Week 21

AP Physics 1

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics 1 Name: ____________ Score: ______

1. A wheel with $I = 4\ \mathrm{kg \cdot m^2}$ spins at $\omega = 3\ \frac{\mathrm{rad}}{\mathrm{s}}$. What is its rotational kinetic energy, in joules?

   ____________ J

2. A constant torque of 5 N · m turns a wheel through 4 rad. How much work is done, in joules?

   ____________ J

3. A disk with $I = 2\ \mathrm{kg \cdot m^2}$ spins at $\omega = 5\ \frac{\mathrm{rad}}{\mathrm{s}}$. What is its angular momentum, in $\mathrm{kg \cdot m^2/s}$?

   ____________ $\mathrm{kg \cdot m^2/s}$

4. A skater with $I_1 = 6\ \mathrm{kg \cdot m^2}$ spins at $2\ \frac{\mathrm{rad}}{\mathrm{s}}$, then pulls in to $I_2 = 2\ \mathrm{kg \cdot m^2}$. What is her new ω, in $\frac{\mathrm{rad}}{\mathrm{s}}$?

   ____________ rad/s

5. The total kinetic energy of a rolling object is:

   - ☐ $\frac{1}{2}mv^2$ only
   - ☐ $\frac{1}{2}I\omega^2$ only
   - ☐ $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
   - ☐ $mv^2 + I\omega^2$

6. What provides the centripetal force that keeps a satellite in a circular orbit?

   - ☐ the satellite's engines
   - ☐ gravity
   - ☐ air resistance
   - ☐ an outward centrifugal force

7. Which is the formula for rotational kinetic energy?

   - ☐ $\frac{1}{2}mv^2$
   - ☐ $\frac{1}{2}I\omega^2$
   - ☐ $I\omega$
   - ☐ mgh

8. A torque of 5 N · m turns a shaft at $\omega = 2\ \frac{\mathrm{rad}}{\mathrm{s}}$. What is the power, in watts?

   ____________ W

9. Angular momentum is the rotational twin of linear momentum $p = mv$. Which is its formula?

- ☐ $\frac{1}{2}I\omega^2$
- ☐ $I\omega$
- ☐ $\tau\theta$
- ☐ $I\alpha$

10. Angular momentum is conserved when there is no external:

- ☐ force
- ☐ torque
- ☐ friction of any kind
- ☐ motion

AP Physics 1

1. **18 J**

   $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}(4)(9) = 18$ J.

2. **20 J**

   $W = \tau\theta = 5 \times 4 = 20$ J.

3. **10 $\mathbf{kg \cdot m^2/s}$**

   $L = I\omega = 2 \times 5 = 10\ \mathrm{kg \cdot m^2/s}$.

4. **6 rad/s**

   $I_1\omega_1 = I_2\omega_2 \Rightarrow 12 = 2\omega_2 \Rightarrow \omega_2 = 6\ \frac{\mathrm{rad}}{\mathrm{s}}$.

5. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

   A rolling body moves and spins, so its KE is the sum $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

6. **gravity**

   Gravity is the centripetal force: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$.

7. $\frac{1}{2}I\omega^2$

   It is $\frac{1}{2}I\omega^2$ — the rotational twin of $\frac{1}{2}mv^2$.

8. **10 W**

   $P = \tau\omega = 5 \times 2 = 10$ W.

9. $I\omega$

   $L = I\omega$ — inertia times angular velocity, the twin of mv.

10. **torque**

    With no external **torque**, L cannot change: $I_1\omega_1 = I_2\omega_2$.

AP PHYSICS 1 • EXERCISE SHEET

6.1 Rotational Kinetic Energy

Name: ______________ Class: _________ Date: ____________ **Total: 11 marks**

KEY WORDS rotational kinetic energy 转动动能

Objective

Build the skills to answer exam questions on **rotational kinetic energy**.

You must be able to:

- define **rotational kinetic energy 转动动能** as $K_{rot} = \frac{1}{2}I\omega^2$
- explain that it depends on both the **rotational inertia** and the **angular velocity**
- find the **total kinetic energy** of an object that both translates and rotates
- apply **conservation of energy** to systems that include rotation

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Rotational kinetic energy**

$K_{rot} = \frac{1}{2}I\omega^2$ — the rotational partner of $\frac{1}{2}mv^2$.

■ **Total kinetic energy of a rolling body**

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

■ **A rolling race**

Rolling down a ramp, a shape with smaller $I/(mR^2)$ ties up less energy in rotation, so it reaches the bottom **faster**. A solid sphere beats a hoop.

2 Practice

2.1 A flywheel of $I = 0.50$ kg m^2 spins at $\omega = 4.0$ rad s^{-1}. Find its rotational kinetic energy. [2]

2.2 A rolling ball has 6.0 J of translational and 2.4 J of rotational kinetic energy. Find its total kinetic energy. [1]

2.3 State why a solid sphere rolls down a ramp faster than a hoop of the same mass and radius. [2]

3 Exam-style questions

3.1 Rotational kinetic energy is [1]

- **A** $\frac{1}{2}I\omega$
- **B** $\frac{1}{2}I\omega^2$
- **C** $I\omega$
- **D** $\frac{1}{2}mv^2$

3.2 A hoop and a disc of equal mass and radius roll from rest down the same ramp. The one reaching the bottom first is the [1]

- **A** hoop
- **B** disc
- **C** they arrive together
- **D** it depends on the mass

3.3 A disc of $I = 0.10$ kg m^2 spins at $\omega = 12$ rad s^{-1}.

(a) Find its rotational kinetic energy. [2]

(b) If this energy came from a 0.20 kg mass falling, find the height it fell (take g =

10 m s^{-2}). [2]

Solutions

2.1 $K_{rot} = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J.

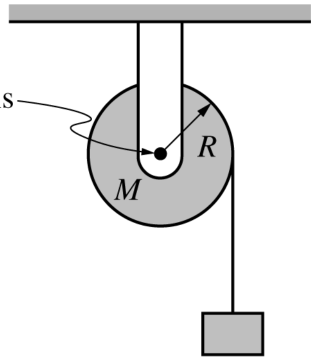
2.2 $K = 6.0 + 2.4 = 8.4$ J.

2.3 The sphere has a smaller rotational inertia for its mass, so less energy goes into rotation and more into translation, giving a larger speed.

3.1 **B**.

3.2 **B** — the disc has the smaller rotational inertia.

3.3 (a) $K_{rot} = \frac{1}{2}(0.10)(12)^2 = 7.2$ J. (b) $mgh = 7.2 \Rightarrow h = \dfrac{7.2}{0.20 \times 10} = 3.6$ m.



. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

(a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M, R, F_T, and physical constants as appropriate.

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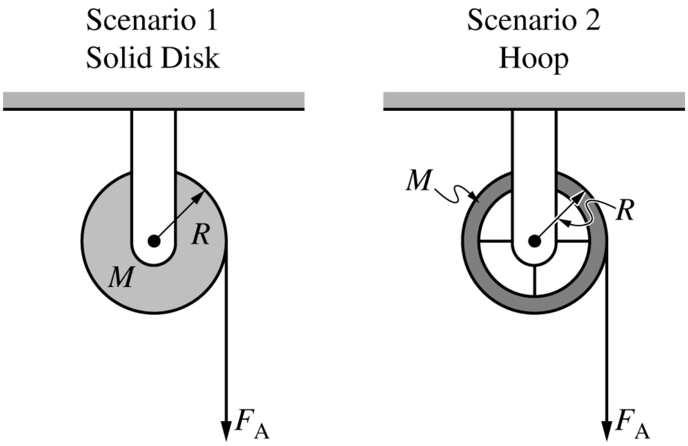
Continue your response to **QUESTION 4** on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

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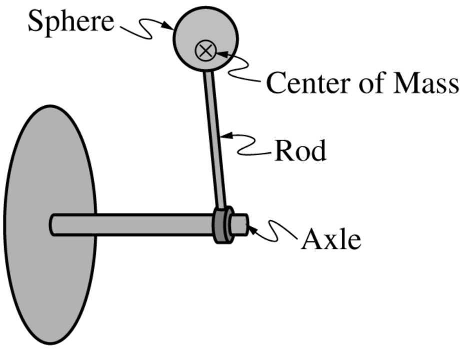

Figure 1

. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

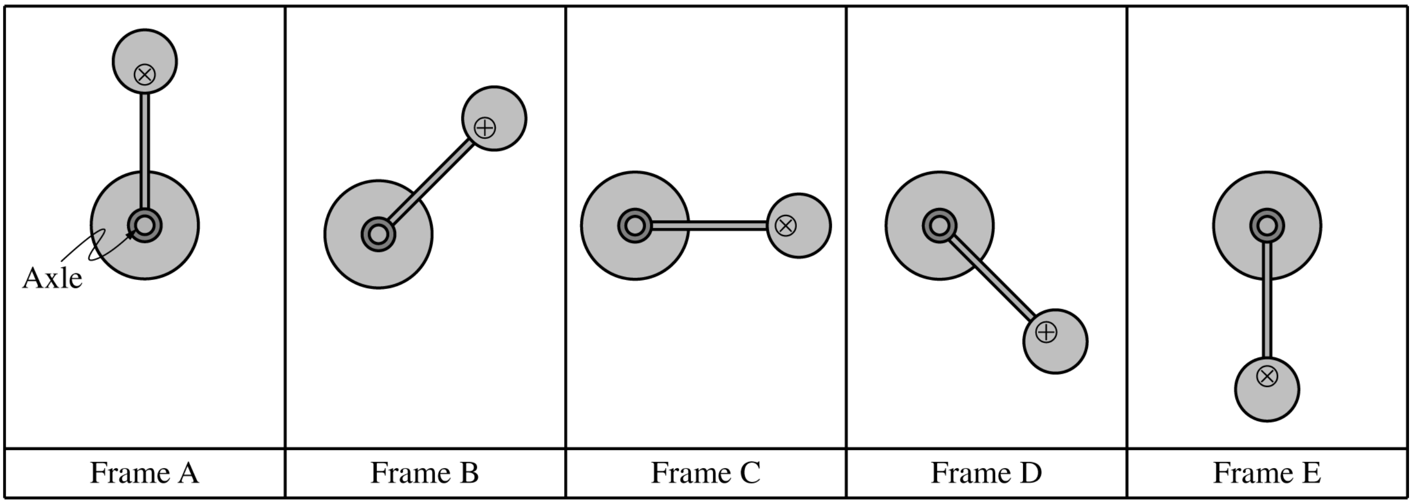

Figure 2

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Continue your response to **QUESTION 5** on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

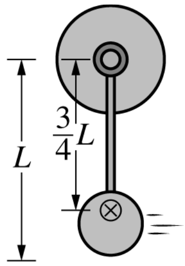

Figure 3

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the <u>rod-sphere-Earth</u> system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

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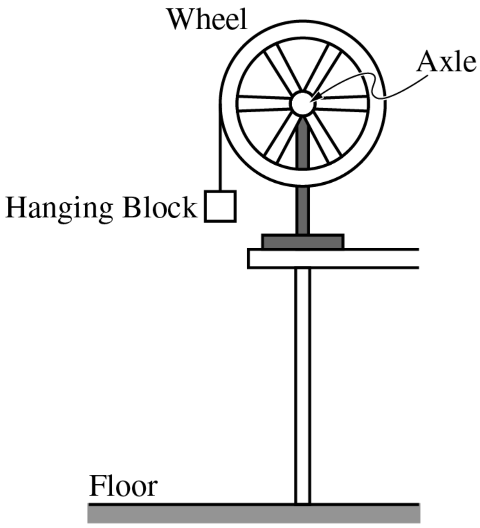
END OF EXAM

Begin your response to **QUESTION 3** on this page.



. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

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Continue your response to **QUESTION 3** on this page.

(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement	Procedure (and diagram, if needed)

(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

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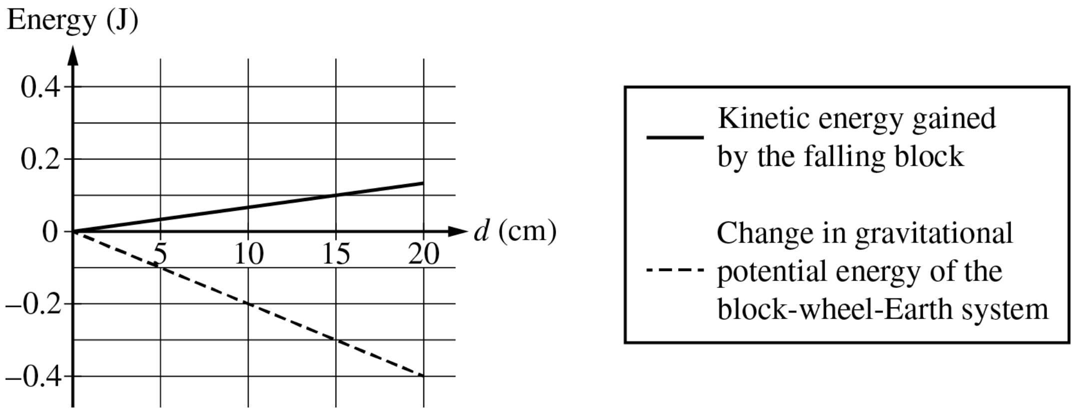
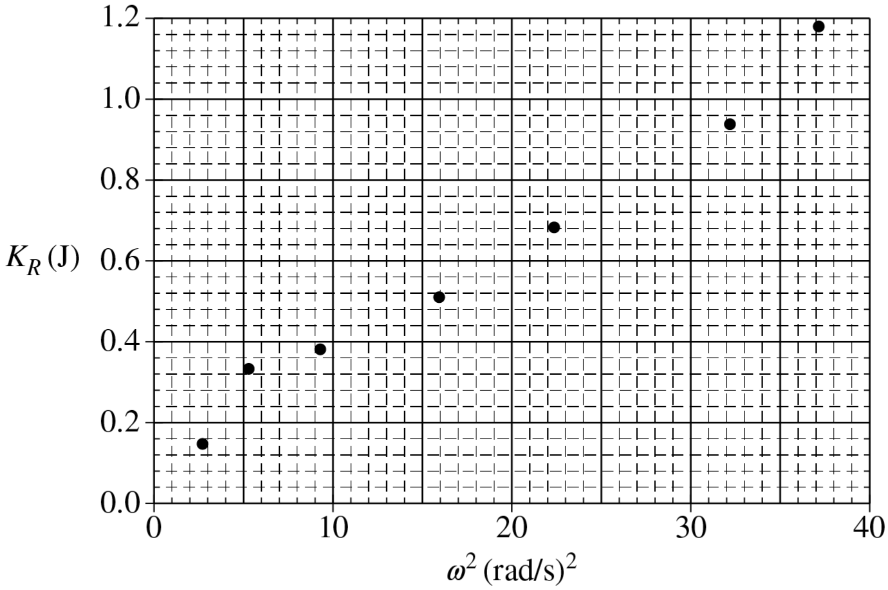


(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d.

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.



i. On the above graph, draw a straight line that best represents the data.

ii. Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

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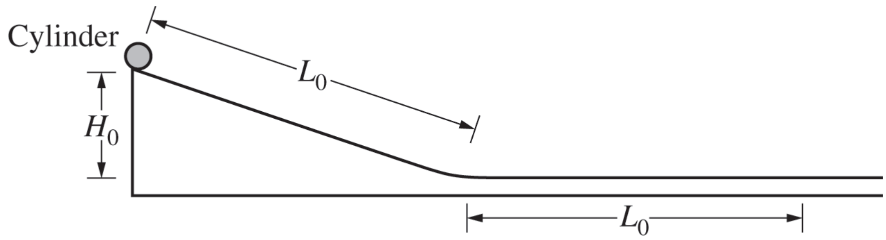
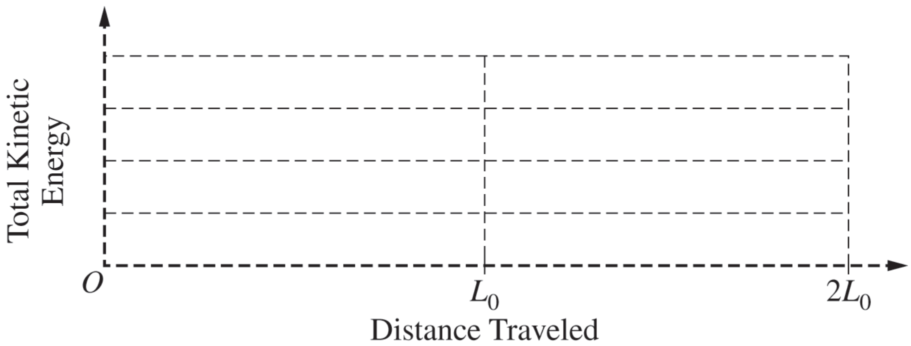


(7 points, suggested time 13 minutes)

A cylinder of mass m_0 is placed at the top of an incline of length L_0 and height H_0, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

(a) On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.



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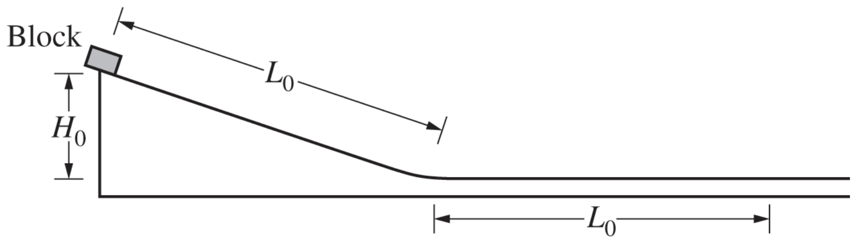

The cylinder is again placed at the top of the incline. A block, also of mass m_0, is placed at the top of a separate rough incline of length L_0 and height H_0, as shown above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

(b) In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

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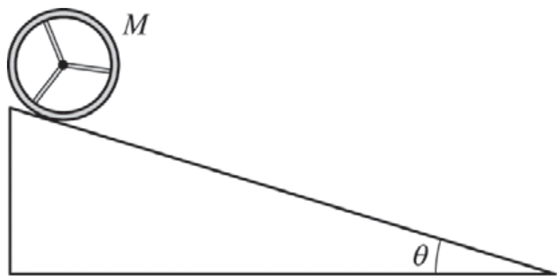
PHYSICS 1

Section II

5 Questions

Time—90 minutes

Directions: Questions 1, 4 and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

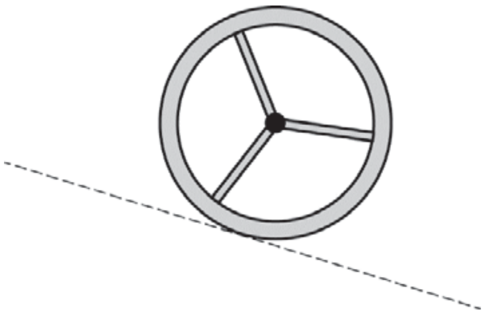


1. (7 points, suggested time 13 minutes)

A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

(a)

i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

(b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

(c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.

6.2 Torque and Work

Name: ____________ Class: ________ Date: __________ **Total: 10 marks**

KEY WORDS rotational kinetic energy 转动动能 • rotational power 转动功率

Objective

Build the skills to answer exam questions on **torque and work**.

You must be able to:

- calculate the **work done by a torque** 力矩做功, $W = \tau\,\Delta\theta$
- relate the work done by a net torque to the change in **rotational kinetic energy** 转动动能
- define **rotational power** 转动功率 as $P = \tau\omega$
- find the work from the **area** under a torque-versus-angle graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Work by a torque

$W = \tau\,\Delta\theta$, with $\Delta\theta$ in radians. This is the rotational partner of $W = Fd$.

The rotational work-energy theorem

$W_{net} = \Delta K_{rot} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$.

Rotational power

$P = \tau\omega$. A torque of 8.0 N m through $\Delta\theta = 2.0$ rad does $W = 16$ J.

2 Practice

2.1 A torque of 5.0 N m turns a wheel through 4.0 rad. Find the work done. [1]

2.2 A motor applies 20 N m at $\omega = 15$ rad s^{-1}. Find its power output. [2]

2.3 A net work of 30 J is done on a flywheel starting from rest. State its final rotational kinetic energy. [1]

3 Exam-style questions

3.1 The work done by a torque is [1]

- **A** $\tau\omega$
- **B** $\tau\,\Delta\theta$
- **C** $I\alpha$
- **D** $\frac{1}{2}I\omega^2$

3.2 Rotational power is equal to [1]

- **A** τ/ω
- **B** $\tau\omega$
- **C** $\tau+\omega$
- **D** $I\alpha$

3.3 A wheel of $I = 0.40$ kg m^2, initially at rest, has a 2.0 N m torque applied through 10 rad.

(a) Find the work done. [2]

(b) Find the final angular velocity. [2]

Solutions

2.1 $W = \tau\,\Delta\theta = 5.0 \times 4.0 = 20$ J.

2.2 $P = \tau\omega = 20 \times 15 = 300$ W.

2.3 30 J (all the net work becomes rotational kinetic energy).

3.1 **B**.

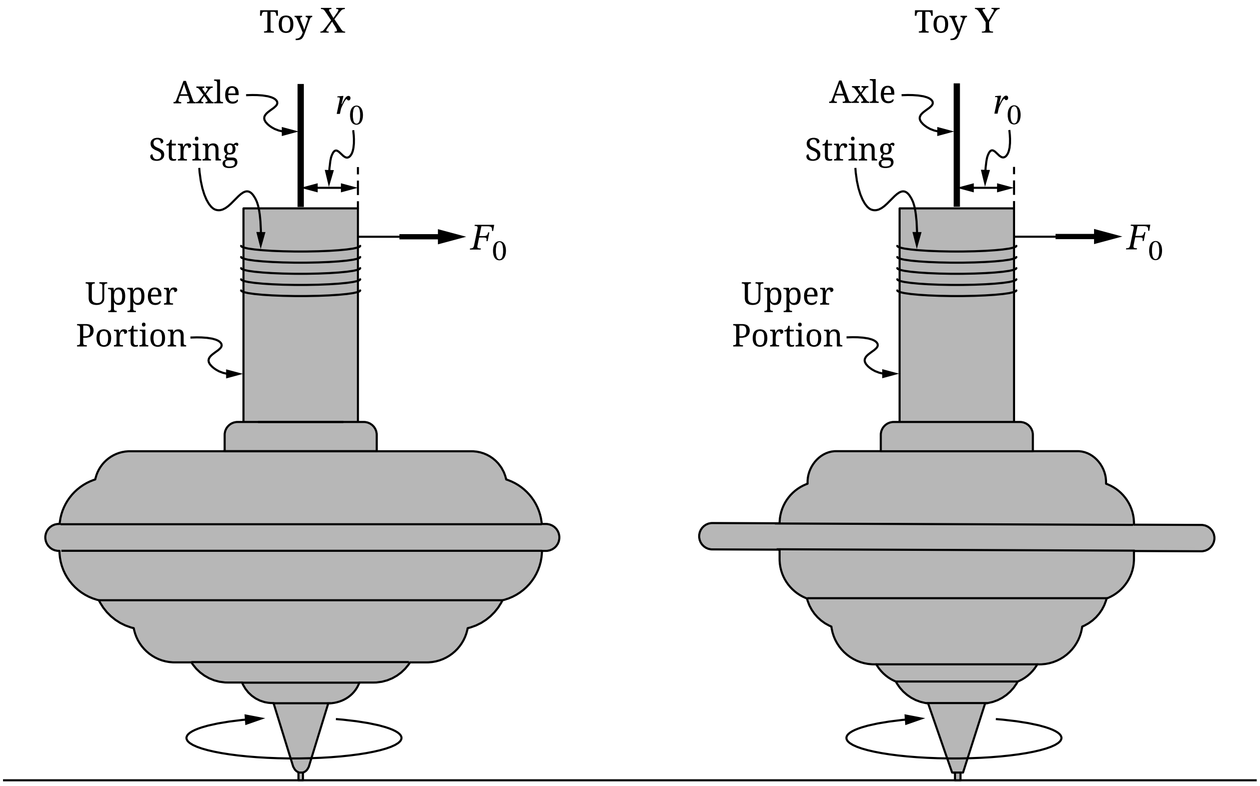
3.2 **B**.

3.3 (a) $W = \tau\,\Delta\theta = 2.0 \times 10 = 20$ J. (b) $\frac{1}{2}I\omega^2 = 20 \Rightarrow \omega = \sqrt{\frac{2(20)}{0.40}} = 10$ rad s^{-1}.

Question 4

4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

Figure 1



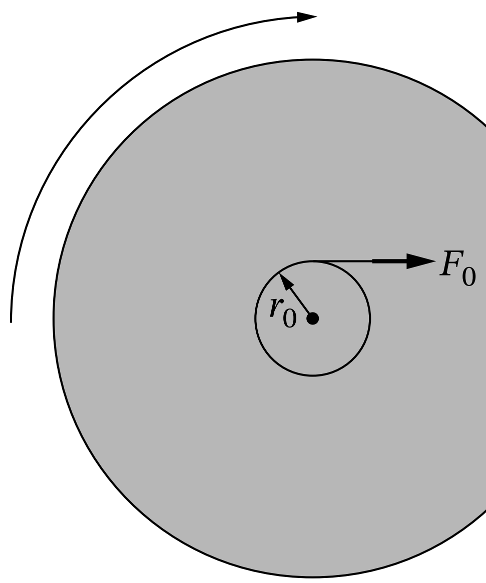
Side View

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

Figure 2



Overhead View

Part A

Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0, I_X, F_0, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

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END OF EXAM

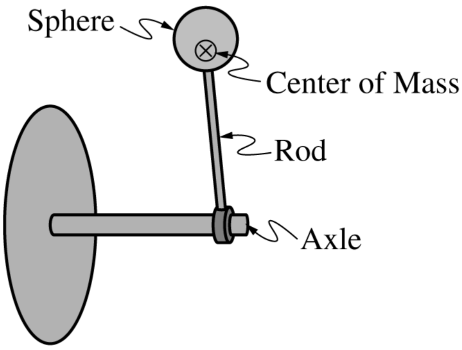

Figure 1

. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

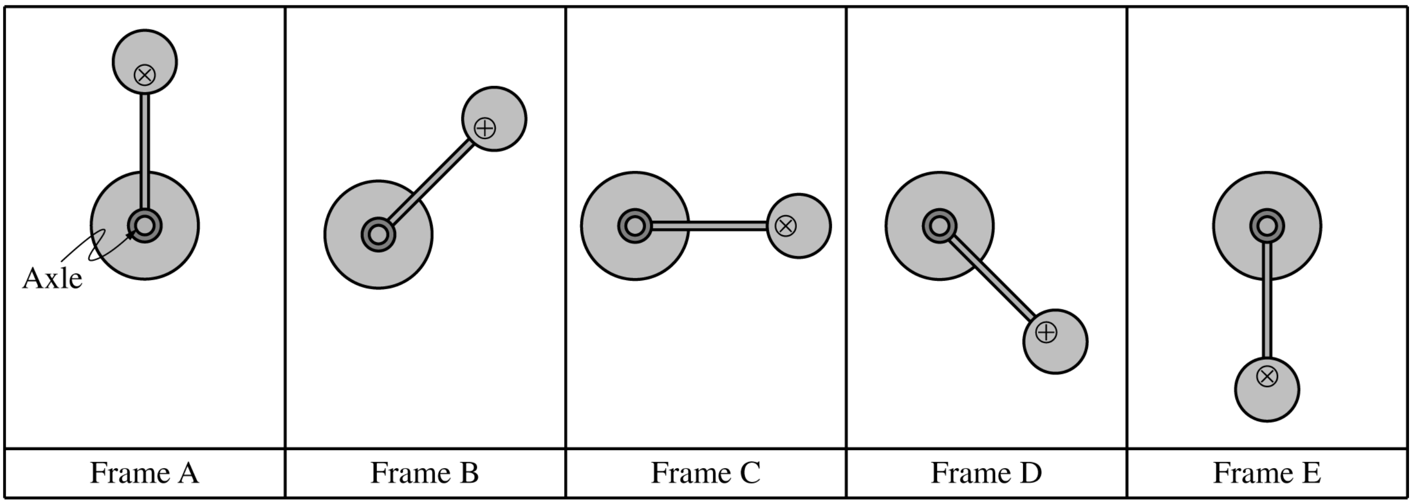

Figure 2

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(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

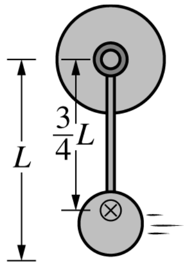

Figure 3

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the <u>rod-sphere-Earth</u> system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

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STOP

END OF EXAM

6.3 Angular Momentum and Angular Impulse

Name: ______ Class: ______ Date: ______ **Total: 9 marks**

KEY WORDS angular momentum 角动量 • angular impulse 角冲量

Objective

Build the skills to answer exam questions on **angular momentum and angular impulse**.

You must be able to:

- define **angular momentum 角动量** of a rigid body as $L = I\omega$
- define **angular impulse 角冲量** as $\Delta L = \tau\,\Delta t$
- relate a net angular impulse to the change in a system's angular momentum

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Angular momentum

$L = I\omega$, measured in kg m^2 s^{-1} — the rotational partner of $p = mv$.

■ Angular impulse

$\Delta L = \tau\,\Delta t$: a net torque acting for a time changes the angular momentum, just as $\vec{J} = \vec{F}\,\Delta t$ changes linear momentum.

■ Example

A disc with $I = 2.0$ kg m^2 at $\omega = 3.0$ rad s^{-1} has $L = 6.0$ kg m^2 s^{-1}.

2 Practice

2.1 A disc of $I = 0.50$ kg m^2 spins at $\omega = 8.0$ rad s^{-1}. Find its angular momentum. [1]

2.2 A torque of 4.0 N m acts for 3.0 s. Find the angular impulse. [1]

2.3 A wheel's angular momentum changes by 12 kg $m^2 s^{-1}$ in 2.0 s. Find the net torque. [2]

3 Exam-style questions

3.1 The angular momentum of a rotating rigid body is [1]

- **A** $I\alpha$
- **B** $I\omega$
- **C** $\frac{1}{2}I\omega^2$
- **D** $\tau\omega$

3.2 Angular impulse is equal to [1]

- **A** $\tau\,\Delta t$
- **B** $I\alpha$
- **C** Fd
- **D** $\frac{1}{2}I\omega^2$

3.3 A flywheel of $I = 0.20$ kg m^2 spins at 30 rad s^{-1}. A constant friction torque of 0.60 N m acts on it.

(a) Find its initial angular momentum. [1]

(b) Find the time for friction to bring it to rest. [2]

Solutions

2.1 $L = I\omega = 0.50 \times 8.0 = 4.0\ \text{kg m}^2\,\text{s}^{-1}$.

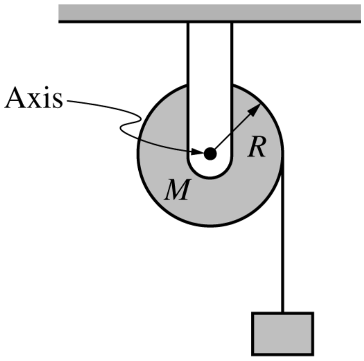
2.2 $\Delta L = \tau\,\Delta t = 4.0 \times 3.0 = 12\ \text{kg m}^2\,\text{s}^{-1}$.

2.3 $\tau = \dfrac{\Delta L}{\Delta t} = \dfrac{12}{2.0} = 6.0\ \text{N m}$.

3.1 **B**.

3.2 **A**.

3.3 (a) $L = I\omega = 0.20 \times 30 = 6.0\ \text{kg m}^2\,\text{s}^{-1}$. (b) $\Delta t = \dfrac{L}{\tau} = \dfrac{6.0}{0.60} = 10\ \text{s}$.



. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

(a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M, R, F_T, and physical constants as appropriate.

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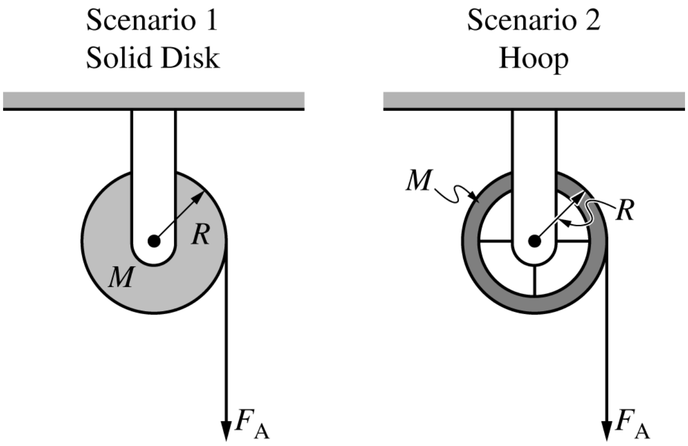
Continue your response to **QUESTION 4** on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

GO ON TO THE NEXT PAGE.

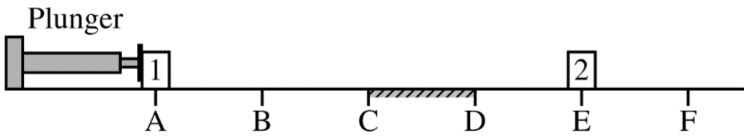
PHYSICS 1

Section II

Time—1 hour and 30 minutes

5 Questions

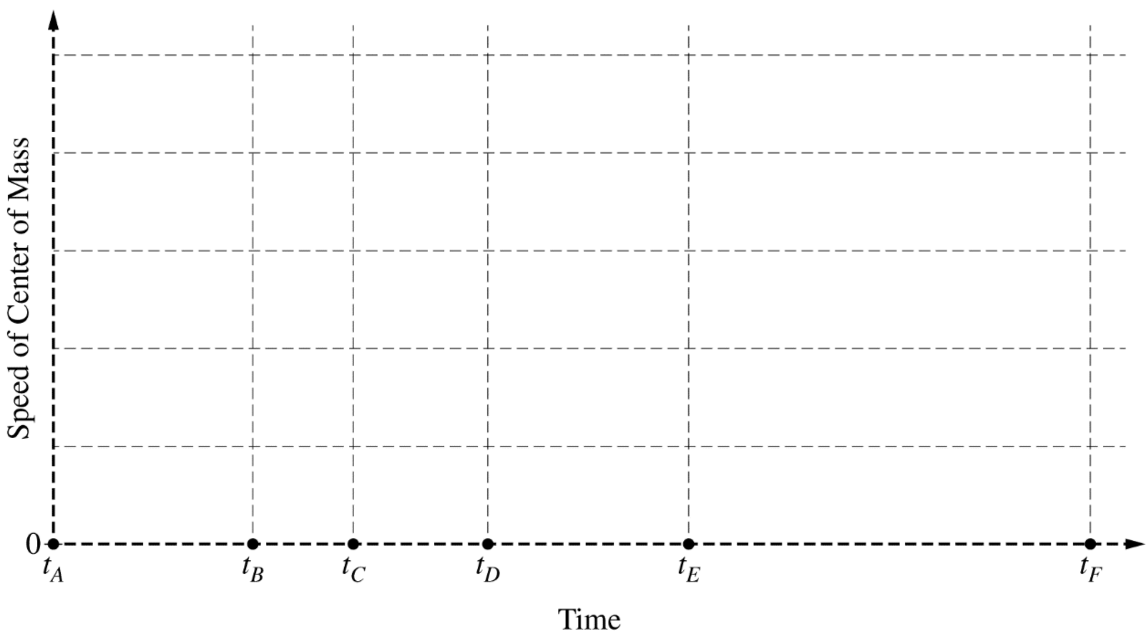
Directions: Questions 1, 4, and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

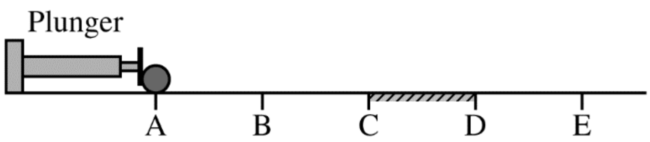


1. (7 points, suggested time 13 minutes)

Identical blocks 1 and 2 are placed on a horizontal surface at points A and E, respectively, as shown. The surface is frictionless except for the region between points C and D, where the surface is rough. Beginning at time t_A, block 1 is pushed with a constant horizontal force from point A to point B by a mechanical plunger. Upon reaching point B, block 1 loses contact with the plunger and continues moving to the right along the horizontal surface toward block 2. Block 1 collides with and sticks to block 2 at point E, after which the two-block system continues moving across the surface, eventually passing point F.

(a) On the axes below, sketch the speed of the center of mass of the two-block system as a function of time, from time t_A until the blocks pass point F at time t_F. The times at which block 1 reaches points A through F are indicated on the time axis.

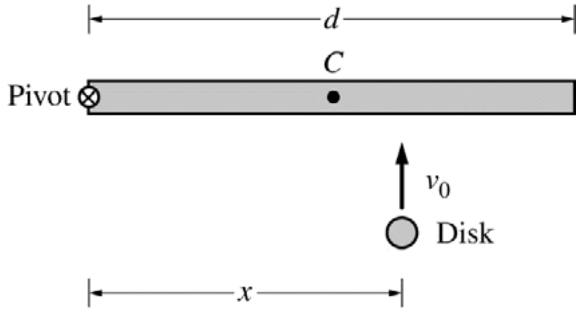




(b) The plunger is returned to its original position, and both blocks are removed. A uniform solid sphere is placed at point A, as shown. The sphere is pushed by the plunger from point A to point B with a constant horizontal force that is directed toward the sphere's center of mass. The sphere loses contact with the plunger at point B and continues moving across the horizontal surface toward point E. In which interval(s), if any, does the sphere's angular momentum about its center of mass change? Check all that apply.

____ A to B ____ B to C ____ C to D ____ D to E _____ None

Briefly explain your reasoning.



Top View

3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

____ To the left of C ____ At C ____ To the right of C

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \dfrac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for ω have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than ____ Less than ____ Equal to

Briefly explain your reasoning.

6.4 Conservation of Angular Momentum

Name: ______ Class: ______ Date: ______ **Total: 11 marks**

KEY WORDS angular momentum 角动量 • conserved 守恒

Objective

Build the skills to answer exam questions on **conservation of angular momentum**.

You must be able to:

- state that **angular momentum 角动量** is conserved when the net external torque is zero
- explain the spinning-skater example (I down, ω up)
- apply conservation of angular momentum to collisions and to a changing rotational inertia

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Conservation of angular momentum

When the net external torque is zero, $I_i\omega_i = I_f\omega_f$.

■ The spinning skater

Pulling the arms in **reduces** I; since $L = I\omega$ stays constant, ω **increases**. Halving I doubles ω.

■ Adding mass to a turntable

A lump landing on a spinning disc raises I (by mr^2), so ω drops — but L is unchanged.

2 Practice

2.1 State the condition for the angular momentum of a system to be conserved. [1]

2.2 A skater with $I = 4.0$ kg m^2 spins at 2.0 rad s^{-1}, then pulls in to $I = 1.0$ kg m^2. Find the new angular velocity. [2]

2.3 Using $L = I\omega$, explain why the skater speeds up. [2]

3 Exam-style questions

3.1 Angular momentum is conserved when [1]

- **A** kinetic energy is constant
- **B** the net external torque is zero
- **C** the angular velocity is constant
- **D** there is no friction anywhere

3.2 A spinning skater pulls in their arms. Their angular speed [1]

- **A** increases
- **B** decreases
- **C** stays the same
- **D** reverses

3.3 A turntable of $I = 0.10$ kg m^2 spins freely at 6.0 rad s^{-1}. A 0.20 kg lump of clay drops onto it 0.30 m from the axis.

(a) Find the rotational inertia added by the clay ($I = mr^2$). [1]

(b) Find the new angular speed. [3]

Solutions

2.1 the net external torque on the system must be zero.

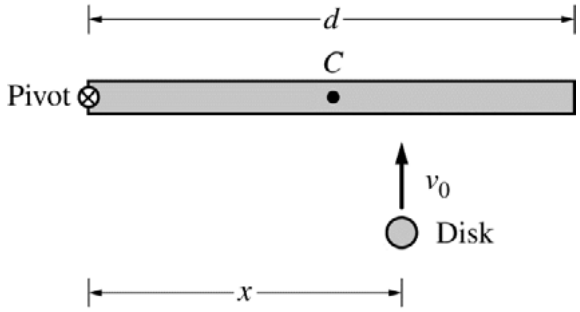
2.2 $I_i\omega_i = I_f\omega_f \Rightarrow 4.0(2.0) = 1.0\,\omega_f$, so $\omega_f = 8.0$ rad s^{-1}.

2.3 $L = I\omega$ is conserved; reducing I must raise ω to keep the product constant.

3.1 **B**.

3.2 **A**.

3.3 (a) $I_{clay} = mr^2 = 0.20(0.30)^2 = 0.018$ kg m^2. (b) $I_i\omega_i = I_f\omega_f \Rightarrow 0.10(6.0) = (0.10 + 0.018)\,\omega_f$, so $\omega_f = \dfrac{0.60}{0.118} = 5.1$ rad s^{-1}.



Top View

3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

____ To the left of C ____ At C ____ To the right of C

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \dfrac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for ω have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than ____ Less than ____ Equal to

Briefly explain your reasoning.

AP PHYSICS 1 • EXERCISE SHEET

6.5 Rolling

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS rolling without slipping 无滑滚动

Objective

Build the skills to answer exam questions on **rolling**.

You must be able to:

- describe **rolling without slipping 无滑滚动** using $v = \omega r$
- split the total kinetic energy of a rolling body into **translational** and **rotational** parts
- compare how objects of different rotational inertia accelerate down an incline

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Rolling without slipping

The contact point is momentarily at rest, so $v = \omega r$ links the linear speed of the centre and the angular speed.

■ Total kinetic energy

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2.$$

■ Down an incline

A smaller $I/(mR^2)$ ties up less energy in spin, so the object accelerates faster: **sphere > disc > hoop**.

2 Practice

2.1 A wheel of radius 0.25 m rolls without slipping at $v = 5.0\ \mathrm{m\ s^{-1}}$. Find its angular speed. [1]

2.2 State the rolling-without-slipping condition. [1]

2.3 A hoop and a solid cylinder roll from rest down the same ramp. State which reaches the bottom first. [1]

3 Exam-style questions

3.1 For an object rolling without slipping, [1]

- **A** $v = \omega r$
- **B** $v = \omega / r$
- **C** $\omega = vr$
- **D** $v = \frac{1}{2}\omega r$

3.2 The total kinetic energy of a rolling object is [1]

- **A** translational only
- **B** rotational only
- **C** the sum of translational and rotational
- **D** neither

3.3 A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0$ m s^{-1}.

(a) Find its angular speed. [1]

(b) Find its total kinetic energy. [3]

Solutions

2.1 $\omega = \frac{v}{r} = \frac{5.0}{0.25} = 20 \text{ rad s}^{-1}$.

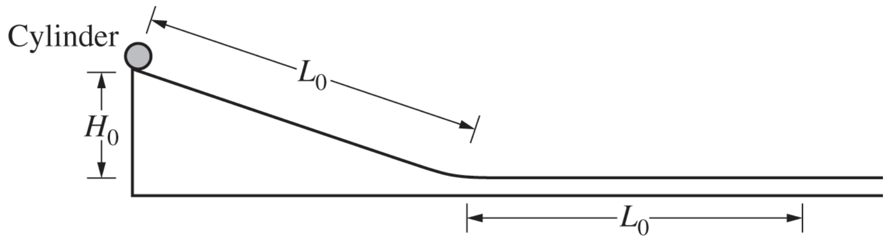
2.2 $v = \omega r$ (the centre's speed equals angular speed times radius).

2.3 the solid cylinder (smaller rotational inertia).

3.1 **A**.

3.2 **C**.

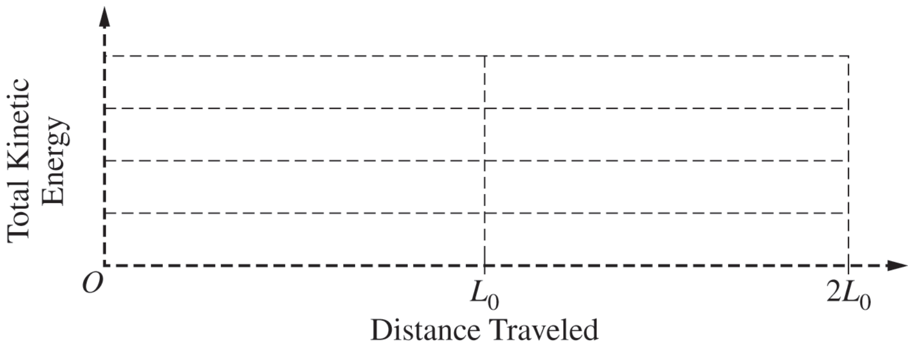
3.3 (a) $\omega = \frac{3.0}{0.10} = 30 \text{ rad s}^{-1}$. (b) $I = \frac{1}{2}(2.0)(0.10)^2 = 0.010 \text{ kg m}^2$; $K = \frac{1}{2}(2.0)(3.0)^2 + \frac{1}{2}(0.010)(30)^2 = 9.0 + 4.5 = 13.5 \text{ J}$.



(7 points, suggested time 13 minutes)

A cylinder of mass m_0 is placed at the top of an incline of length L_0 and height H_0, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

(a) On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.



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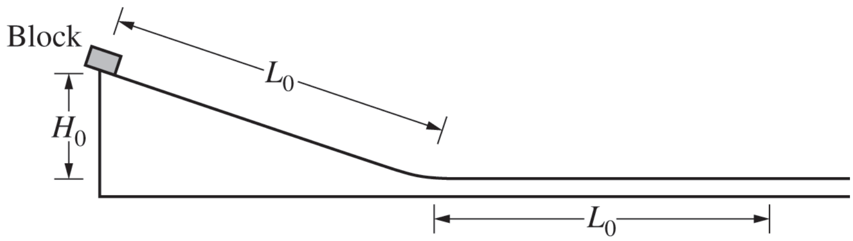


The cylinder is again placed at the top of the incline. A block, also of mass m_0, is placed at the top of a separate rough incline of length L_0 and height H_0, as shown above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

(b) In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

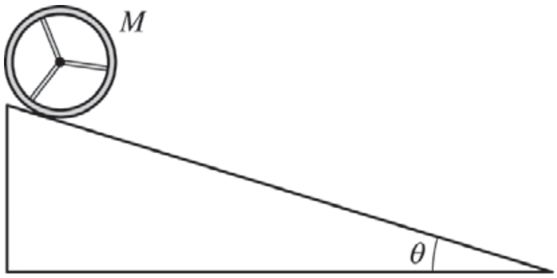
PHYSICS 1

Section II

5 Questions

Time—90 minutes

Directions: Questions 1, 4 and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

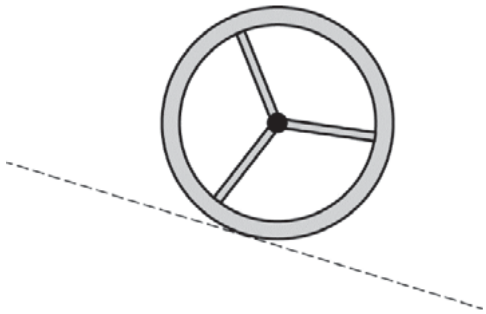


1. (7 points, suggested time 13 minutes)

   A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

   (a)

   i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



   ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

   Briefly explain your reasoning.

   (b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

(c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.

6.6 Motion of Orbiting Satellites

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS satellite 卫星 • orbital speed 轨道速度

Objective

Build the skills to answer exam questions on **the motion of orbiting satellites**.

You must be able to:

- explain how **gravity** provides the centripetal force that keeps a satellite in orbit
- relate the **orbital speed** 轨道速度 to the radius with $\frac{GMm}{r^2} = \frac{mv^2}{r}$
- describe the energy of a satellite as kinetic plus gravitational potential energy

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Gravity is the centripetal force

Setting the gravitational force equal to the required centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}.$$

Closer orbits are faster

Because $v \propto 1/\sqrt{r}$, a satellite in a **lower** orbit moves **faster**.

Energy

A satellite has kinetic energy $\frac{1}{2}mv^2$ and (negative) gravitational potential energy; the two together give its total orbital energy.

2 Practice

2.1 State what provides the centripetal force on an orbiting satellite. [1]

2.2 State how the orbital speed changes if the orbital radius increases. [1]

2.3 Write the equation obtained by setting the gravitational force equal to the centripetal force. [1]

3 Exam-style questions

3.1 The centripetal force on a satellite in orbit is provided by [1]

- **A** its engine
- **B** gravity
- **C** air resistance
- **D** the normal force

3.2 As the orbital radius increases, the orbital speed [1]

- **A** increases
- **B** decreases
- **C** stays constant
- **D** becomes zero

3.3 A satellite orbits Earth ($M = 6.0 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}) at radius $r = 7.0 \times 10^6$ m.

(a) Write the expression $v = \sqrt{GM/r}$. [1]

(b) Find the orbital speed. [3]

Solutions

2.1 the gravitational pull of the planet on the satellite.

2.2 it decreases ($v \propto 1/\sqrt{r}$).

2.3 $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$.

3.1 B.

3.2 B.

3.3 (a) $v = \sqrt{\dfrac{GM}{r}}$. (b) $v = \sqrt{\dfrac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{7.0 \times 10^{6}}} = \sqrt{5.7 \times 10^{7}} = 7.6 \times 10^{3}$ m s^{-1}.