**Week 17**

# AP Physics 1

Homework bundle for this week

本周作业合集

exercises.lol

## Contents 目录

AP Physics 1 Name: ____________________ Score: ________

1. A wheel goes from rest to $\omega = 12\ \frac{\text{rad}}{\text{s}}$ in 4 s. What is its angular acceleration, in $\frac{\text{rad}}{\text{s}^2}$?

   ______________ rad/s$^2$

2. A point is 0.3 m from a wheel's axis, spinning at $\omega = 10\ \frac{\text{rad}}{\text{s}}$. What is its linear speed, in $\frac{\text{m}}{\text{s}}$?

   ______________ m/s

3. A 20 N force acts at right angles, 0.4 m from a bolt. What is the torque, in $\text{N} \cdot \text{m}$?

   ______________ $\text{N} \cdot \text{m}$

4. Two 0.5 kg balls sit 2 m from the axis on a light rod. What is the rotational inertia, in $\text{kg} \cdot \text{m}^2$?

   ______________ $\text{kg} \cdot \text{m}^2$

5. A 300 N child sits 2 m from a seesaw pivot. How far from the pivot must a 400 N child sit to balance it, in metres?

   ______________ m

6. How many radians are there in one complete turn?

   ☐ 360

   ☐ $\pi$

   ☐ $2\pi$

   ☐ 1

7. On a rigid spinning disk, how do a rim point and a point near the hub compare?

   ☐ same $\omega$ and same $v$

   ☐ same $\omega$, but the rim has larger $v$

   ☐ same $v$, but the rim has larger $\omega$

   ☐ the hub point does not move

8. A force is aimed straight through the pivot. What torque does it produce?

   ☐ the maximum

   ☐ half the maximum

   ☐ zero

   ☐ a negative torque

9. Rotational inertia depends on:

- ☐ the total mass only
- ☐ the mass and how it is distributed about the axis
- ☐ the colour of the object
- ☐ the speed of rotation

10. What is the condition for rotational equilibrium?
    - ☐ the net force is zero
    - ☐ the net torque is zero
    - ☐ the object is at rest
    - ☐ all forces are equal

AP Physics 1

1. **3 rad/s$^2$**

   $\alpha = \Delta\omega/\Delta t = 12/4 = 3\ \frac{\text{rad}}{\text{s}^2}$.

2. **3 m/s**

   $v = r\omega = 0.3 \times 10 = 3\ \frac{\text{m}}{\text{s}}$.

3. **8 N · m**

   $\tau = rF\sin 90^\circ = 0.4 \times 20 \times 1 = 8\ \text{N}\cdot\text{m}$.

4. **4 kg · m$^2$**

   $I = \sum mr^2 = 2 \times (0.5 \times 2^2) = 4\ \text{kg}\cdot\text{m}^2$.

5. **1.5 m**

   $300 \times 2 = 400 \times d_2 \Rightarrow d_2 = 600/400 = 1.5$ m.

6. $2\pi$

   One full circle is $2\pi$ radians ($\approx 6.28$).

7. **same $\omega$, but the rim has larger $v$**

   All points share $\omega$; the rim has a larger radius, so $v = r\omega$ is larger there.

8. **zero**

   $\theta = 0$ means $\sin\theta = 0$, so $\tau = 0$ — a force through the pivot cannot turn it.

9. **the mass and how it is distributed about the axis**

   $I = \sum mr^2$ depends on both the mass and its distance from the axis (squared).

10. **the net torque is zero**

    Rotational equilibrium means $\sum \tau = 0$ — the turning effects balance.

# 5.1 Rotational Kinematics

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

**KEY WORDS** angular velocity 角速度 • torque 力矩 • angular acceleration 角加速度 • angular displacement 角位移 • radians 弧度

## Objective

Build the skills to answer exam questions on **rotational kinematics**.

**You must be able to:**

- define **angular displacement** 角位移 $\theta$, **angular velocity** 角速度 $\omega$, and **angular acceleration** 角加速度 $\alpha$
- describe the direction of angular quantities as clockwise or counterclockwise
- apply the **rotational kinematic equations** for constant angular acceleration
- interpret graphs of angular position, velocity, and acceleration

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Angular quantities

$\theta$ in radians, $\omega = \dfrac{\Delta\theta}{\Delta t}$ in rad s$^{-1}$, and $\alpha = \dfrac{\Delta\omega}{\Delta t}$ in rad s$^{-2}$. Counterclockwise is taken as positive.

### ■ The rotational kinematic equations (constant $\alpha$)

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2, \qquad \omega^2 = \omega_0^2 + 2\alpha\theta.$$

These mirror the linear equations, with $x \to \theta$, $v \to \omega$, $a \to \alpha$.

### ■ Example

A wheel starts from rest with $\alpha = 2.0$ rad s$^{-2}$. After 3.0 s: $\omega = 0 + 2.0(3.0) = 6.0$ rad s$^{-1}$.

## 2 Practice

**2.1** A disc accelerates from rest at $\alpha = 4.0$ rad s$^{-2}$ for 5.0 s. Find its final angular velocity. [1]

**2.2** Find the angle turned by the disc in that time. [2]

**2.3** State the rotational quantity that plays the role of velocity in the linear equations. [1]

## 3 Exam-style questions

**3.1** Angular acceleration is the rate of change of [1]

- **A** angular displacement
- **B** angular velocity
- **C** torque
- **D** period

**3.2** A wheel spinning at 10 rad s$^{-1}$ decelerates at 2.0 rad s$^{-2}$. The time to stop is [1]

- **A** 2.0 s
- **B** 5.0 s
- **C** 8.0 s
- **D** 20 s

**3.3** A fan blade slows from 20 rad s$^{-1}$ to 8.0 rad s$^{-1}$ in 4.0 s.

(a) Find the angular acceleration. [2]

(b) Find the angle turned during this time. [2]

## Solutions

**2.1** $\omega = 0 + 4.0(5.0) = 20$ rad s$^{-1}$.

**2.2** $\theta = \frac{1}{2}(4.0)(5.0)^2 = 50$ rad.

**2.3** angular velocity $\omega$.

**3.1** **B**.

**3.2** **B** — $t = \dfrac{0 - 10}{-2.0} = 5.0$ s.

**3.3** (a) $\alpha = \dfrac{8.0 - 20}{4.0} = -3.0$ rad s$^{-2}$. (b) $\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(20 + 8.0)(4.0) = 56$ rad.

Begin your response to **QUESTION 3** on this page.

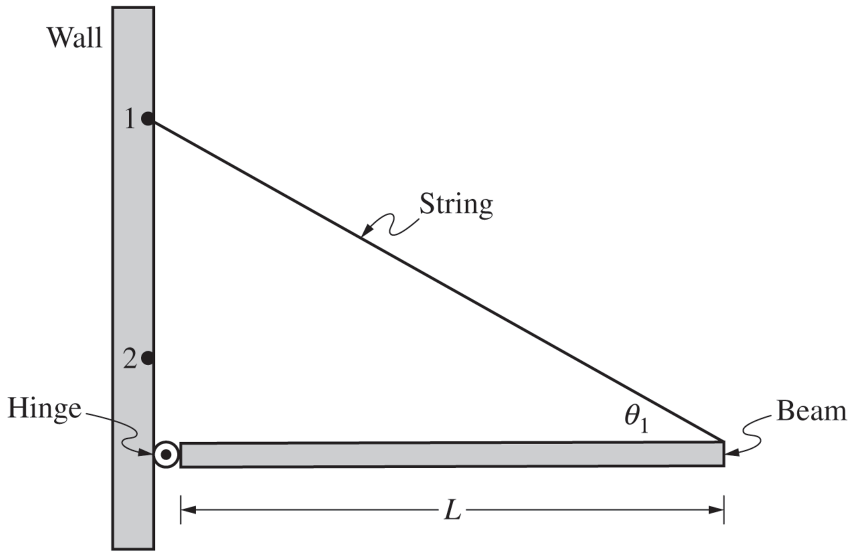



Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass $M$ and length $L$ is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude $F_H$, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

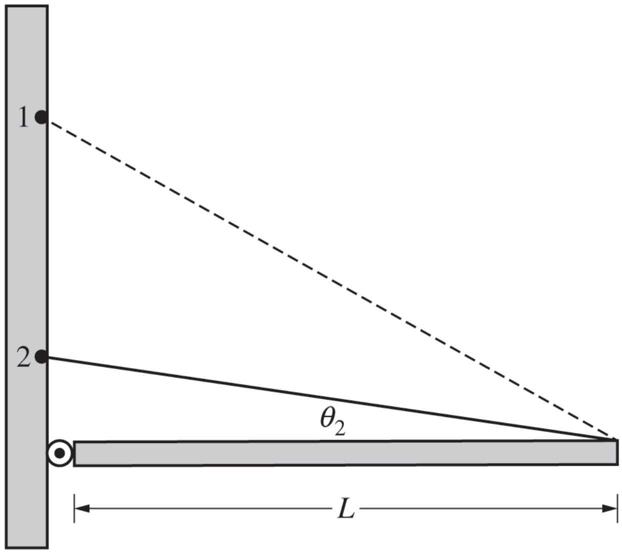



Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is $F_{T1}$. The magnitude of the tension in the string shown in Figure 2 is $F_{T2}$. **Indicate** which of the following correctly compares $F_{T2}$ with $F_{T1}$.

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

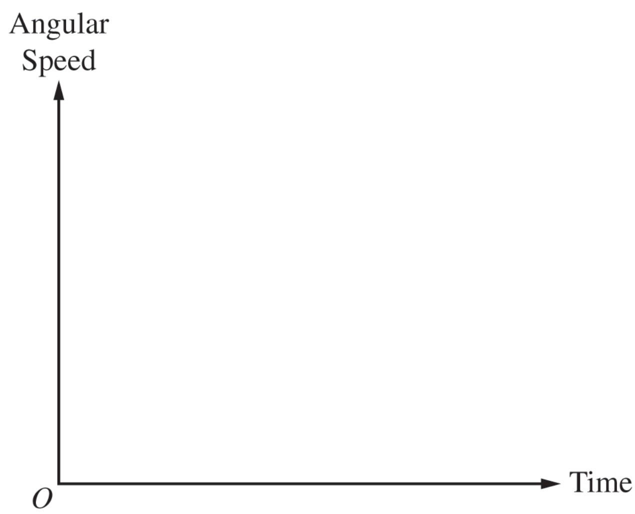
(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of $M$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

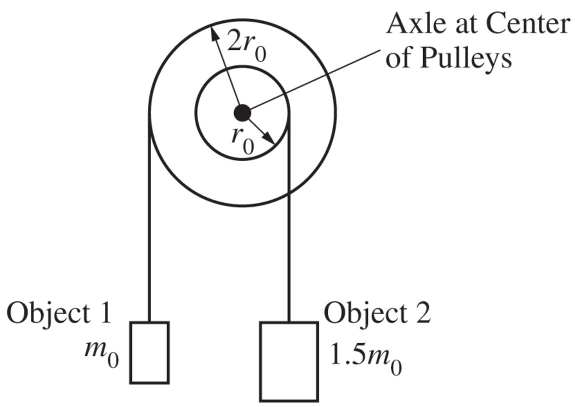
(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.




**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 5** on this page.




5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

$m_0$ is the mass of object 1.
$1.5m_0$ is the mass of object 2.
$r_0$ is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_0$, $r_0$, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

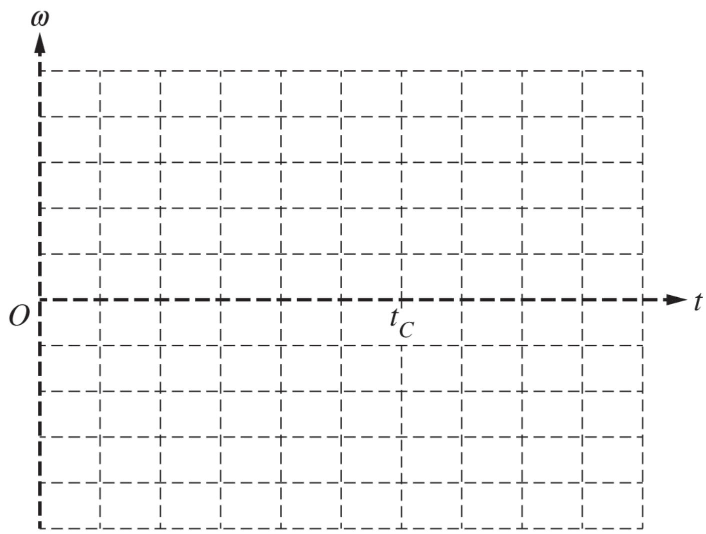
**GO ON TO THE NEXT PAGE.**

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction          ___ Opposite directions

Briefly explain your reasoning.

(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.




**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**

# 5.2 Connecting Linear and Rotational Motion

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

**KEY WORDS** tangential acceleration 切向加速度 • arc length 弧长 • centripetal acceleration 向心加速度 • tangential velocity 切向速度 • angular acceleration 角加速度

## Objective

Build the skills to answer exam questions on **connecting linear and rotational motion**.

**You must be able to:**

- relate **tangential velocity 切向速度** to angular velocity, $v = r\omega$
- relate **tangential acceleration 切向加速度** to angular acceleration, $a_t = r\alpha$
- distinguish tangential from **centripetal acceleration 向心加速度** for a point in a circle
- calculate **arc length 弧长** with $s = r\theta$

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Linear from angular

$v = r\omega$ and $a_t = r\alpha$. On a rigid body every point has the **same** $\omega$, but a point farther from the axis has a **larger** linear speed.

### ■ Two accelerations

**Tangential** $a_t = r\alpha$ changes the speed; **centripetal** $a_c = \dfrac{v^2}{r} = \omega^2 r$ changes only the direction.

### ■ Example

A wheel of radius 0.30 m spins at $\omega = 10$ rad s$^{-1}$: a rim point moves at $v = 0.30\times 10 = 3.0$ m s$^{-1}$.

## 2 Practice

**2.1** A point 0.50 m from the axis rotates at $\omega = 6.0$ rad s$^{-1}$. Find its linear speed. [1]

**2.2** A wheel of radius 0.20 m has angular acceleration 5.0 rad s$^{-2}$. Find the tangential acceleration of its rim. [1]

**2.3** Two points on a disc are 0.10 m and 0.30 m from the axis. State which has the greater (a) angular speed and (b) linear speed. [2]

**2.4** Find the arc length swept by a point 2.0 m from the axis when it turns through 3.0 rad. [1]

## 3 Exam-style questions

**3.1** On a rigid rotating disc, every point has the same [1]

- **A** linear speed
- **B** angular speed
- **C** tangential acceleration
- **D** arc length

**3.2** The tangential acceleration of a point at radius $r$ is [1]

- **A** $r\omega$
- **B** $r\alpha$
- **C** $v^2/r$
- **D** $\omega^2 r$

**3.3** A car tyre of radius 0.35 m rotates at $\omega = 20$ rad s$^{-1}$ while rolling without slipping.

(a) Find the linear speed of a point on the tread. [2]

(b) State the speed of the car. [1]

## Solutions

**2.1** $v = r\omega = 0.50 \times 6.0 = 3.0 \text{ m s}^{-1}$.

**2.2** $a_t = r\alpha = 0.20 \times 5.0 = 1.0 \text{ m s}^{-2}$.

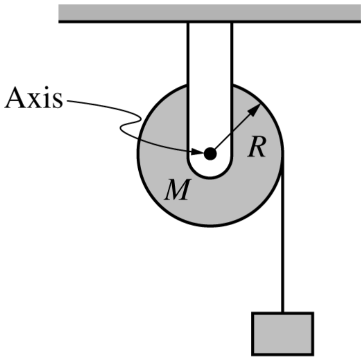
**2.3** (a) equal — same angular speed for both. (b) the point at 0.30 m, since $v = r\omega$.

**2.4** $s = r\theta = 2.0 \times 3.0 = 6.0$ m.

**3.1** **B**.

**3.2** **B**.

**3.3** (a) $v = r\omega = 0.35 \times 20 = 7.0 \text{ m s}^{-1}$. (b) $7.0 \text{ m s}^{-1}$ (rolling without slipping).




. (7 points, suggested time 13 minutes)

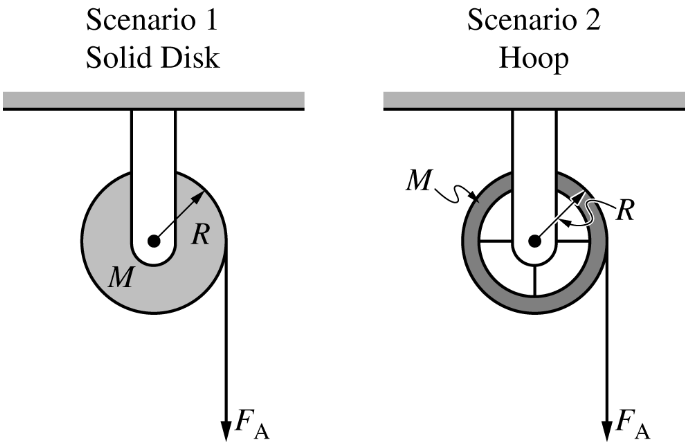
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass $M$ and radius $R$. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is $F_T$.

(a) Determine an expression for the magnitude of the angular acceleration $\alpha_D$ of the disk as the block travels downward. Express your answer in terms of $M$, $R$, $F_T$, and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 4** on this page.




Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass $M$ and radius $R$ as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force $F_A$ for the same time interval $\Delta t$, causing the pulleys to rotate without the string slipping. After time interval $\Delta t$, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval $\Delta t$. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

**GO ON TO THE NEXT PAGE.**

AP PHYSICS 1 • EXERCISE SHEET

# 5.3 Torque

Name: ____________ Class: ________ Date: __________ **Total: 12 marks**

**KEY WORDS** torque 力矩 • lever arm 力臂

## Objective

Build the skills to answer exam questions on **torque**.

**You must be able to:**

- define **torque 力矩** as the tendency of a force to cause rotation about an axis
- calculate a torque with $\tau = rF\sin\theta$
- identify the **lever arm 力臂** as the perpendicular distance from the axis to the line of action
- determine the **direction** of a torque (clockwise or counterclockwise)

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### Torque formula

$\tau = rF\sin\theta = F\times(\text{lever arm})$, measured in N m. The **lever arm** is the perpendicular distance from the axis to the force's line of action.

### Maximum torque

Torque is greatest when the force is **perpendicular** to $r$ ($\theta = 90°$) and zero when the force points along $r$.

### Example

A 20 N force at the end of a 0.50 m wrench, perpendicular to it: $\tau = 20 \times 0.50 = 10$ N m.

## 2 Practice

**2.1** A 30 N force acts perpendicular to a door, 0.80 m from the hinge. Find the torque. [1]

**2.2** State the angle between the force and the lever arm that gives the maximum torque. [1]

**2.3** A 40 N force acts at 30° to a 0.60 m arm. Find the torque. [2]

**2.4** Explain why a longer wrench makes it easier to loosen a tight bolt. [2]

## 3 Exam-style questions

**3.1** Torque is greatest when the force is applied [1]

- **A** along the arm
- **B** at 45° to the arm
- **C** perpendicular to the arm
- **D** at the axis

**3.2** A 10 N force acts at a perpendicular distance of 0.20 m from an axis. The torque is [1]

- **A** 0.02 N m
- **B** 2.0 N m
- **C** 10 N m
- **D** 50 N m

**3.3** A seesaw pivots at its centre. A 300 N child sits 1.5 m from the pivot.

(a) Find the torque this child produces. [2]

(b) Find where a 450 N child must sit to balance the seesaw. [2]

## Solutions

**2.1** $\tau = 30 \times 0.80 = 24$ N m.

**2.2** $90°$ (perpendicular).

**2.3** $\tau = rF \sin\theta = 0.60 \times 40 \times \sin 30° = 12$ N m.

**2.4** A longer arm is a larger lever arm $r$, so the same force produces a bigger torque $\tau = rF$.

**3.1** **C**.

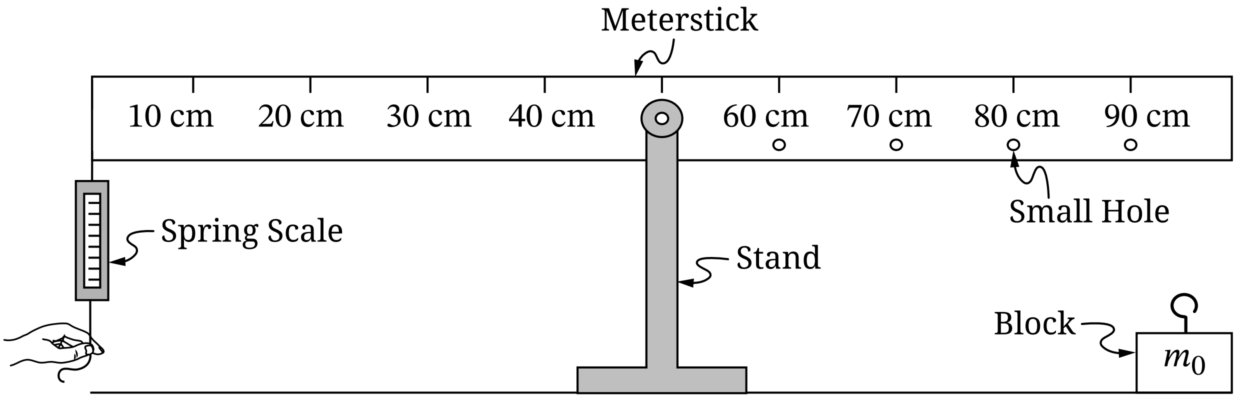
**3.2** **B** — $\tau = 10 \times 0.20 = 2.0$ N m.

**3.3** (a) $\tau = 300 \times 1.5 = 450$ N m. (b) balance needs equal torque: $450 \times d = 450 \Rightarrow d = 1.0$ m from the pivot.

**Question 3: Version J**

**3.** Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass $m_0$. The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

**Figure 1**



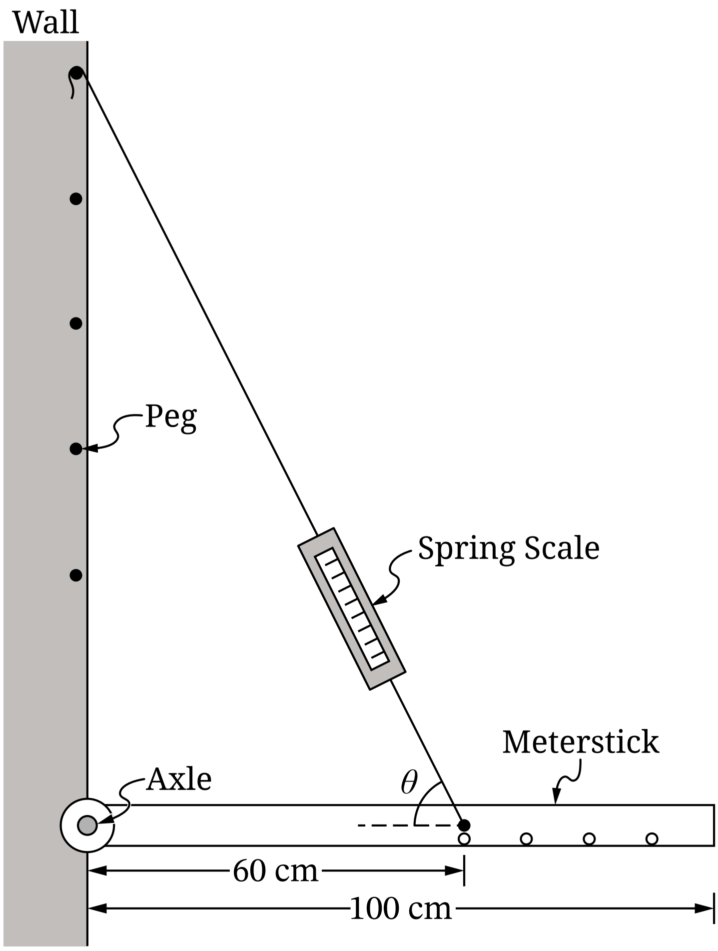

The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass $m_0$ of the block.

**A. Describe** an experimental procedure to collect data that would allow the students to determine $m_0$. Include any steps necessary to reduce experimental uncertainty.

**B. Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine $m_0$.

The students have an identical meterstick of mass $M$ that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

**Figure 2**




The angle $\theta$ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension $F_T$ required to hold the meterstick horizontal. Table 1 shows the measured values of $\theta$ and $F_T$.

Table 1

| $\theta$ (degrees) | $F_T$ (N) |
|---|---|
| 22 | 21 |
| 31 | 17 |
| 36 | 13 |
| 45 | 12 |
| 80 | 8 |

The students correctly determine that the relationship between $F_T$ and $\theta$ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

**C.**

i. **Indicate** what measured or calculated quantity could be plotted on the vertical axis to yield a linear graph whose slope can be used to calculate an experimental value for the mass $M$ of the meterstick.

Vertical axis: ______ Horizontal axis: $\frac{1}{\sin\theta}$

ii. On the blank grid provided, create a graph of the quantities indicated in part C (i) that can be used to determine $M$.

- Use Table 2 to record the data points or calculated quantities that you will plot.
- Clearly **label** the vertical axis, including units as appropriate.
- **Plot** the points you recorded in Table 2.

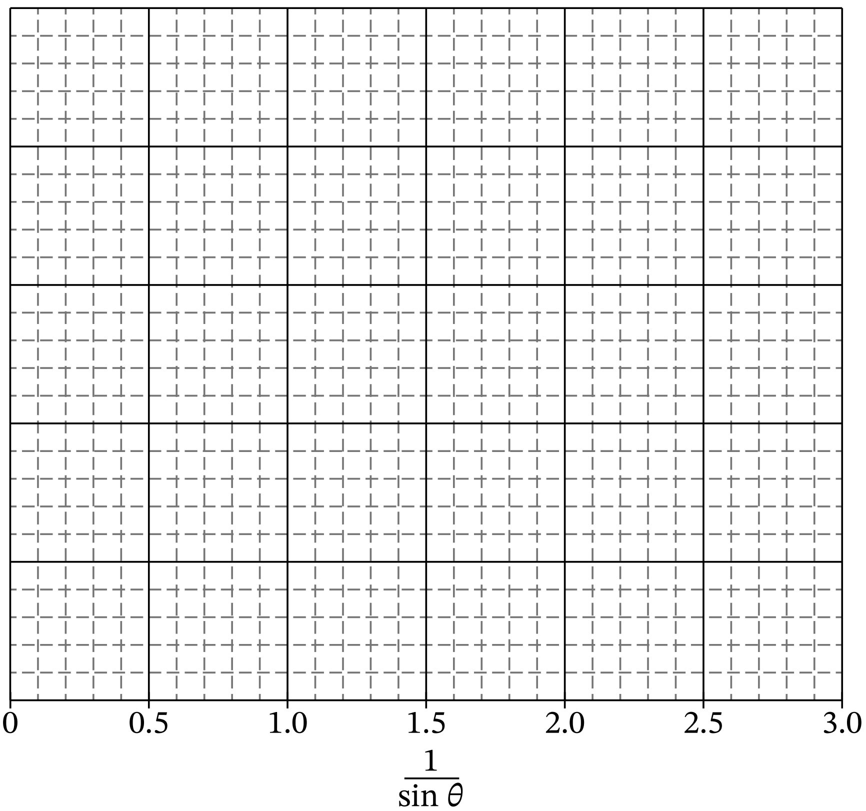



Figure 3

iii. **Draw** a straight best-fit line for the data graphed in part C (ii).

**D.** Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for the mass $M$ of the meterstick.

Begin your response to **QUESTION 3** on this page.

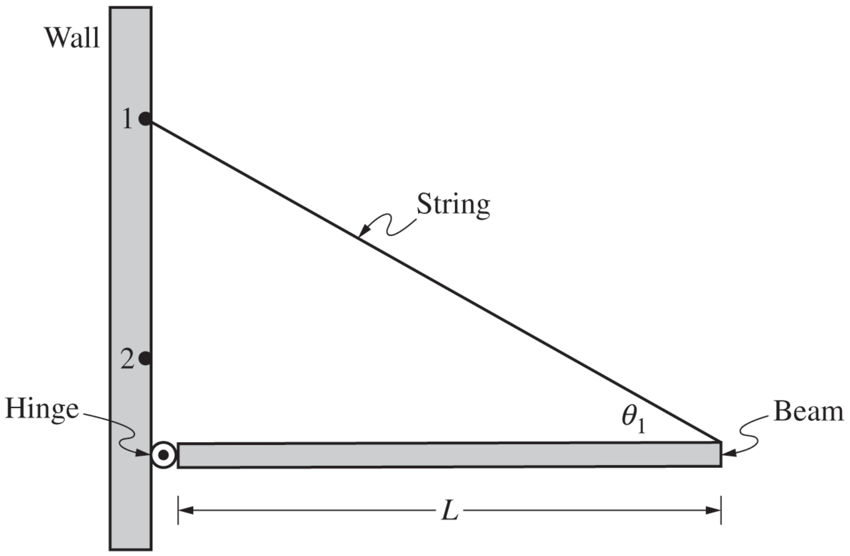



Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass $M$ and length $L$ is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude $F_H$, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

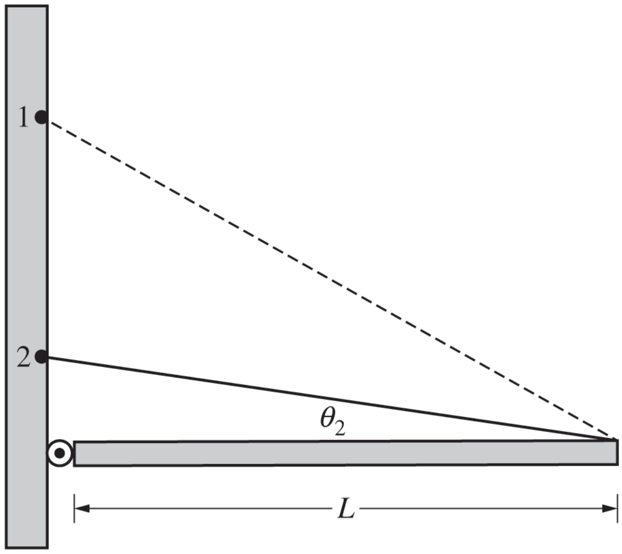



Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is $F_{T1}$. The magnitude of the tension in the string shown in Figure 2 is $F_{T2}$. **Indicate** which of the following correctly compares $F_{T2}$ with $F_{T1}$.

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

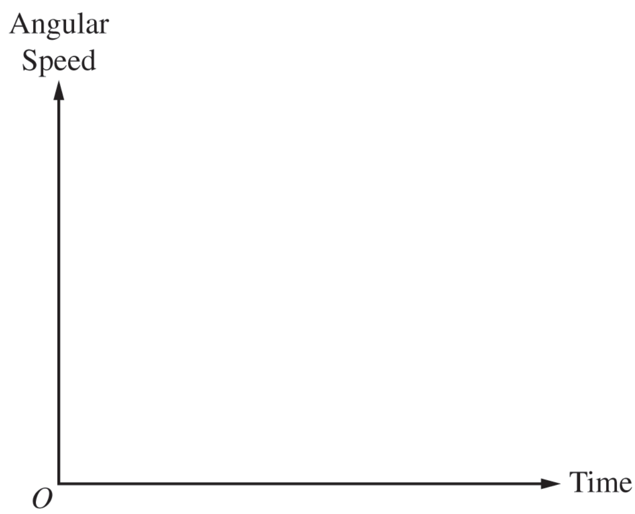
(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of $M$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.




**GO ON TO THE NEXT PAGE.**

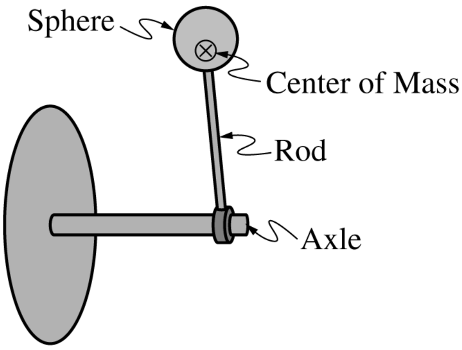



Figure 1

. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

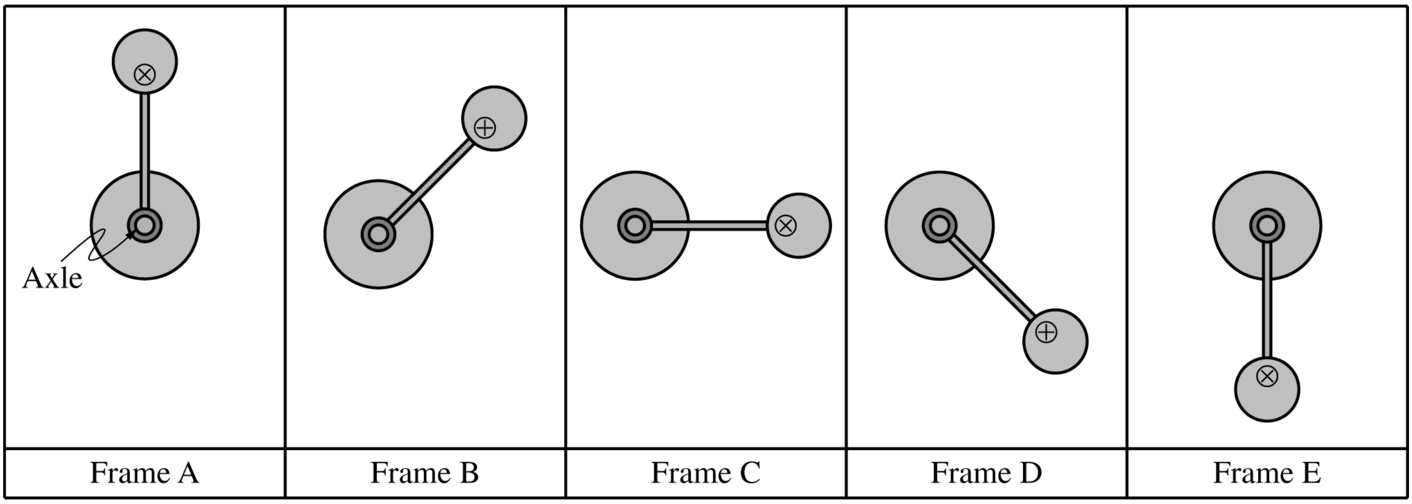



Figure 2

**GO ON TO THE NEXT PAGE.**

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

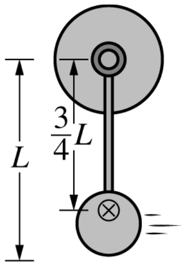



Figure 3

(b) The rod-sphere system has mass $M$ and length $L$, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of $M$, $L$, and fundamental constants, as appropriate.

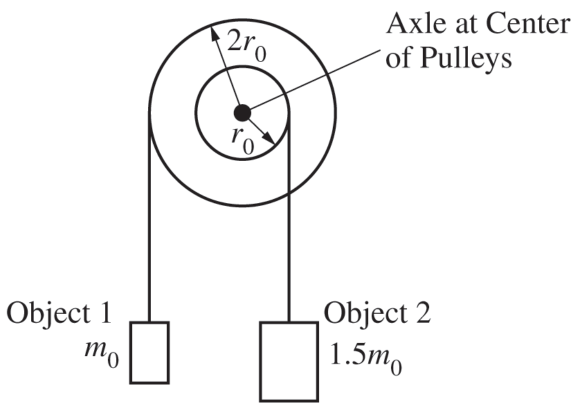
ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**

Begin your response to **QUESTION 5** on this page.




5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

$m_0$ is the mass of object 1.
$1.5m_0$ is the mass of object 2.
$r_0$ is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_0$, $r_0$, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

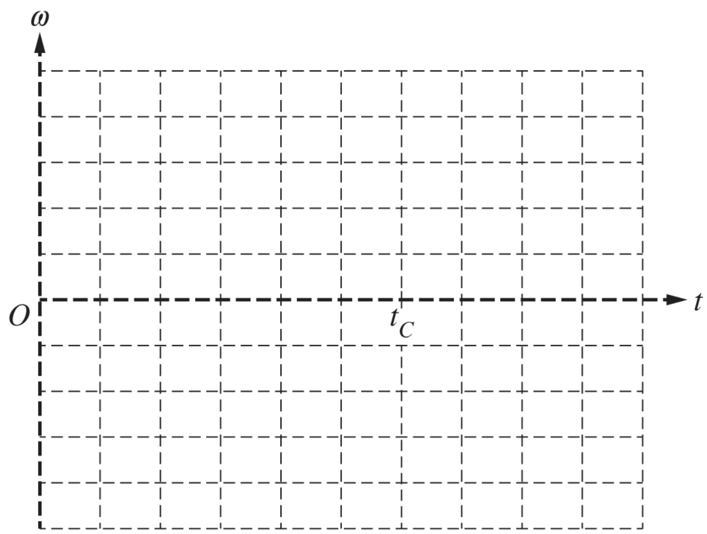
**GO ON TO THE NEXT PAGE.**

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction ___ Opposite directions

Briefly explain your reasoning.

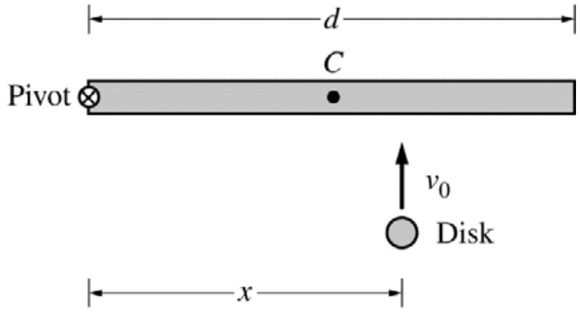
(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.




**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**




Top View

3. (12 points, suggested time 25 minutes)

The left end of a rod of length $d$ and rotational inertia $I$ is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point $C$ marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass $m_{disk}$ toward the rod with velocity $v_0$ perpendicular to the rod, and the disk will stick to the rod a distance $x$ from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point $C$, at point $C$, or to the right of point $C$ ?

____ To the left of $C$ ____ At $C$ ____ To the right of $C$

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed $\omega$ of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for $\omega$ have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for $\omega$.

(c) Another student deriving an equation for the postcollision angular speed $\omega$ of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{disk} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{disk} x^2$ and its angular momentum with respect to the pivot is $m_{disk} v_0 x$. Derive an equation for the postcollision angular speed $\omega$ of the rod. Express your answer in terms of $d$, $m_{disk}$, $I$, $x$, $v_0$, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than ____ Less than ____ Equal to

Briefly explain your reasoning.

# 5.4 Rotational Inertia

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

**KEY WORDS** rotational inertia 转动惯量 • parallel-axis theorem 平行轴定理 • torque 力矩

## Objective

Build the skills to answer exam questions on **rotational inertia**.

**You must be able to:**

- define **rotational inertia 转动惯量** (moment of inertia) as the resistance to a change in rotation
- explain that it depends on both the **mass** and its **distribution** about the axis
- calculate the rotational inertia of point masses with $I = \sum mr^2$
- use the **parallel-axis theorem 平行轴定理**, $I = I_{cm} + Md^2$

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Rotational inertia of point masses**

$I = \sum mr^2$. Mass **far** from the axis contributes most (the $r^2$).

■ **Common rigid bodies**

Solid disc $I = \frac{1}{2}MR^2$; hoop $I = MR^2$; solid sphere $I = \frac{2}{5}MR^2$; rod about its centre $I = \frac{1}{12}ML^2$.

■ **Parallel-axis theorem**

$I = I_{cm} + Md^2$, where $d$ is the distance from the centre of mass to the new axis.

## 2 Practice

**2.1** Two 0.50 kg masses sit 0.40 m from an axis. Find the total rotational inertia. [2]

**2.2** State how the rotational inertia changes if the masses are moved farther from the axis. [1]

**2.3** A solid disc has $M = 4.0$ kg and $R = 0.30$ m. Find its moment of inertia ($I = \frac{1}{2}MR^2$). [2]

## 3 Exam-style questions

**3.1** The rotational inertia of an object depends on [1]

- **A** its mass only
- **B** its mass and how the mass is distributed about the axis
- **C** its speed only
- **D** the applied torque

**3.2** For the same mass and radius, which shape has the **largest** rotational inertia? [1]

- **A** solid disc
- **B** hoop
- **C** solid sphere
- **D** rod about its centre

**3.3** A 0.20 kg point mass is 0.50 m from an axis.

(a) Find its rotational inertia about that axis. [2]

(b) State its rotational inertia about a parallel axis passing through the mass itself. [1]

## Solutions

**2.1** $I = \sum mr^2 = 2 \times (0.50)(0.40)^2 = 0.16$ kg m$^2$.

**2.2** it increases (each contributes $mr^2$, which grows with $r$).

**2.3** $I = \frac{1}{2}(4.0)(0.30)^2 = 0.18$ kg m$^2$.

**3.1** **B**.

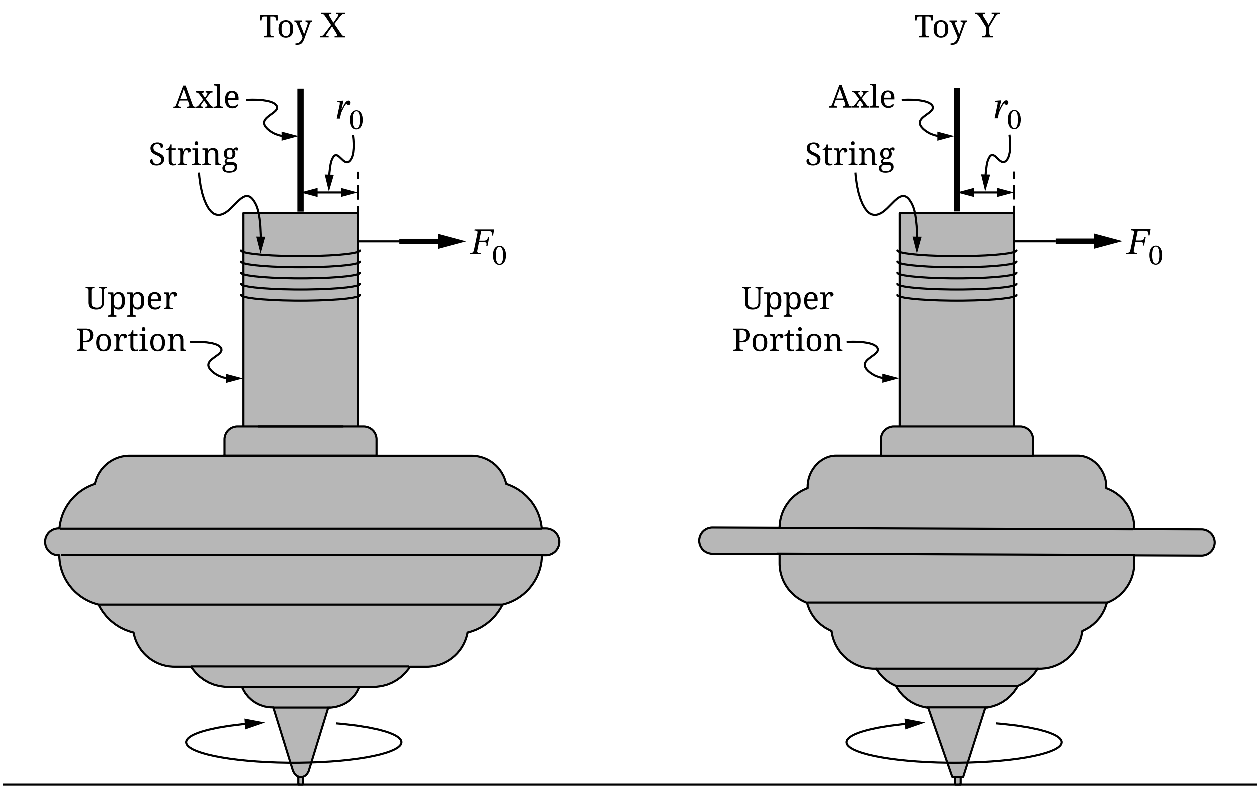
**3.2** **B** — a hoop has all its mass at the rim, $I = MR^2$.

**3.3** (a) $I = mr^2 = 0.20 \times (0.50)^2 = 0.050$ kg m$^2$. (b) 0 — a point mass has no size, so $I = 0$ about an axis through it.

**Question 4**

**4.** Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius $r_0$, as shown in the side view in Figure 1. A string of length $\ell_0$ is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is $I_X$, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

**Figure 1**



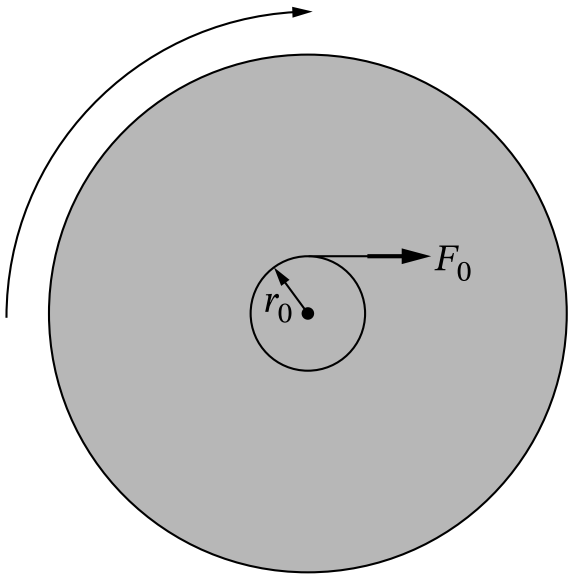

Side View

Toys X and Y are initially at rest. A constant horizontal force of magnitude $F_0$ is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is $\omega_X$.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is $\omega_Y$.

**Figure 2**




Overhead View

**Part A**

**Indicate** whether $\omega_Y$ is greater than, less than, or equal to $\omega_X$ by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

**Justify** your answer using qualitative reasoning beyond referencing equations.

**Part B**

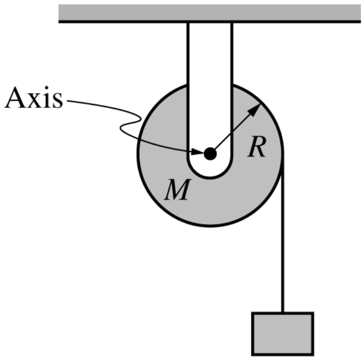
Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed $\omega_X$ of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of $\ell_0$, $I_X$, $F_0$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

**Part C**

**Justify** how your derived equation in part B is or is not consistent with your reasoning in part A.

**STOP**

**END OF EXAM**



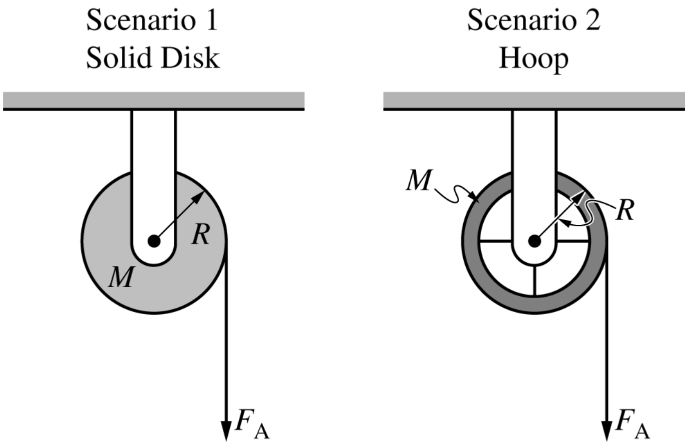

. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass $M$ and radius $R$. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is $F_T$.

(a) Determine an expression for the magnitude of the angular acceleration $\alpha_D$ of the disk as the block travels downward. Express your answer in terms of $M$, $R$, $F_T$, and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**



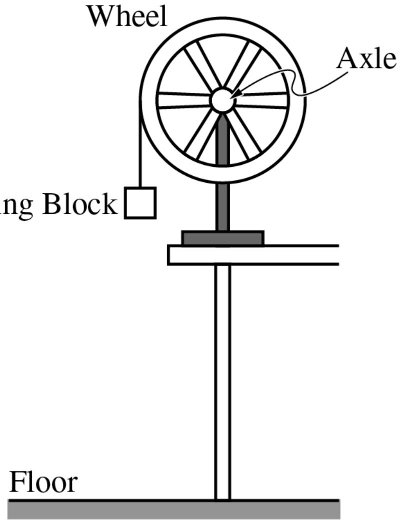

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass $M$ and radius $R$ as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force $F_A$ for the same time interval $\Delta t$, causing the pulleys to rotate without the string slipping. After time interval $\Delta t$, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval $\Delta t$. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 3** on this page.




. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

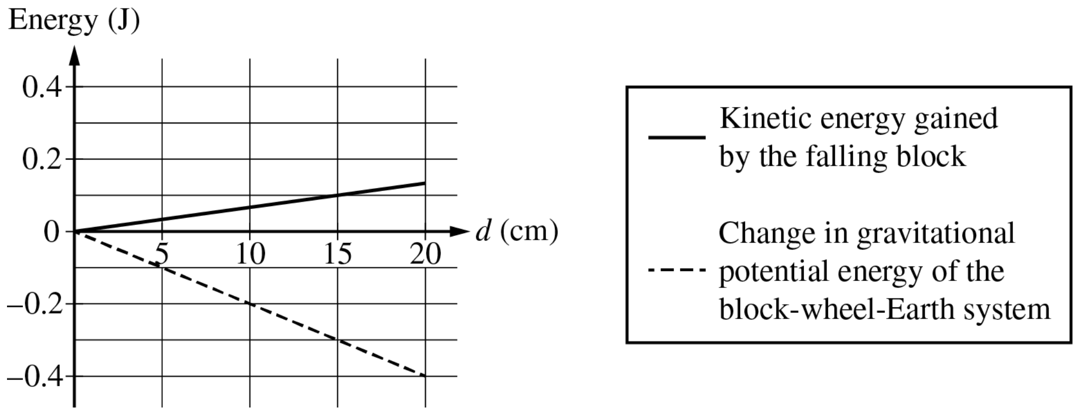
If needed, you may include a simple diagram of the setup with your procedure.

| Quantity to Be Measured | Symbol for Quantity | Equipment for Measurement | Procedure (and diagram, if needed) |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

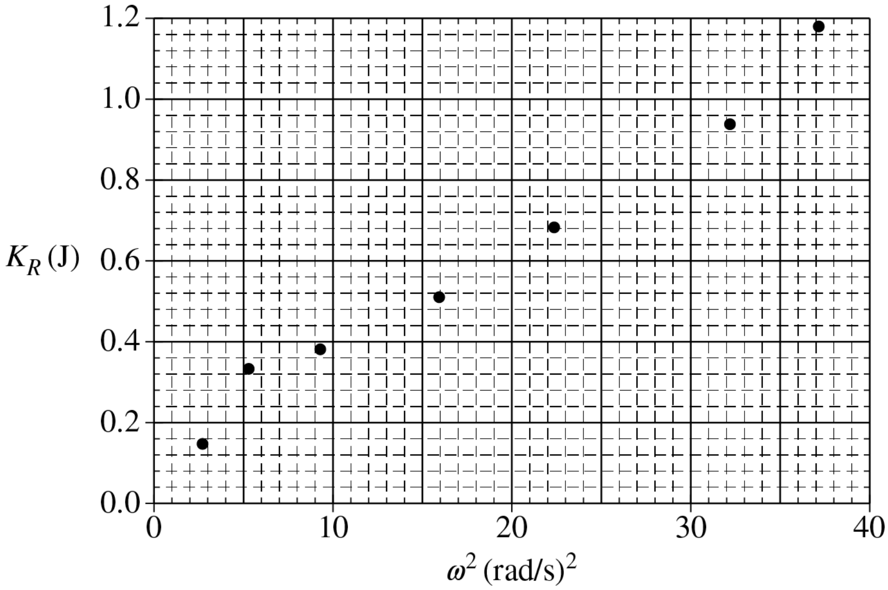
**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.




(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance $d$. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance $d$.

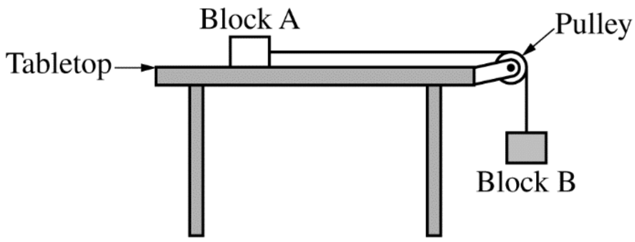
(d) The students also measure the angular velocity $\omega$ of the wheel as the block falls and determine the rotational kinetic energy $K_R$ of the wheel. The students then make a graph of $K_R$ as a function of $\omega^2$, as shown.




i. On the above graph, draw a straight line that best represents the data.

ii. Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

**GO ON TO THE NEXT PAGE.**




2. (12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass $m_A$, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass $m_B$, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much <u>greater</u> than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much <u>less</u> than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Block A Block B

(c) Derive an equation for the acceleration of the blocks after release in terms of $m_A$, $m_B$, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much <u>less</u> than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

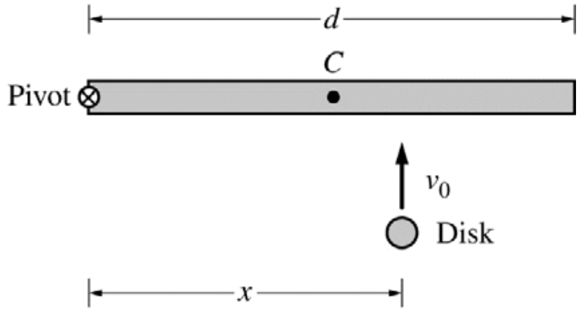
____ Yes ____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when $m_A$ is much <u>less</u> than $m_B$.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is $T_1$. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is $T_2$. How do the two tensions compare to each other?

____ $T_2 > T_1$ ____ $T_2 = T_1$ ____ $T_2 < T_1$

Briefly explain your reasoning.




Top View

3. (12 points, suggested time 25 minutes)

The left end of a rod of length $d$ and rotational inertia $I$ is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point $C$ marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass $m_{\text{disk}}$ toward the rod with velocity $v_0$ perpendicular to the rod, and the disk will stick to the rod a distance $x$ from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point $C$, at point $C$, or to the right of point $C$ ?

____ To the left of $C$ ____ At $C$ ____ To the right of $C$

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed $\omega$ of the rod in this situation: $\omega = \dfrac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for $\omega$ have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for $\omega$.

(c) Another student deriving an equation for the postcollision angular speed $\omega$ of the rod makes a mistake and comes up with $\omega = \dfrac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular speed $\omega$ of the rod. Express your answer in terms of $d$, $m_{\text{disk}}$, $I$, $x$, $v_0$, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than ____ Less than ____ Equal to

Briefly explain your reasoning.

# 5.5 Rotational Equilibrium and Newton's First Law in Rotational Form

Name: ________ Class: ________ Date: ________ **Total: 11 marks**

**KEY WORDS** net torque 净力矩 • rotational equilibrium 转动平衡 • static equilibrium 静平衡 • torque 力矩

## Objective

Build the skills to answer exam questions on **rotational equilibrium** and Newton's first law in rotational form.

**You must be able to:**

- state that **rotational equilibrium 转动平衡** needs zero **net torque 净力矩**
- describe **static equilibrium 静平衡** as zero net force **and** zero net torque
- apply the equilibrium conditions to beams, seesaws, and hanging signs
- choose a convenient **axis** to solve for unknown forces or distances

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Two conditions for static equilibrium

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \tau = 0.$$

### ■ Choosing the axis

Take torques about the point where an **unknown** force acts — that force then has zero lever arm and drops out, leaving one unknown.

### ■ Example

A uniform 60 N beam 3.0 m long rests on a support at each end; a 30 N weight sits 1.0 m from the left support. Torques about the right support: $L(3.0) = 60(1.5) + 30(2.0) = 150$, so $L = 50$ N.

## 2 Practice

**2.1** State the two conditions for an object to be in static equilibrium. [2]

**2.2** A seesaw balances with a 200 N child 1.0 m left of the pivot. Find the clockwise torque needed on the right to balance it. [1]

**2.3** A uniform 40 N plank 4.0 m long rests on a support at each end. State the force on each support, with reasoning. [2]

## 3 Exam-style questions

**3.1** An object in rotational equilibrium has [1]

- **A** zero net force
- **B** zero net torque
- **C** constant speed
- **D** the maximum possible torque

**3.2** To balance a seesaw, the clockwise and counterclockwise torques about the pivot must be [1]

- **A** both zero
- **B** equal in size
- **C** as large as possible
- **D** opposite in sign only

**3.3** A uniform 80 N beam 4.0 m long rests on a support at each end. A 120 N box sits 1.0 m from the left support.

(a) Taking torques about the right support, find the force on the left support. [3]

(b) Find the force on the right support. [1]

## Solutions

**2.1** the net force is zero **and** the net torque is zero.

**2.2** $200 \times 1.0 = 200$ N m clockwise.

**2.3** 20 N on each — the plank is uniform and symmetric, so its weight splits equally.

**3.1 B**.

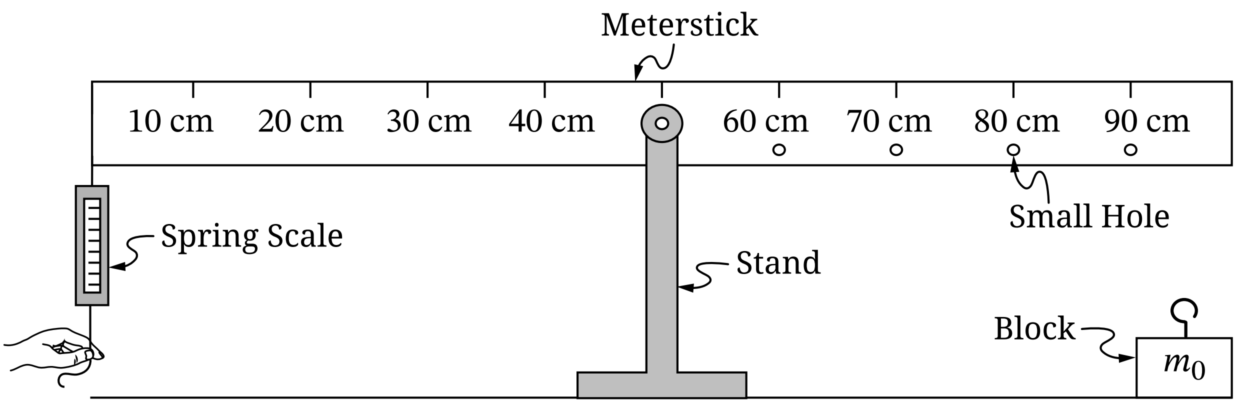
**3.2 B**.

**3.3** (a) torques about the right support: $L(4.0) = 80(2.0) + 120(3.0) = 160 + 360 = 520$, so $L = 130$ N. (b) $R = (80 + 120) - 130 = 70$ N.

**Question 3: Version J**

**3.** Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass $m_0$. The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

**Figure 1**



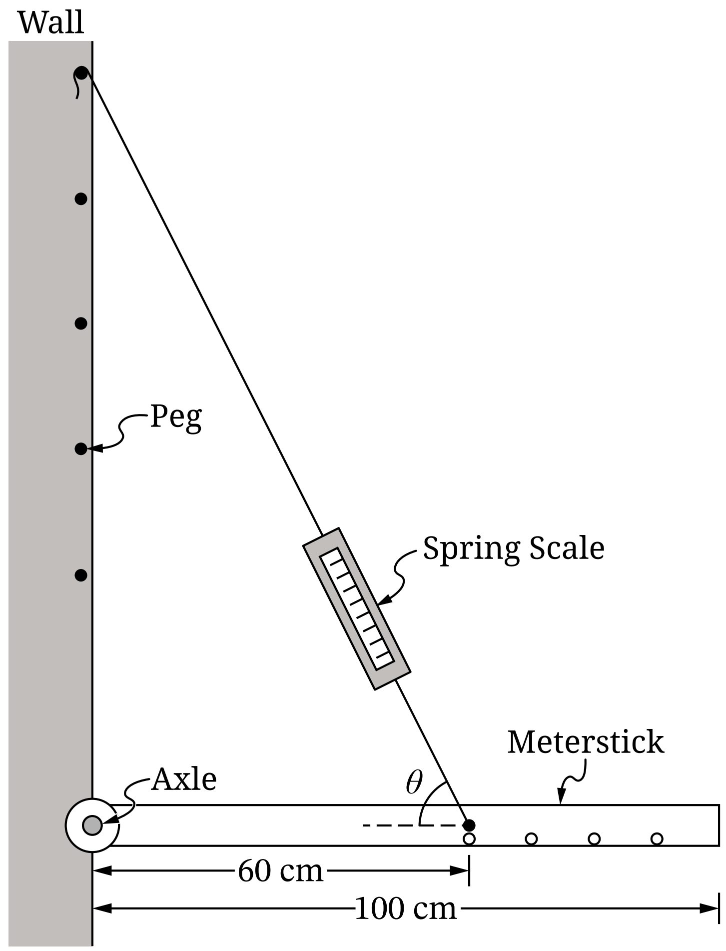

The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass $m_0$ of the block.

**A. Describe** an experimental procedure to collect data that would allow the students to determine $m_0$. Include any steps necessary to reduce experimental uncertainty.

**B. Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine $m_0$.

The students have an identical meterstick of mass $M$ that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

**Figure 2**




The angle $\theta$ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension $F_T$ required to hold the meterstick horizontal. Table 1 shows the measured values of $\theta$ and $F_T$.

Table 1

| $\theta$ (degrees) | $F_T$ (N) |
|---|---|
| 22 | 21 |
| 31 | 17 |
| 36 | 13 |
| 45 | 12 |
| 80 | 8 |

The students correctly determine that the relationship between $F_T$ and $\theta$ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

**C.**

i. **Indicate** what measured or calculated quantity could be plotted on the vertical axis to yield a linear graph whose slope can be used to calculate an experimental value for the mass $M$ of the meterstick.

Vertical axis: ______ Horizontal axis: $\frac{1}{\sin\theta}$

ii. On the blank grid provided, create a graph of the quantities indicated in part C (i) that can be used to determine $M$.

- Use Table 2 to record the data points or calculated quantities that you will plot.
- Clearly **label** the vertical axis, including units as appropriate.
- **Plot** the points you recorded in Table 2.

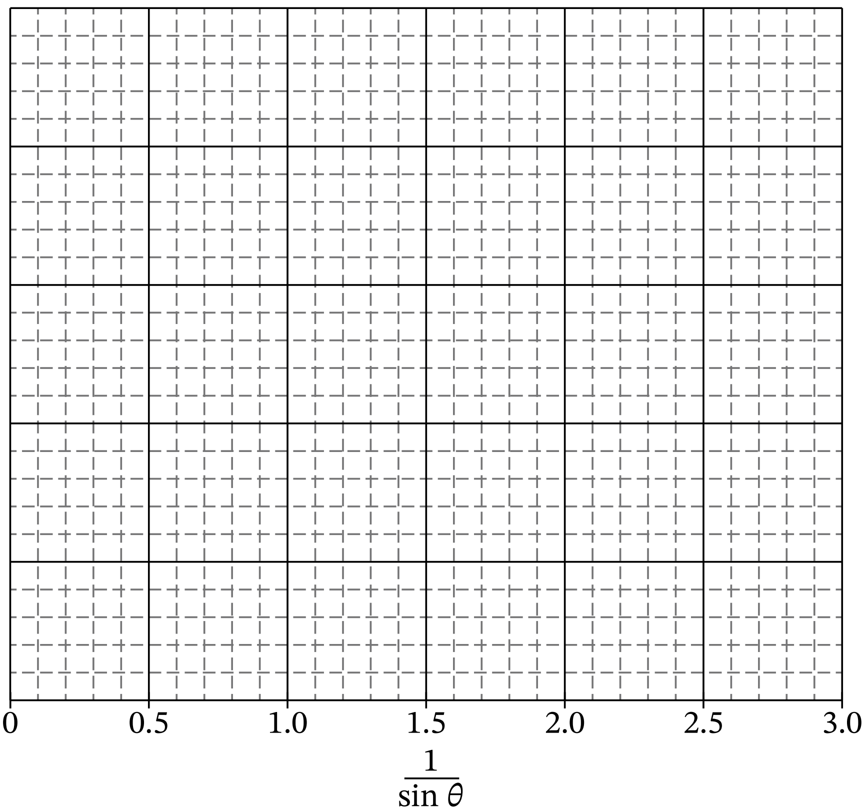



Figure 3

iii. **Draw** a straight best-fit line for the data graphed in part C (ii).

**D.** Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for the mass $M$ of the meterstick.

Begin your response to **QUESTION 3** on this page.

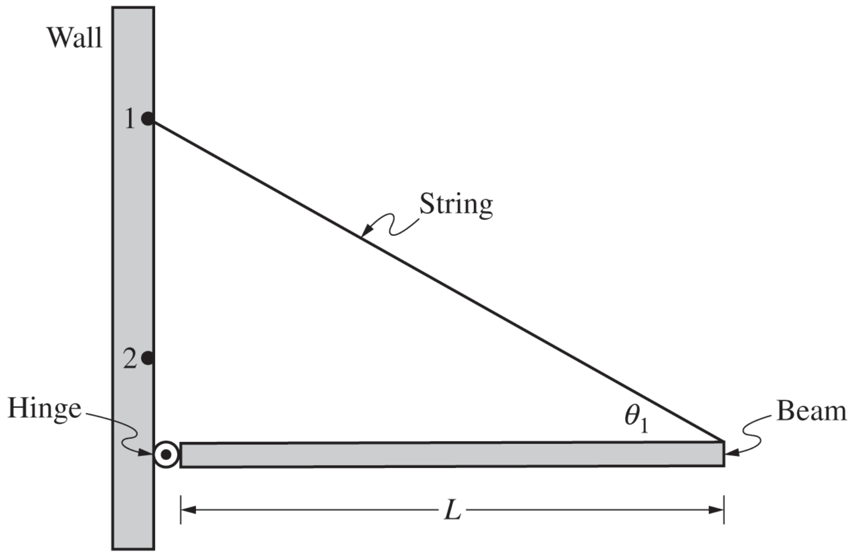



Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass $M$ and length $L$ is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude $F_H$, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

Continue your response to **QUESTION 3** on this page.

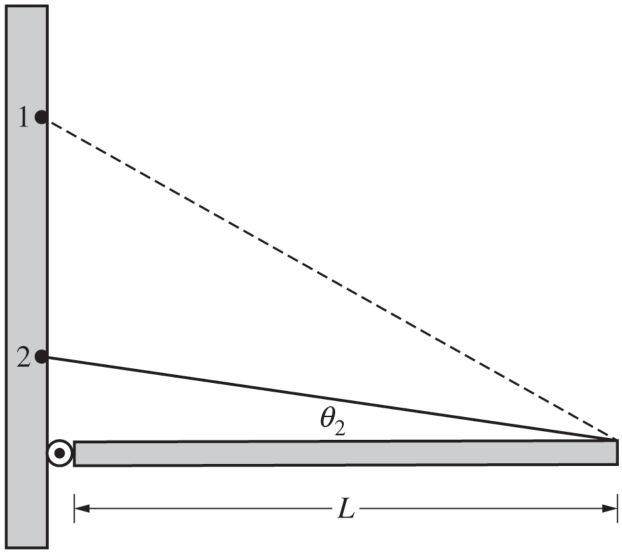



Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is $F_{T1}$. The magnitude of the tension in the string shown in Figure 2 is $F_{T2}$. **Indicate** which of the following correctly compares $F_{T2}$ with $F_{T1}$.

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

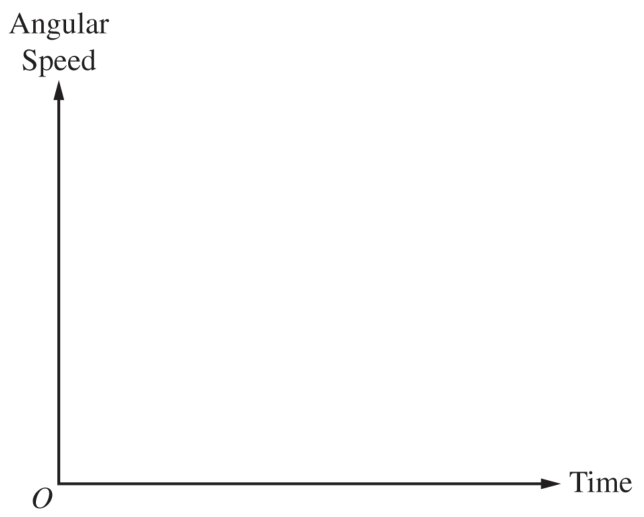
(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of $M$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.




**GO ON TO THE NEXT PAGE.**

# 5.6 Newton's Second Law in Rotational Form

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

**KEY WORDS** angular velocity 角速度 • torque 力矩 • angular acceleration 角加速度 • rotational inertia 转动惯量

## Objective

Build the skills to answer exam questions on **Newton's second law in rotational form**.

**You must be able to:**

- state **Newton's second law for rotation** 转动的牛顿第二定律, $\tau_{net} = I\alpha$
- determine the **angular acceleration** produced by a given net torque
- explain that a larger **rotational inertia** gives a smaller angular acceleration for the same torque
- analyse systems combining translation and rotation, such as a mass on a pulley

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Rotational Newton's second law

$\tau_{net} = I\alpha$ — the rotational analogue of $F = ma$ (with $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$).

### ■ Larger inertia, smaller acceleration

For a fixed net torque, a bigger $I$ gives a smaller $\alpha$.

### ■ Example

A net torque of 6.0 N m on a body of $I = 2.0$ kg m$^2$ gives $\alpha = \dfrac{6.0}{2.0} = 3.0$ rad s$^{-2}$.

## 2 Practice

**2.1** A net torque of 12 N m acts on a wheel of $I = 3.0$ kg m$^2$. Find its angular acceleration. [1]

---

**2.2** State how the angular acceleration changes if the rotational inertia doubles for the same torque. [1]

---

**2.3** A disc of $I = 0.50$ kg m$^2$ gains an angular acceleration of 8.0 rad s$^{-2}$. Find the net torque acting on it. [1]

---

## 3 Exam-style questions

**3.1** The rotational form of Newton's second law is [1]

- **A** $F = ma$
- **B** $\tau = I\alpha$
- **C** $p = mv$
- **D** $L = I\omega$

---

**3.2** For a fixed net torque, doubling the rotational inertia makes the angular acceleration [1]

- **A** twice as large
- **B** half as large
- **C** unchanged
- **D** four times as large

---

**3.3** A torque of 5.0 N m acts on a disc of rotational inertia 0.25 kg m$^2$, initially at rest.

(a) Find the angular acceleration. [2]

(b) Find the angular velocity after 4.0 s. [2]

## Solutions

**2.1** $\alpha = \dfrac{\tau}{I} = \dfrac{12}{3.0} = 4.0\text{ rad s}^{-2}$.

**2.2** it halves ($\alpha \propto 1/I$).

**2.3** $\tau = I\alpha = 0.50 \times 8.0 = 4.0\text{ N m}$.

**3.1** **B**.

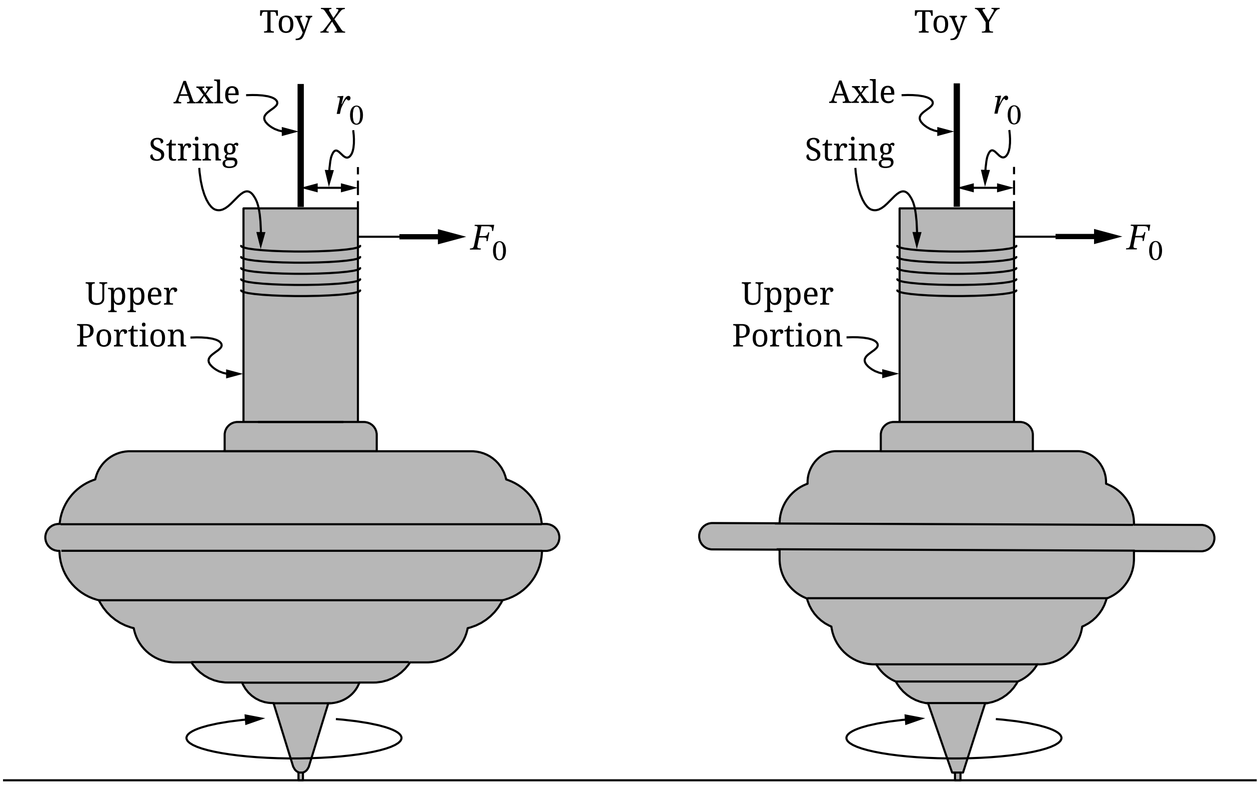
**3.2** **B**.

**3.3** (a) $\alpha = \dfrac{5.0}{0.25} = 20\text{ rad s}^{-2}$. (b) $\omega = \omega_0 + \alpha t = 0 + 20(4.0) = 80\text{ rad s}^{-1}$.

## Question 4

**4.** Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius $r_0$, as shown in the side view in Figure 1. A string of length $\ell_0$ is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is $I_X$, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

**Figure 1**



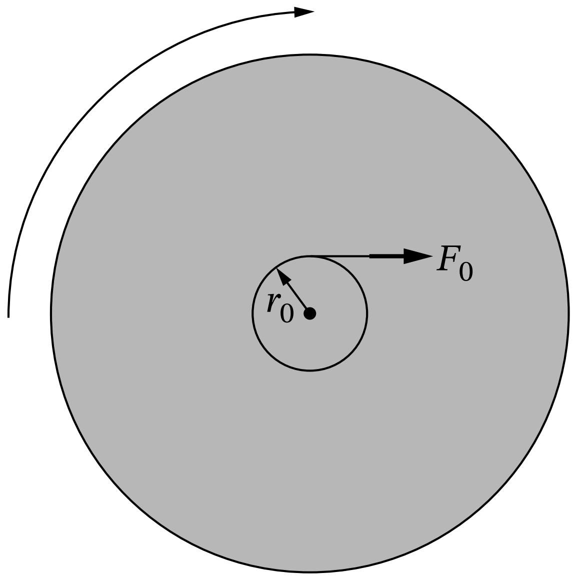

Side View

Toys X and Y are initially at rest. A constant horizontal force of magnitude $F_0$ is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is $\omega_X$.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is $\omega_Y$.

**Figure 2**




Overhead View

### Part A

**Indicate** whether $\omega_Y$ is greater than, less than, or equal to $\omega_X$ by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

**Justify** your answer using qualitative reasoning beyond referencing equations.

### Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed $\omega_X$ of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of $\ell_0$, $I_X$, $F_0$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

### Part C

**Justify** how your derived equation in part B is or is not consistent with your reasoning in part A.

**STOP**

**END OF EXAM**

Begin your response to **QUESTION 3** on this page.

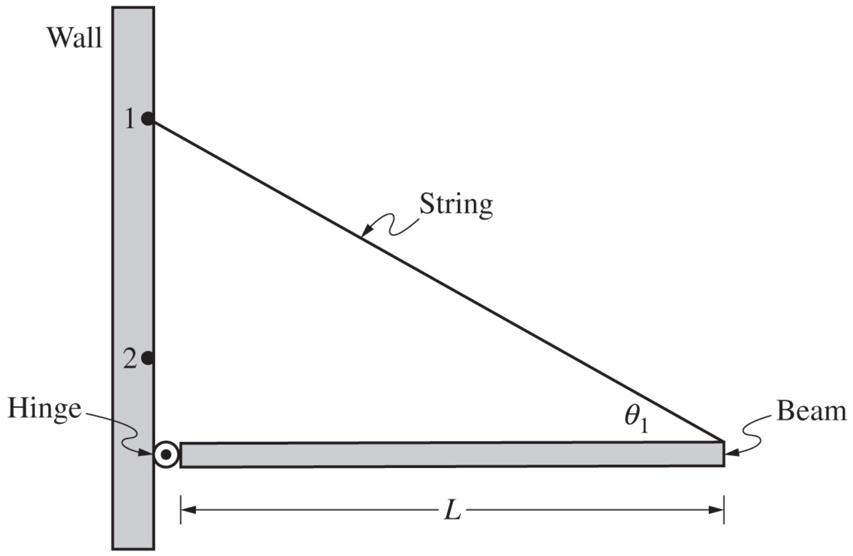



Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass $M$ and length $L$ is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude $F_H$, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

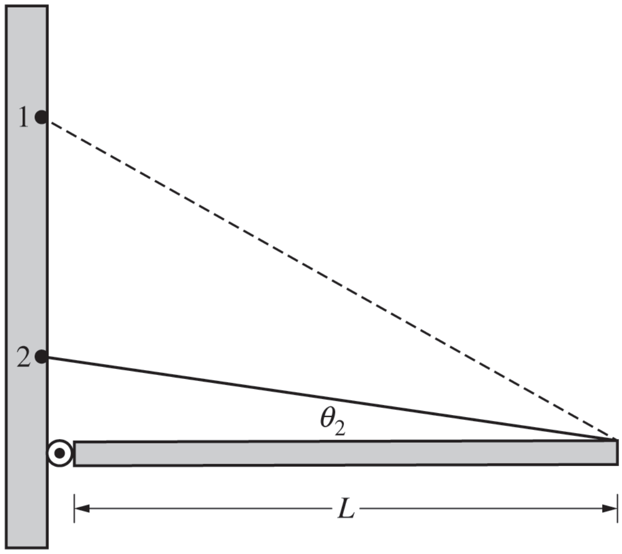



Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is $F_{T1}$. The magnitude of the tension in the string shown in Figure 2 is $F_{T2}$. **Indicate** which of the following correctly compares $F_{T2}$ with $F_{T1}$.

_____ $F_{T2} > F_{T1}$     _____ $F_{T2} < F_{T1}$     _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

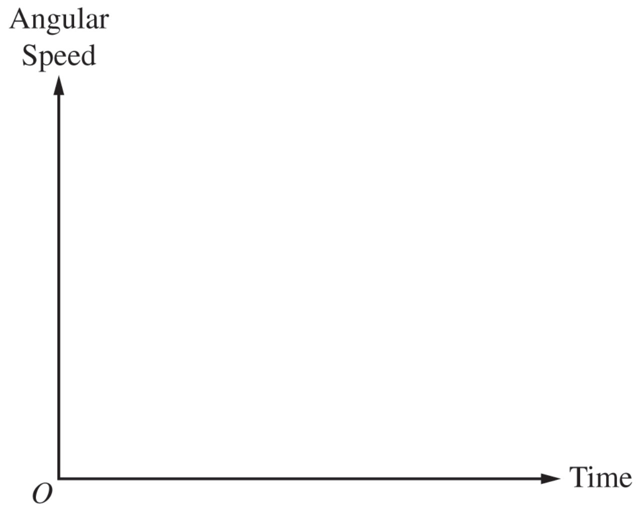
(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of $M$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

**GO ON TO THE NEXT PAGE.**

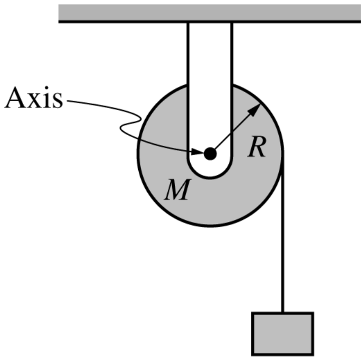
Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.




**GO ON TO THE NEXT PAGE.**




. (7 points, suggested time 13 minutes)

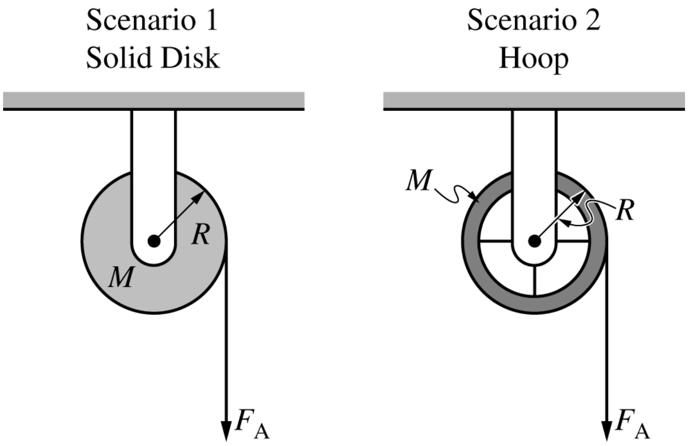
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass $M$ and radius $R$. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is $F_T$.

(a) Determine an expression for the magnitude of the angular acceleration $\alpha_D$ of the disk as the block travels downward. Express your answer in terms of $M$, $R$, $F_T$, and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 4** on this page.



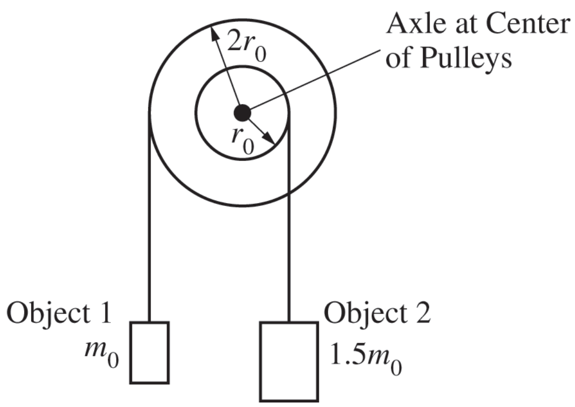

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass $M$ and radius $R$ as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force $F_A$ for the same time interval $\Delta t$, causing the pulleys to rotate without the string slipping. After time interval $\Delta t$, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval $\Delta t$. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 5** on this page.




5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

$m_0$ is the mass of object 1.
$1.5m_0$ is the mass of object 2.
$r_0$ is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_0$, $r_0$, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

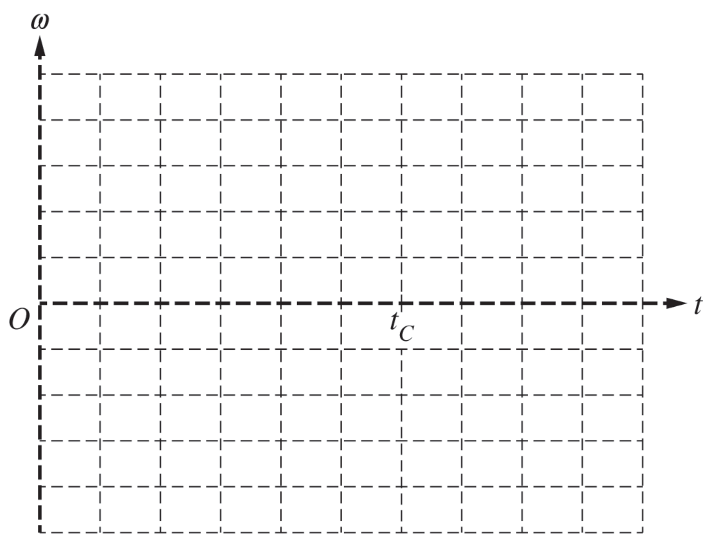
**GO ON TO THE NEXT PAGE.**

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction          ___ Opposite directions

Briefly explain your reasoning.

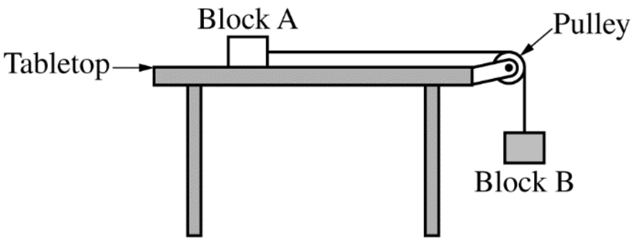
(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.




**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**




2. (12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass $m_A$, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass $m_B$, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Block A　　　　Block B

(c) Derive an equation for the acceleration of the blocks after release in terms of $m_A$, $m_B$, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much <u>less</u> than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

____ Yes ____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when $m_A$ is much <u>less</u> than $m_B$.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is $T_1$. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is $T_2$. How do the two tensions compare to each other?

____ $T_2 > T_1$ ____ $T_2 = T_1$ ____ $T_2 < T_1$

Briefly explain your reasoning.

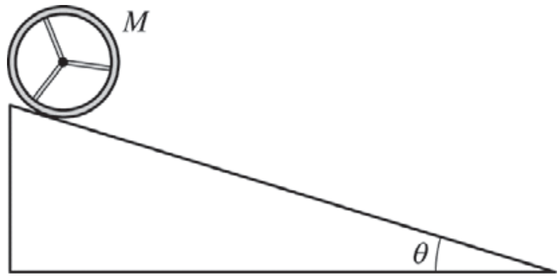
# PHYSICS 1

## Section II

## 5 Questions

## Time—90 minutes

**Directions:** Questions 1, 4 and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

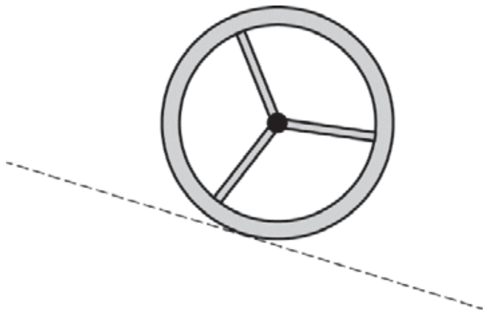



1. (7 points, suggested time 13 minutes)

A wooden wheel of mass $M$, consisting of a rim with spokes, rolls down a ramp that makes an angle $\theta$ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

(a)

i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

(b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of $M$, $\theta$, and physical constants, as appropriate.

(c) In a second experiment on the same ramp, a block of ice, also with mass $M$, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.