

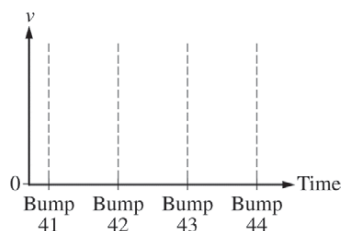
Note: Figure not drawn to scale.

3. (12 points, suggested time 25 minutes)

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg} . This means the time interval taken to move from one bump to the next bump becomes constant.

(a) Consider the cart's motion between bump 41 and bump 44.

- In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.
- Over the same time interval, draw a dashed horizontal line at $v = v_{\text{avg}}$. Label this line " v_{avg} ".



(b) Suppose the distance between the bumps is increased but everything else stays the same.

Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

☐ Greater than ☐ Less than ☐ The same as

Briefly explain your reasoning.

(c) With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ . Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

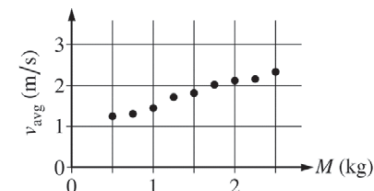
☐ Greater than ☐ Less than ☐ The same as

Briefly explain your reasoning.

- (d) Before deriving an equation for a quantity such as v_{avg} , it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed:

$$v_{\text{avg}} = C \frac{Mg \sin \theta}{d}, \text{ where } C \text{ is a positive constant.}$$

- To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M .



Are these data consistent with the student's equation?

☐ Yes ☐ No

Briefly explain your reasoning.

- Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

☐ Yes ☐ No

Briefly explain your reasoning.