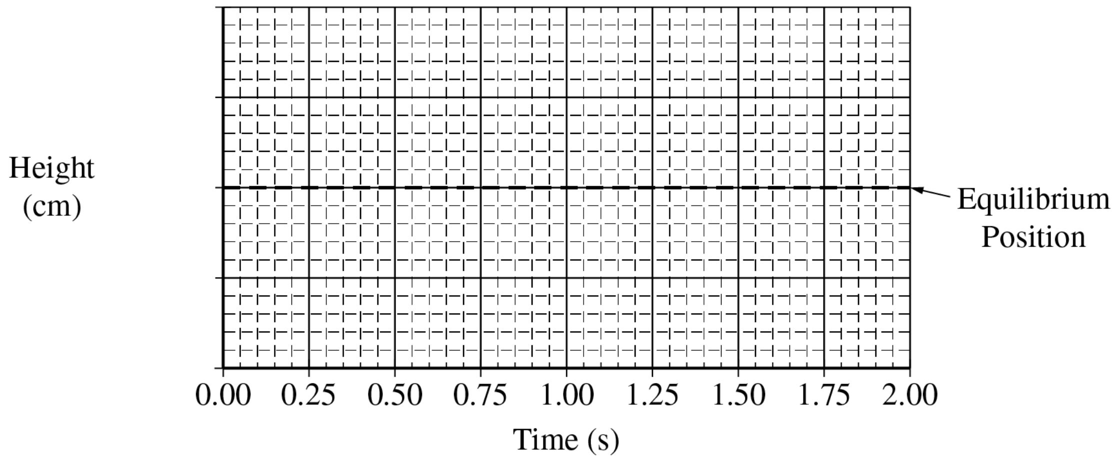

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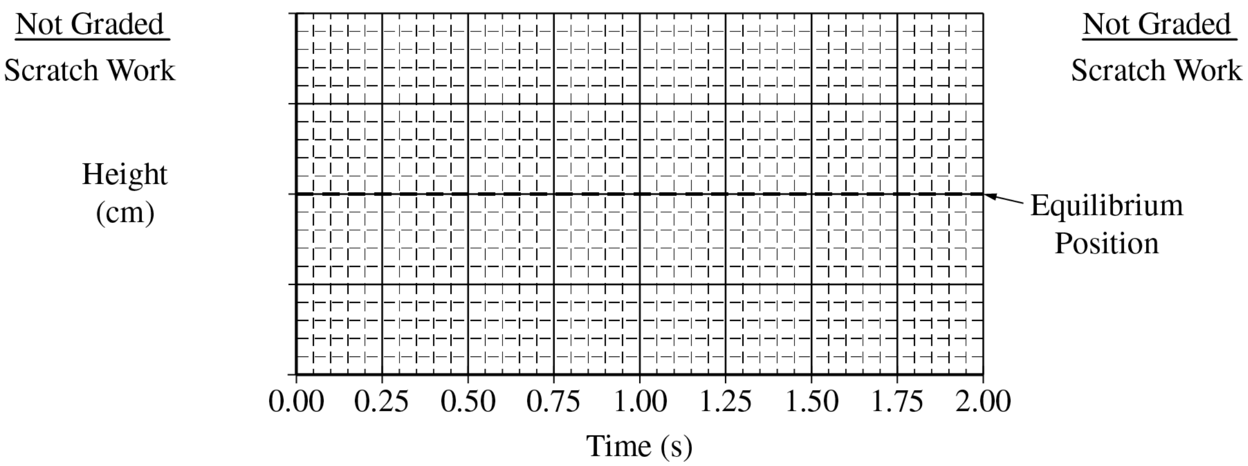

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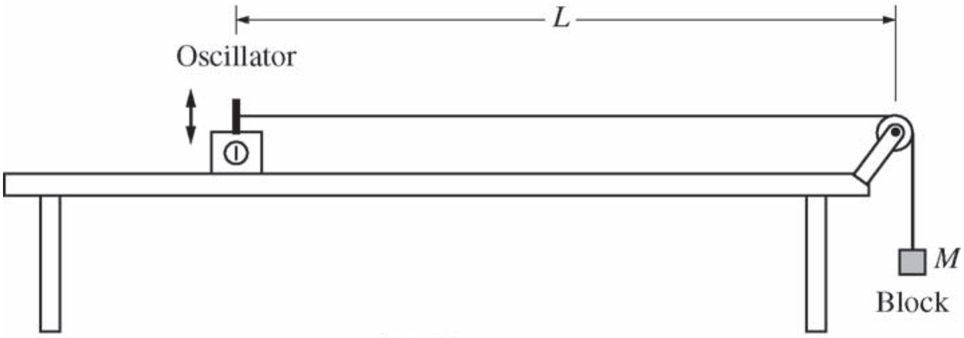

Side View

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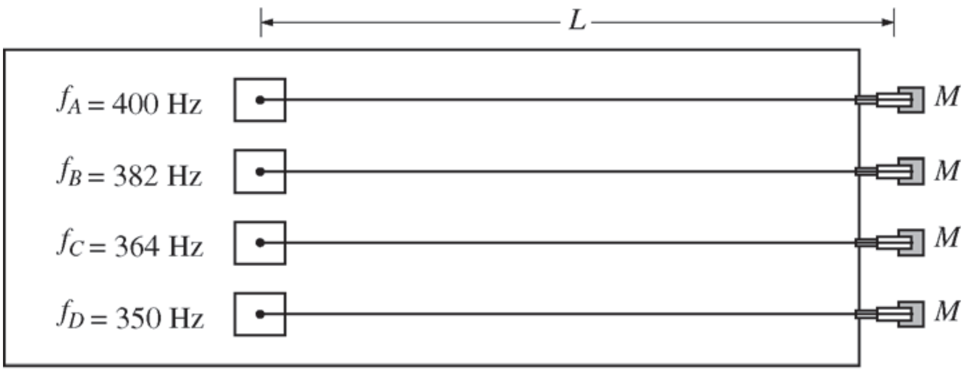

Top View

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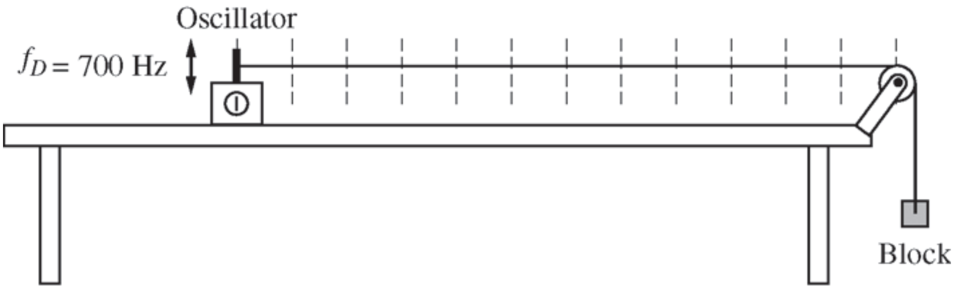

Side View

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END OF EXAM