# Past-paper walk-through

Representing and Analyzing SHM ▪ 7.3

eXercise

# Questions in this set

- 2024 Q2
- 2022 Q5
- 2019 Q5
- 2016 Q5
- 2015 Q5

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 2

2024 ▪ Q2

[Figure 1: A stand with a horizontal support arm (labeled "Stand") has a spring hanging vertically from the arm, with a small loop at the bottom of the spring. To the right, a set of stacked cylindrical masses of various sizes (labeled "Cylinders") sits on the table, available for hanging on the spring's loop.]

A student hangs a spring of unknown spring constant $k$ vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

# Question 2

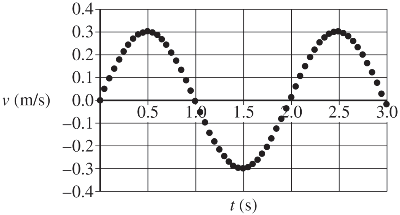
**(a)** (a) Design an experimental procedure the student could use to determine the spring constant $k$ of the spring.
In the following table, list the quantities that would be measured using only the provided equipment in your experiment. Define a symbol to represent each quantity.
In the space below the table, **describe** the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. If needed, you may include a simple diagram with your procedure.

| Quantity to Be Measured | Symbol for Quantity | Equipment for Measurement |
|---|---|---|
| | | Stopwatch<br>Digital scale |

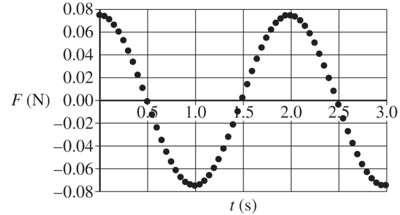
# Question 2

Procedure (and diagram, if needed):

# Question 2



0.4
0.3
0.2
0.1
0.0
–0.1
–0.2
–0.3
–0.4
v (m/s)
0.5
1.0
1.5
2.0
2.5
3.0
t (s)

0.08
0.06
0.04
0.02
0.00
–0.02
–0.04
–0.06
–0.08
F (N)
0.5
1.0
1.5
2.0
2.5
3.0
t (s)

# Question 2

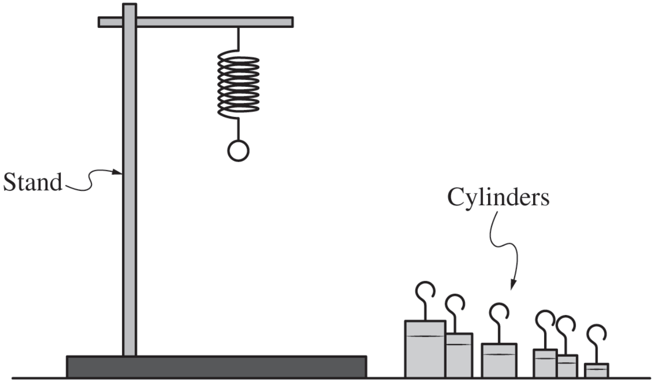



Figure 1

# Question 2

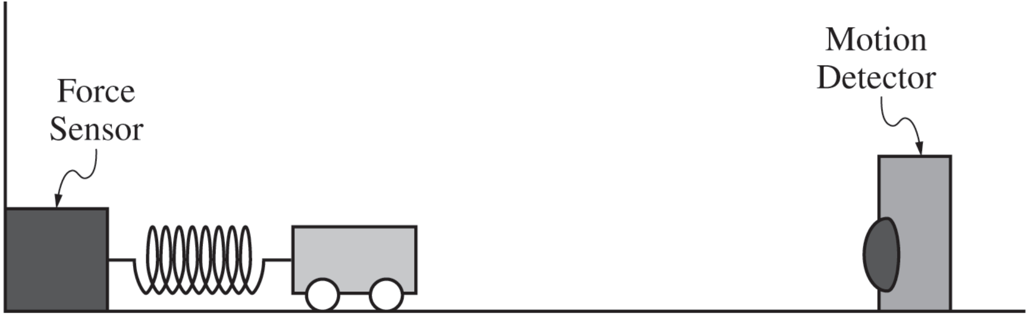



Figure 2

# Question 2

2024 ▪ Q2

[Figure 1: A stand with a horizontal support arm (labeled "Stand") has a spring hanging vertically from the arm, with a small loop at the bottom of the spring. To the right, a set of stacked cylindrical masses of various sizes (labeled "Cylinders") sits on the table, available for hanging on the spring's loop.]

A student hangs a spring of unknown spring constant $k$ vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

**(b)(i)** (b)

i. **Indicate** the quantities that could be plotted to produce a linear graph whose slope can be used to determine the spring constant $k$ of the spring.

Vertical axis: ____________________ Horizontal axis: ____________________

# Question 2

**(b)(ii)** ii. Briefly **describe** how the slope of the graph would be analyzed to determine the spring constant $k$ of the spring.

# Question 2

2024 ▪ Q2

[Figure 1: A stand with a horizontal support arm (labeled "Stand") has a spring hanging vertically from the arm, with a small loop at the bottom of the spring. To the right, a set of stacked cylindrical masses of various sizes (labeled "Cylinders") sits on the table, available for hanging on the spring's loop.]

A student hangs a spring of unknown spring constant $k$ vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

**(c)(i)** [Figure 2: A force sensor is mounted on a wall on the left. A spring connects the force sensor to a cart of mass $m = 0.25$ kg on wheels, sitting on a horizontal track. To the right of the cart is a motion detector facing the cart.]

# Question 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity $v$ of the cart and the force $F$ exerted on the cart by the spring as functions of time $t$.

[Graph 1: velocity $v$ (m/s) vs. time $t$ (s); y-axis from $-0.4$ to $0.4$ in increments of $0.1$; x-axis from $0$ to $3.0$ s with gridlines at $0.5, 1.0, 1.5, 2.0, 2.5, 3.0$. The curve is a smooth cosine-like oscillation: starting near $v \approx 0$ at $t = 0$, rising to a positive peak near $v \approx 0.3$–$0.35$ around $t \approx 0.5$ s, back down through zero near $t \approx 1.0$–$1.2$ s, down to a negative peak near $v \approx -0.3$–$-0.35$ around $t \approx 1.7$ s, back up through zero near

# Question 2

$t \approx 2.2$–$2.4$ s, and up to another positive peak near $t \approx 2.9$–$3.0$ s. Period of oscillation is approximately $1.7$–$1.8$ s.]

[Graph 2: force $F$ (N) vs. time $t$ (s); y-axis from $-0.08$ to $0.08$ in increments of $0.02$; x-axis from $0$ to $3.0$ s with gridlines at $0.5, 1.0, 1.5, 2.0, 2.5, 3.0$. The curve is a smooth oscillation with the same period as the velocity graph, appearing approximately $90°$ out of phase with $v(t)$: starting near $F \approx 0.06$–$0.07$ (near a peak) at $t = 0$, decreasing through zero around $t \approx 0.4$–$0.5$ s, reaching a negative peak near $F \approx -0.07$ around $t \approx 0.8$–$0.9$ s, back through zero near $t \approx 1.3$ s, reaching a positive peak near $t \approx 1.7$–$1.8$ s, and continuing to oscillate with the same period through $t = 3.0$ s.]

i. Using the data in the velocity-time graph, **calculate** the change in kinetic energy of the cart from $t = 0.5$ s to $t = 2.0$ s. Show your steps and substitutions.

# Question 2

**(c)(ii)** ii. Using the data in the force-time graph, **estimate** the change in momentum of the cart from $t = 0.5$ s to $t = 2.5$ s. Briefly **explain** how you arrived at your estimation.
**(c)(iii)** iii. Do the data from the velocity-time graph confirm your estimation from part (c)(ii)? Briefly **explain**.

# Solution 2

## (a) Mark scheme

- Procedure: place a cylinder on the digital scale and record its mass. Hang the cylinder from the spring and pull it down a small distance so the spring is stretched, then release it so it oscillates. Use the stopwatch to measure the time for the cylinder to complete many (e.g. ten) full cycles (from maximum stretch back to maximum stretch), then divide by the number of oscillations to get the period. Repeat the procedure for cylinders of different masses. Points (4 total, 1 each): measuring the mass of at least one cylinder with the digital scale; measuring the period of oscillation with the stopwatch; a procedure that indicates the cylinder is set into oscillatory motion; a procedure that reduces experimental uncertainty (accept any one of: using multiple masses, doing multiple trials with a

# Solution 2

single mass, or measuring multiple oscillations and dividing by the number of oscillations).

# Solution 2

## (b)(i) Mark scheme

- List a linear-izing pair of axes whose slope gives $k$, e.g. Vertical axis: $m$, Horizontal axis: $T^2$ (from $T = 2\pi\sqrt{m/k}$, so $m = \frac{k}{4\pi^2}T^2$). Any of the following pairs earns the point: $m$ vs. $T^2$; $T^2$ vs. $m$; $4\pi^2 m$ vs. $T^2$; $T^2$ vs. $4\pi^2 m$; $\frac{T^2}{4\pi^2}$ vs. $m$; $m$ vs. $\frac{T^2}{4\pi^2}$; $T$ vs. $\sqrt{m}$; $\sqrt{m}$ vs. $T$; $\frac{T}{2\pi}$ vs. $\sqrt{m}$; $\sqrt{m}$ vs. $\frac{T}{2\pi}$; $T$ vs. $2\pi\sqrt{m}$; $2\pi\sqrt{m}$ vs. $T$ (any bullet may also substitute $1/f$ for $T$).

# Solution 2

## (b)(ii) Mark scheme

- Describe how the slope relates to $k$, consistent with the axes chosen in (b)(i). Using $T = 2\pi\sqrt{m/k}$: e.g. for $m$ (vertical) vs. $T^2$ (horizontal), slope $= \dfrac{k}{4\pi^2}$, so $k = (\text{slope})\,4\pi^2$. (Other axis choices give the corresponding relation, e.g. $T^2$ vs. $m$ gives slope $= 4\pi^2/k$ so $k = 4\pi^2/\text{slope}$; $4\pi^2 m$ vs. $T^2$ gives slope $= k$; etc.) Full credit (1 point) for correctly relating the slope of the best-fit line to $k$ given the student's chosen axes.

# Solution 2

## (c)(i)

**Mark scheme**

- Estimate the change in kinetic energy of the cart using $K = \frac{1}{2}mv^2$:
  $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(0.25)(0)^2 - \frac{1}{2}(0.25)(0.3)^2 \approx -0.0113$ J.
  Points: using the kinetic energy equation $K = \frac{1}{2}mv^2$ (1 point); substituting one of the given values — mass $0.25$ kg or initial velocity $0.3$ m/s (1 point); an answer that approximates $\Delta K \approx -0.0113$ J (unit and sign not required) — a correct numeric answer with no supporting work earns this point alone (1 point).

# Solution 2

## (c)(ii)

**Mark scheme**

- Because the cart's speed returns to the same value ($0.3$ m/s) before and after the interaction, the magnitude of the change in momentum is zero. This is estimated from the force-time graph: the area under a force-vs-time curve equals the impulse, i.e. the change in momentum; the area under the curve from $t = 0.5$ s to $t = 2.5$ s is zero (equal positive and negative areas cancel), so $\Delta p \approx 0$. Points: indicating the magnitude of the change in momentum is zero (1 point); indicating that the area under the force-time graph represents the value of the change in momentum (1 point).

# Solution 2

## (c)(iii)

**Mark scheme**

- Yes — the velocity-time graph confirms the estimate from (c)(ii): it shows the velocity is $0.3$ m/s at both $t = 0.5$ s and $t = 2.5$ s, and since momentum is mass times velocity, the momentum is the same at both times, giving a change in momentum of zero. This agrees with the (c)(ii) estimate that $\Delta p \approx 0$. Full credit (1 point) for an explanation that compares the (c)(ii) estimate to the velocity-time data in this way.

# Question 5

2022 ▪ Q5

[Figure: A spring hangs vertically from a ceiling (shown as a hatched surface at top). A lightweight hanger is attached to the lower end of the spring. Below the hanger, a vertical distance of "1.00 m" is marked down to a "Motion Detector" resting on the floor, facing upward directly under the hanger.]

A spring of unknown spring constant $k_0$ is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance $d_0$ from equilibrium and released at time $t = 0$. The motion detector records the height of the

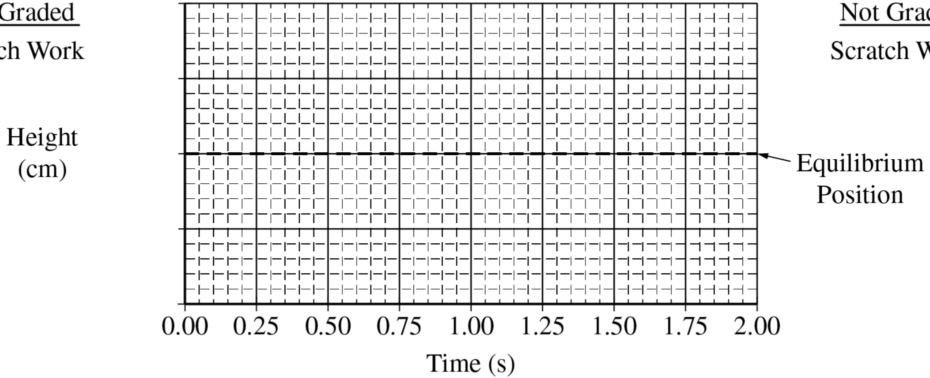
# Question 5

bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.
[Graph: "Height (cm)" on the y-axis from 50 to 70 cm (gridlines at 50, 55, 60, 65, 70), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). A smooth sinusoidal curve oscillates between about 55 cm and 65 cm, starting near a minimum around $t = 0$, with "Equilibrium Position" labeled at the horizontal midline around 60 cm. The curve appears to have a period of about 1.00 s.]
**(a)** Using the information given and information taken from the graph, calculate the spring constant.

# Question 5

Not Graded

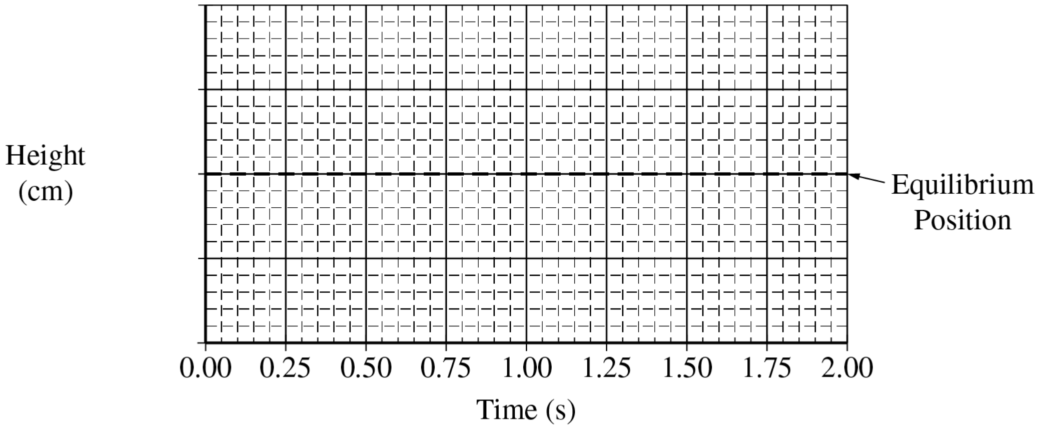
Scratch Work



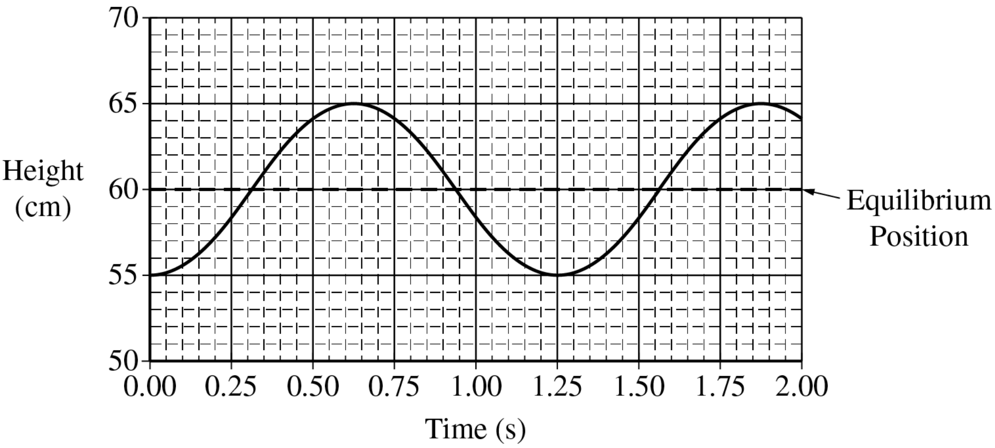

Not Graded

Scratch Work

# Question 5


Height
(cm)
Equilibrium
Position
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Time (s)

# Question 5



70
65
60
55
50
Height (cm)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Time (s)
Equilibrium Position

# Question 5

2022 ▪ Q5

[Figure: A spring hangs vertically from a ceiling (shown as a hatched surface at top). A lightweight hanger is attached to the lower end of the spring. Below the hanger, a vertical distance of "1.00 m" is marked down to a "Motion Detector" resting on the floor, facing upward directly under the hanger.]

A spring of unknown spring constant $k_0$ is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance $d_0$ from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

# Question 5

[Graph: "Height (cm)" on the y-axis from 50 to 70 cm (gridlines at 50, 55, 60, 65, 70), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). A smooth sinusoidal curve oscillates between about 55 cm and 65 cm, starting near a minimum around $t = 0$, with "Equilibrium Position" labeled at the horizontal midline around 60 cm. The curve appears to have a period of about 1.00 s.]

**(b)(i)** At time 0.75 s, the object-spring-Earth system has a total kinetic energy $K_0$ and a total potential energy $U_0$. At 1.13 s, the object-spring-Earth system again has a total kinetic energy $K_0$ and a total potential energy $U_0$.

Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

**(b)(ii)** Briefly explain why the total potential energy of the system is the same at these two times.

# Question 5

2022 ▪ Q5

[Figure: A spring hangs vertically from a ceiling (shown as a hatched surface at top). A lightweight hanger is attached to the lower end of the spring. Below the hanger, a vertical distance of "1.00 m" is marked down to a "Motion Detector" resting on the floor, facing upward directly under the hanger.]

A spring of unknown spring constant $k_0$ is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance $d_0$ from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

# Question 5

[Graph: "Height (cm)" on the y-axis from 50 to 70 cm (gridlines at 50, 55, 60, 65, 70), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). A smooth sinusoidal curve oscillates between about 55 cm and 65 cm, starting near a minimum around $t = 0$, with "Equilibrium Position" labeled at the horizontal midline around 60 cm. The curve appears to have a period of about 1.00 s.]

**(c)** The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

**(c)(i)** Determine the new equilibrium position above the motion detector.

**(c)(ii)** The object is again pulled down the same distance $d_0$ from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.

# Question 5

[Graph: "Height (cm)" on the y-axis (unlabeled scale, to be filled in by student), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). "Equilibrium Position" labeled at the horizontal midline. Blank grid for the student to draw the curve.]
The following graph is provided for scratch work only and will not be graded.
[Two "Not Graded / Scratch Work" grids follow, each labeled "Height (cm)" vs. "Time (s)" from 0.00 to 2.00 s, with "Equilibrium Position" marked at the midline — for student scratch work, not graded.]

# Solution 5

## (a) Mark scheme

- From the graph, the period is T = 1.25 s. Using T = $2\pi\sqrt{m/k}, solvefork: k_0 = \frac{m}{(T/2\pi)^2} = \frac{0.50\ \mathrm{kg}}{(1.25\ \mathrm{s}/2\pi)^2} \approx 12.6\ \mathrm{N/m}. (Alternatesolution: thespringstretches 0.40 mtoreachequilibrium - -sincetheequilibriumheightis 60 cmandtheoriginalunstretchedheightwas 100 cm - -andatequilibriummg = kx, sok = mg/x = 5\ \mathrm{N}/0.40\ \mathrm{m} = 12.5\ \mathrm{N/m}.) Scoring: 1 pointforobtainingaperiodof 1.25 sfromthegraph (or, inthealternatemethod, 1 pointforcorrectlyfinc$

# Solution 5

## (b)(i)

### Mark scheme

- The magnitude of the slope of the height-vs-time graph (the object's speed) is the same at $t=0.75$ s and $t=1.13$ s, so the kinetic energy (which depends only on speed) is the same at both times. (Equivalently: the object is the same distance from the equilibrium position at both times, so its speed – and hence KE – must be the same.) Scoring: 1 point for a valid reason why the kinetic energy is the same at both times.

# Solution 5

## (b)(ii)

**Mark scheme**

- The total mechanical energy of the object-spring-Earth system is constant, so since the kinetic energy K_0 is the same at both times, the potential energy U_0 (= total energy minus K_0) must also be the same at both times. (Equivalently: equal energy is transferred between gravitational and spring potential energy as the object oscillates.) Scoring: 1 point for a valid reason why the total potential energy is the same at both times.

# Solution 5

## (c)

**Mark scheme**

- This is the introductory stem describing the repeated experiment with a new spring of constant 4k_0 (same natural length) – it poses no scorable question itself; the two scored sub-parts are (c)(i) and (c)(ii), which together carry all 3 points allocated to part (c) in the scoring guidelines.

# Solution 5

## (c)(i)

**Mark scheme**

- The new equilibrium position is 90 cm (0.90 m) above the motion detector – with a spring 4 times stiffer, the same weight stretches it only 1/4 as far (0.10 m instead of 0.40 m), so the equilibrium position rises from 60 cm to 90 cm. Scoring: 1 point for writing 90 cm or 0.90 m.

# Solution 5

## (c)(ii) Mark scheme

- Draw a sinusoidal curve centered on the new equilibrium height of 90 cm, with the same amplitude as the original graph (the object is pulled down the same distance $d_0 fromequilibrium), andwithhalftheperiodoftheoriginalmotion(since T = 2\pi\sqrt{m/k} andkincreasedbyafactorof 4, T_new = T_old/2).Labeltheverticalaxiswithanappropriatenumericalscalereflectingthe 90 cmcenterandsamea$
$1 pointforboth(agraphwiththesameamplitudeastheoriginal AND centeredonthenew 90 cmequilibriu$

# Question 5

2019 ▪ Q5

[Diagram: A tuning fork labeled "Tuning Fork" is shown to the left of a cylindrical tube labeled "Tube", open at both ends (shown with dashed lines at the right end indicating the opening). The length of the tube is labeled $L$, marked by a double-headed arrow spanning its full length.]

A tuning fork vibrating at 512 Hz is held near one end of a tube of length $L$ that is open at both ends, as shown above. The column of air in the tube resonates at its fundamental frequency. The speed of sound in air is 340 m/s.

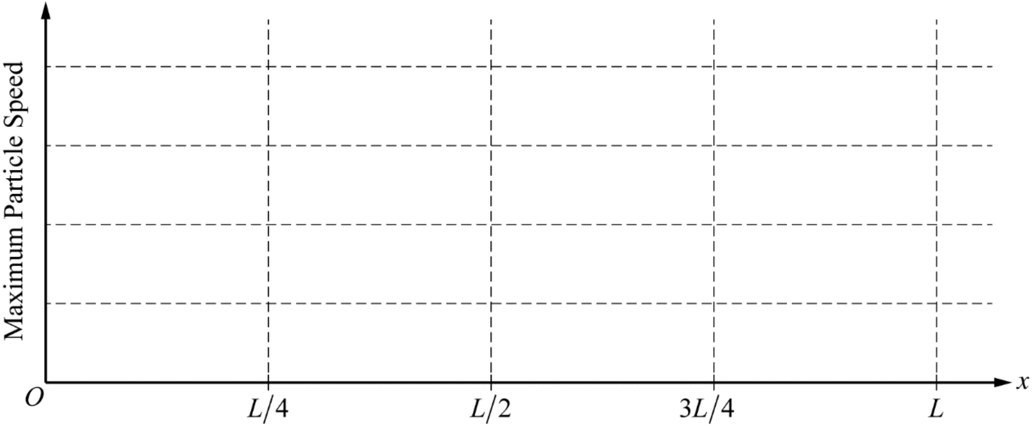
**(a)** Calculate the length $L$ of the tube.

**(b)** The column of air in the tube is still resonating at its fundamental frequency. On the axes below, sketch a graph of the maximum speed of air molecules as they oscillate in the tube, as a function of position $x$, from $x = 0$ (left end of tube) to $x = L$ (right end of tube). (Ignore random thermal motion of the air molecules.)

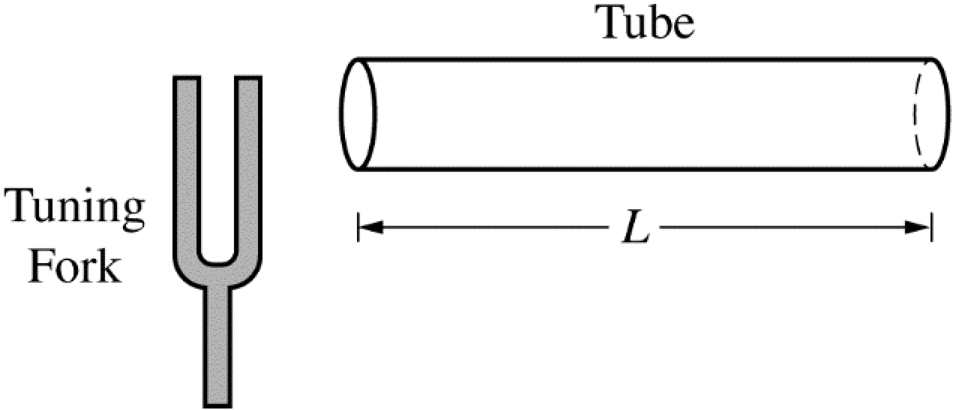
# Question 5

[Graph: y-axis "Maximum Particle Speed", no numeric scale shown, with four evenly spaced horizontal dashed gridlines above 0; x-axis labeled $x$ with tick marks at $O$ (origin), $L/4$, $L/2$, $3L/4$, $L$; axes otherwise blank dashed for the student to sketch on.]
**(c)** The right end of the tube is now capped shut, and the tube is placed in a chamber that is filled with another gas in which the speed of sound is 1005 m/s. Calculate the new fundamental frequency of the tube.

# Question 5



# Question 5



# Solution 5

## (a)

**Mark scheme**

- For a tube open at both ends resonating at its fundamental frequency, $L = \lambda/2$. $\lambda = v/f = (340 \text{ m/s})/(512 \text{ Hz}) = 0.66$ m, so $L = \lambda/2 = 0.33$ m. Points: 1 for using $\lambda = v/f$; 1 for correctly taking L as half of the calculated wavelength with correct units, $L = 0.33$ m.

# Solution 5

## (b) Mark scheme

- Sketch of maximum air-molecule speed vs. position x (tube open at both ends, fundamental mode): a symmetric curve with MAXIMA (velocity antinodes) at both ends $x = 0$ and $x = L$, decreasing smoothly and symmetrically to a single NODE (zero) exactly at the midpoint $x = L/2$, and nonnegative everywhere – reflecting that both open ends are displacement/velocity antinodes and the fundamental standing wave has exactly one node, at the center. Points: 1 for a curve with a node (value zero) at $x = L/2$; 1 for a curve with maxima at $x = 0$ and $x = L$ and no other local maxima; 1 for a nonhorizontal curve that is symmetric about $x = L/2$ and nonnegative everywhere.

# Solution 5

## (c)

**Mark scheme**

- With the right end now capped (closed), the tube is closed-open, so the fundamental wavelength becomes $\lambda = 4L = 4(0.33 \text{ m}) = 1.32 \text{ m}$ (rather than $2L$). Using the new speed of sound, $f = v/\lambda = (1005 \text{ m/s})/(1.32 \text{ m}) \approx 757$ Hz. Correct final answer: 757 Hz. Points: 1 for an indication that the fundamental wavelength is now $4L$; 1 for substituting the new sound speed into $v = \lambda f$ to obtain the numeric answer.

# Question 5

2016 ▪ Q5

[Figure at top of page: Two diagrams of a rope hanging vertically from an oscillator mounted at the top. The figure on the left shows the rope hanging straight down (oscillator off), with two labeled points $P$ (upper) and $Q$ (lower) marked on the rope. The figure on the right shows the same rope forming a standing wave (oscillator on at a certain frequency): the rope traces out two loops (antinodes) with the envelope shown as solid and dashed curves bulging outward from the vertical, node at the top (near the oscillator), a node between the two loops, and a node at the free bottom end; point $P$ is again marked, located near the top loop, and point $Q$ is marked, located near the node between the two loops.]
The figure above on the left shows a uniformly thick rope hanging vertically from an oscillator that is turned off. When the oscillator is on and set at a certain frequency,

# Question 5

the rope forms the standing wave shown above on the right. $P$ and $Q$ are two points on the rope.

**(a)** The tension at point $P$ is greater than the tension at point $Q$. Briefly explain why.

**(b)** A student hypothesizes that increasing the tension in a rope increases the speed at which waves travel along the rope. In a clear, coherent paragraph-length response that may also contain figures and/or equations, explain why the standing wave shown above supports the student's hypothesis.

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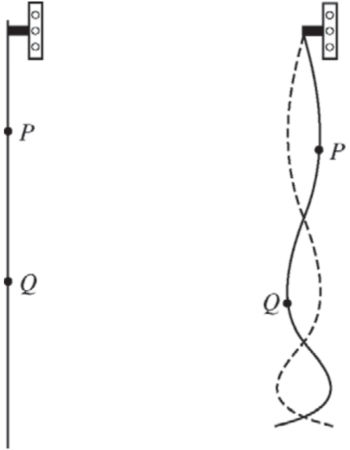

# Solution 5

## (a) Mark scheme

- Explain why the tension at point $P$ is greater than the tension at point $Q$. Reasoning: there is more rope (and hence more weight) hanging below $P$ than below $Q$; at any point in the rope, the tension there supports/counteracts the weight of everything hanging below that point, and since more weight hangs below $P$, the tension there must be greater. Example: the section of rope below $P$ has an upward force from the rope above it and a downward gravitational force; the same is true below $Q$. Because the gravitational force is greater on the longer section (the one below $P$), the upward force — the tension — must be greater at $P$. Scoring: 1 pt for indicating there is more rope or weight below one point than

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the other; 1 pt for indicating (explicitly or implicitly) that the tension at a point counteracts or supports the weight hanging below that point.

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## (b) Mark scheme

- Paragraph-length response explaining why the standing wave shown supports the student's hypothesis that increasing the tension in a rope increases the wave speed. The standing wave has a longer wavelength near the top of the rope than near the bottom (loops more widely spaced at the top, closer together at the bottom). Because it is a single standing wave on a continuous rope, the frequency $f$ is the same throughout the rope. Using $v = \lambda f$, a longer wavelength at the same frequency means the wave speed $v$ is greater near the top than near the bottom. From part (a), the tension is greater near the top of the rope (more weight hangs below that region) than near the bottom. So the region of greater tension (top) is also the region of greater wave speed, and the region of lesser tension (bottom)

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has the lesser wave speed — which supports the hypothesis that greater tension means greater wave speed. Scoring: 1 pt for indicating the wavelength is longer near the top of the rope (or shorter near the bottom); 1 pt for indicating (explicitly or implicitly) that the frequency is the same throughout the rope; 1 pt for using $v = \lambda f$ to conclude the wave speed is greater near the top (or less near the bottom), based on the difference in wavelength; 1 pt for indicating (explicitly or implicitly), consistent with part (a), that the tension is greater near the top (or less near the bottom); 1 pt for a response with sufficient paragraph structure, as described in the published requirements for the paragraph-length response.

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[Figure: Side view of a string with one end attached to an oscillator and the other end attached to a block, with the string passing over a pulley mounted at the edge of a table. The distance between the oscillator and the pulley is labeled $L$. The block (mass $M$) hangs below the pulley.]
The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, $A$, $B$, $C$, and $D$, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency $f$. The distance between each oscillator and pulley $L$ is the same, and the mass $M$ of each block is the same. However, the fundamental frequency of each string is different.

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[Figure: Top view of the four parallel string setups, each spanning the same distance $L$ from its oscillator to its pulley, each with a hanging block of mass $M$. Labeled fundamental frequencies: $f_A = 400$ Hz, $f_B = 382$ Hz, $f_C = 364$ Hz, $f_D = 350$ Hz.]

The equation for the velocity $v$ of a wave on a string is $v = \sqrt{\dfrac{F_T}{m/L}}$, where $F_T$ is the tension of the string and $m/L$ is the mass per unit length (linear mass density) of the string.

**(a)** What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

**(b)** A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

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**(c)** The frequency of the oscillator connected to string $D$ is changed so that the string vibrates in its second harmonic. On the side view of string $D$ below, mark and label the points on the string that have the greatest average vertical speed.
[Figure: Side view of string $D$, now labeled $f_D = 700$ Hz, drawn taut between the oscillator and the pulley with a dashed pattern of evenly spaced vertical tick marks along its length, for the student to mark the antinode points on. Block hangs below the pulley as before.]

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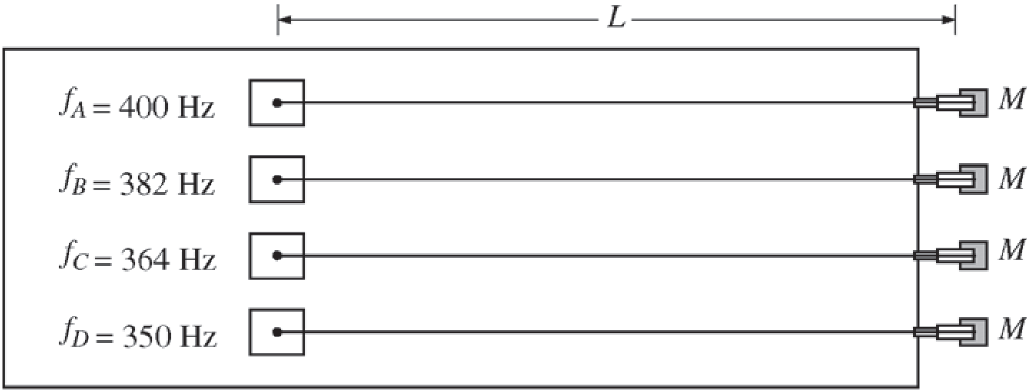


Top View

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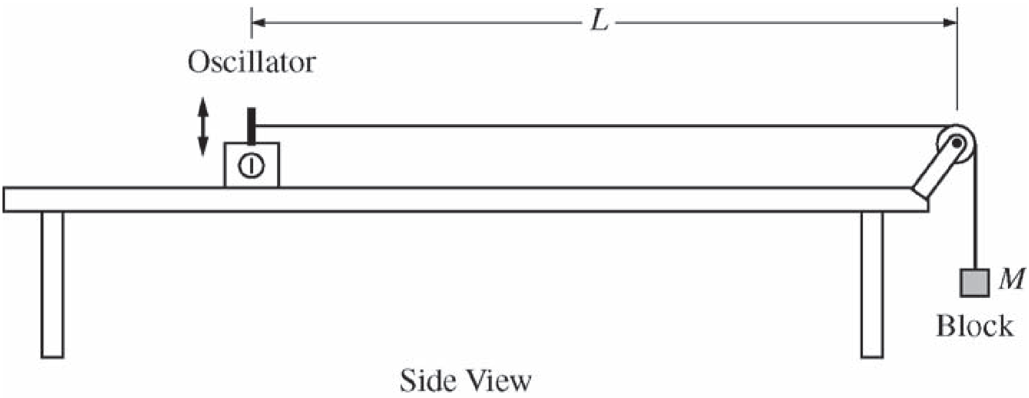

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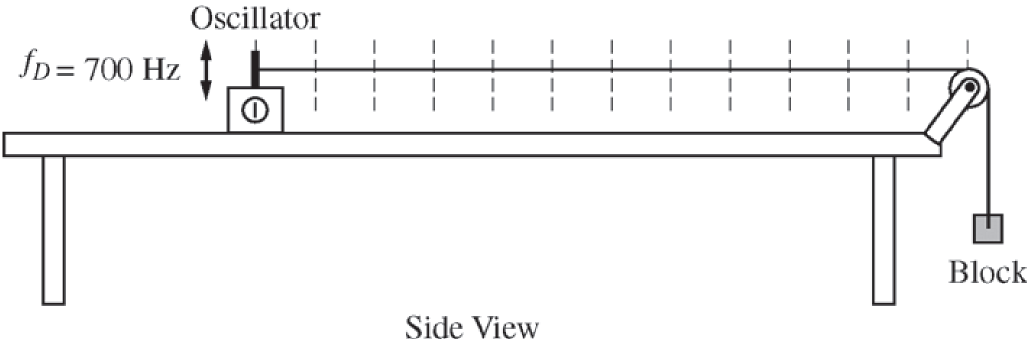

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## (a) Mark scheme

- Four strings of equal length $L$, each vibrating at a different fundamental frequency, each under tension from an identical hanging mass $M$. Since all strings have the same length, and the fundamental wavelength depends only on length ($\lambda_1 = 2L$), all four waves have the SAME wavelength. Since the wavelengths are equal but the frequencies differ, and $v = f\lambda$, the wave SPEEDS must differ between strings. Since the string tensions are equal (same mass $M$ on each) and $v = \sqrt{F_T/(m/L)}$, the LINEAR MASS DENSITIES $m/L$ of the four strings must differ to produce the different wave speeds (equivalently: different linear mass densities give different vertical accelerations of a string element under the same tension, producing different frequencies). Scoring (3 points, 1 each): (1) reasoning that

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equal string lengths give equal fundamental wavelengths ($\lambda_1 = 2L$); (2) reasoning that equal wavelengths with different frequencies imply different wave velocities; (3) reasoning that equal tensions (same $M$) require different linear mass densities to give different velocities, via $v = \sqrt{F_T/(m/L)}$. Note: responses may describe the physical difference between strings in various equivalent ways (different linear mass density / total mass / thickness of the same material).

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## (b) Mark scheme

- Combine $v = f\lambda$ with $v = \sqrt{F_T/(m/L)}$ to get $f = \frac{1}{\lambda}\sqrt{\frac{F_T}{m/L}}$, i.e. $f \propto \sqrt{1/(m/L)}$ – frequency is proportional to the SQUARE ROOT of the inverse linear mass density, not directly proportional to it. So a graph of frequency vs. $1/(m/L)$ would NOT be linear (it would curve like a square root). Scoring (2 points, 1 each): (1) combining $v = f\lambda$ with $v = \sqrt{F_T/(m/L)}$ (or referencing an equivalent combined equation from part (a)); (2) indicating how the combined equation shows frequency is not proportional to (not linear in) the inverse of the linear mass density.

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## (c) Mark scheme

- String D ($f_D = 700$ Hz) is driven at its SECOND harmonic, so its wavelength is $\lambda_2 = L$ (the string shows 2 antinodes / loops, with nodes at both ends and at the midpoint). The points of greatest average vertical speed are the ANTINODES of this standing wave, located at $\frac{1}{4}L$ and $\frac{3}{4}L$ from the oscillator. Scoring (2 points, 1 each): (1) any indication of the second harmonic on the string (a wave shape such that $\lambda_2 = L$); (2) marking points at the antinodes of the second harmonic (or of whatever standing wave was drawn). Full credit requires two points located at one-fourth and three-fourths of the string's length from the oscillator; one point is deducted for each incorrectly marked point on the figure.