Past-paper walk-through

Frequency and Period of SHM ▪ 7.2

eXercise

Questions in this set

- 2024 Q4
- 2023 Q1
- 2022 Q5
- 2019 Q5
- 2015 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2024 ▪ Q4

[Figure showing a pendulum: a rigid horizontal support bracket attached to a wall/ceiling structure, from which a string hangs. The string makes angle θ with the vertical dashed line. At the end of the string is a small sphere labeled "Small Sphere", at position A. A dashed circle labeled B marks the lowest point of the swing, directly below the pivot along the vertical dashed line.]

A simple pendulum consists of a small sphere that hangs from a string with negligible mass. The top end of the string is fixed. The sphere is pulled to Point A so that the string makes a small angle θ with the vertical, as shown. The sphere is then released from rest and swings through its lowest point at Point B. The work done on the sphere by Earth between points A and B is W_E.

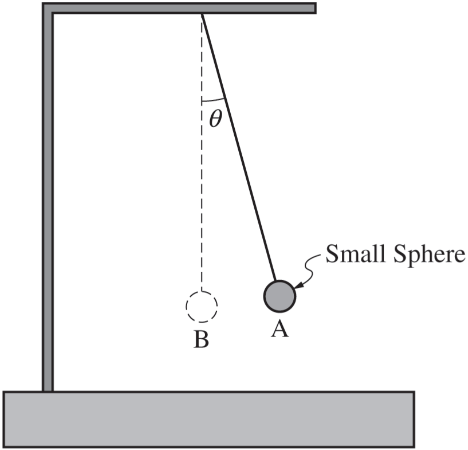
The pendulum is then taken to Planet X. The mass of Planet X is the same as the mass of Earth, but the radius of Planet X is greater than the radius of Earth. The

Question 4

sphere is again brought to Point A (displaced θ from the vertical), released from rest, and swings through its lowest point at Point B. The work done on the sphere by Planet X between points A and B is W_X.

(a) (a) **Justify** why W_X is less than W_E.

Question 4



Question 4

2024 ▪ Q4

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The pendulum is then taken to Planet X. The mass of Planet X is the same as the mass of Earth, but the radius of Planet X is greater than the radius of Earth. The sphere is again brought to Point A (displaced θ from the vertical), released from rest, and swings through its

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lowest point at Point B. The work done on the sphere by Planet X between points A and B is W_X.

(b) A new pendulum is made by hanging the same small sphere from a different string with negligible mass. The new string is slightly elastic, and the length of the string may increase or decrease depending on the tension applied to the string. On Earth, when the sphere is again displaced θ from the vertical and released from rest, the new pendulum oscillates with period T_E.

The new pendulum is then taken to a different planet, Planet Y. The radius of Planet Y is the same as the radius of Earth, but the mass of Planet Y is larger than the mass of Earth. On Planet Y, when the sphere is again displaced from the vertical and released from rest, the new pendulum oscillates with period T_Y.

(b) In a clear, coherent paragraph-length response that may also contain drawings, **explain** how T_Y <u>could be larger</u> than T_E but also <u>could be smaller</u> than T_E.

Solution 4

(a) Mark scheme

- Because Planet X has a larger radius than Earth (same mass sphere/planet setup), the gravitational force (equivalently the gravitational field strength or acceleration due to gravity) is smaller on Planet X. Since the sphere travels the same vertical distance in both scenarios, and work done equals force times distance, the smaller gravitational force on Planet X means less work is done on the sphere there — so $W_X < W_E$. Points: indicating one of — gravitational force, gravitational field strength, or acceleration due to gravity would be smaller for the greater radius (1 point); indicating one of — the sphere travels the same vertical distance in both scenarios, the work done depends on the magnitude of the gravitational force, or the change in gravitational potential energy is less on Planet X (1 point).

Solution 4

(b) Mark scheme

- Argument: the pendulum period is $T = 2\pi\sqrt{\ell/g}$. On Planet Y, since the planet's mass is larger, the gravitational force on the sphere (and hence its weight, and the gravitational field strength g) is larger than on Earth, so the sphere experiences a larger acceleration due to gravity on Planet Y. Because g is in the denominator of the period equation, a larger g alone would tend toward a smaller period. However, the larger gravitational force on the sphere also means a greater stretching force on the (slightly elastic) string, so the string — and hence the pendulum length ℓ — could increase. Since T increases with ℓ, a longer string could push the period back up. So the net effect on T depends on which change (larger g vs. longer ℓ) dominates. Points (5 total, 1 each): relating a larger

Solution 4

planetary mass to a larger weight of the sphere / larger g / larger gravitational field strength; indicating the period is inversely related to g (or gravitational field strength); indicating the amount of string stretch depends on the weight of the sphere / g / gravitational field strength; relating the length of the string to the period of the pendulum; and an overall logical, relevant, internally consistent argument that addresses the question and follows the paragraph-response format.

Question 1

2023 ▪ Q1

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.
[Figure 1: A cart attached to a spring coiled between a wall on the left and the cart on the right, resting on a horizontal surface. Positions marked below: $x = -L$ (at the wall end), $x = 0$ (middle), $x = +L$ (at the cart's rest position shown).]
[Figure 2: Graph of kinetic energy K (J) on the vertical axis, from 0 to 6, versus potential energy U (J) on the horizontal axis, from 0 to 6. A straight diagonal line runs from approximately $(0, 4)$ down to $(4, 0)$, i.e., the line has x-intercept and y-intercept both equal in value.]

Question 1

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

Question 1

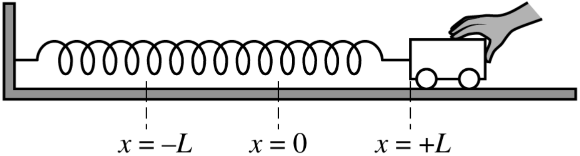

Figure 1

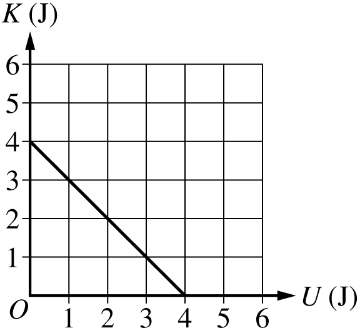

Figure 2

Question 1

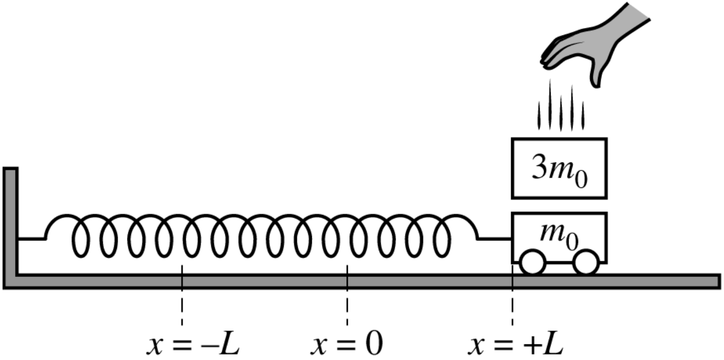

Figure 3

Question 1

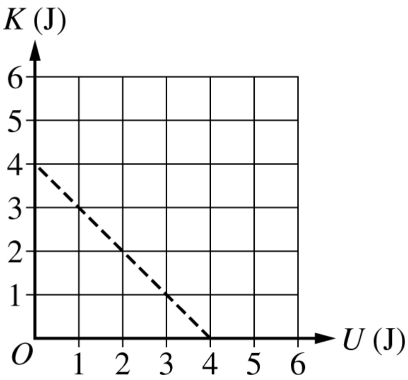

Figure 4

Question 1

2023 ▪ Q1

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

[Figure 1: A cart attached to a spring coiled between a wall on the left and the cart on the right, resting on a horizontal surface. Positions marked below: $x = -L$ (at the wall end), $x = 0$ (middle), $x = +L$ (at the cart's rest position shown).]

[Figure 2: Graph of kinetic energy K (J) on the vertical axis, from 0 to 6, versus potential energy U (J) on the horizontal axis, from 0 to 6. A straight diagonal line runs from approximately $(0, 4)$ down to $(4, 0)$, i.e., the line has x-intercept and y-intercept both equal in value.]

Question 1

(b) [Continuation of Question 1.] When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.
[Figure 3: Same spring-and-wall setup as Figure 1, but now a block labeled $3m_0$ falls (shown with motion marks) onto a cart labeled m_0 sitting atop the cart at position $x = +L$. Positions marked: $x = -L$, $x = 0$, $x = +L$.]
The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

Question 1

2023 ▪ Q1

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[Figure 2: Graph of kinetic energy K (J) on the vertical axis, from 0 to 6, versus potential energy U (J) on the horizontal axis, from 0 to 6. A straight diagonal line runs from approximately $(0, 4)$ down to $(4, 0)$, i.e., the line has x-intercept and y-intercept both equal in value.]

Question 1

(c)(i) [Continuation of Question 1.] The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.
[Figure 4: Graph of kinetic energy K (J), vertical axis 0 to 6, versus potential energy U (J), horizontal axis 0 to 6. A dashed diagonal line runs from approximately $(0, 4)$ down to $(4, 0)$ — identical to the solid line in Figure 2.]
i. Briefly explain why the two graphs must be the same, using physics principles.
(c)(ii) ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

Solution 1

(a)

Mark scheme

- Maximum kinetic energy equals maximum potential energy (both 4 J) because energy is conserved in the cart-spring system: the graph's x-intercept (max U, $K = 0$) and y-intercept (max K, $U = 0$) both represent the same total mechanical energy. Full credit for simply stating 'conservation of energy.'

Solution 1

(b) Mark scheme

- Set up the ratio of frequencies (or periods): $T = 2\pi\sqrt{m/k}$, $f = \frac{1}{2\pi}\sqrt{k/m}$, so $\frac{f_2}{f_1} = \sqrt{\frac{m_1}{m_2}}$ (1 point for using the frequency/period equation in a valid ratio — any equivalent orientation of f_1/f_2, T_1/T_2, etc. is accepted). Substitute the total mass after the block sticks, $4m_0$ (since $m_0 + 3m_0 = 4m_0$), for m_2: $\frac{f_2}{f_1} = \sqrt{\frac{m_0}{4m_0}} = \frac{1}{2}$ (1 point for correctly substituting the total mass $4m_0$ into the ratio). Final answer: the new frequency is half the old frequency, $f_2/f_1 = 1/2$ (equivalently $T_1/T_2 = 1/2$).

Solution 1

(c)(i)

Mark scheme

- Accept any one valid explanation: (1) no (net external) work is done on the system, so its total mechanical energy is unchanged; (2) the maximum spring potential energy does not depend on the mass of the system, so it is unchanged when the block is added; (3) the force exerted on the system (by the wall/spring at the point of interest) is perpendicular to the direction of motion. Example: 'The maximum potential energy of the system does not depend upon the mass of the system, therefore there will be no change when the block is added.'

Solution 1

(c)(ii) Mark scheme

- Sketch a single solid straight line on the K vs U graph from $(U, K) = (0, 1)$ to $(4, 0)$. Three points: (1) the line's horizontal (U) intercept is the same as the original dashed graph's, at $U = 4$ J (max spring PE is unchanged — it doesn't depend on mass); (2) the line's vertical (K) intercept is less than the original graph's vertical intercept (max KE decreases, since the perfectly inelastic collision between the block and cart dissipates some kinetic energy); (3) the vertical intercept is the exact, correct value of 1 J (total mechanical energy after the collision, found from conservation of momentum in the collision then energy conservation in the subsequent oscillation).

Question 5

2022 ▪ Q5

[Figure: A spring hangs vertically from a ceiling (shown as a hatched surface at top). A lightweight hanger is attached to the lower end of the spring. Below the hanger, a vertical distance of "1.00 m" is marked down to a "Motion Detector" resting on the floor, facing upward directly under the hanger.]

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the

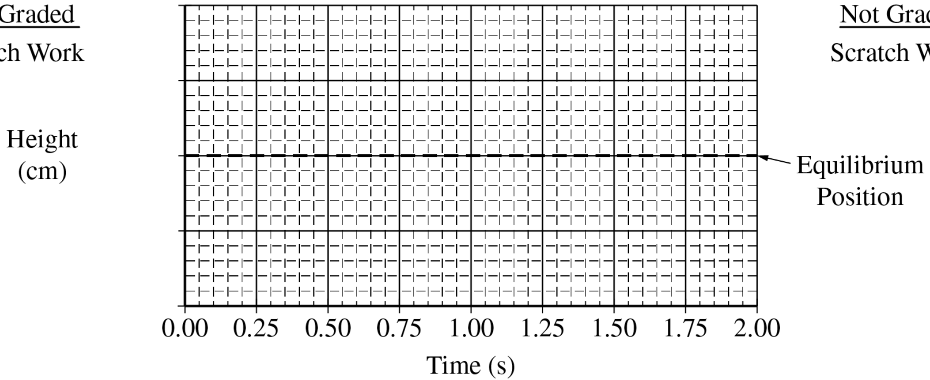
Question 5

bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.
[Graph: "Height (cm)" on the y-axis from 50 to 70 cm (gridlines at 50, 55, 60, 65, 70), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). A smooth sinusoidal curve oscillates between about 55 cm and 65 cm, starting near a minimum around $t = 0$, with "Equilibrium Position" labeled at the horizontal midline around 60 cm. The curve appears to have a period of about 1.00 s.]
(a) Using the information given and information taken from the graph, calculate the spring constant.

Question 5

Not Graded

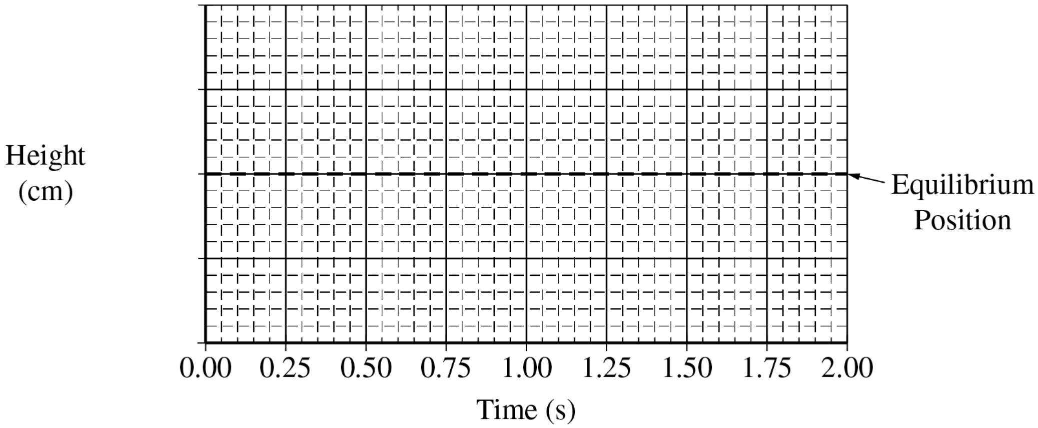
Scratch Work



Not Graded

Scratch Work

Question 5


Height
(cm)
Equilibrium
Position
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Time (s)

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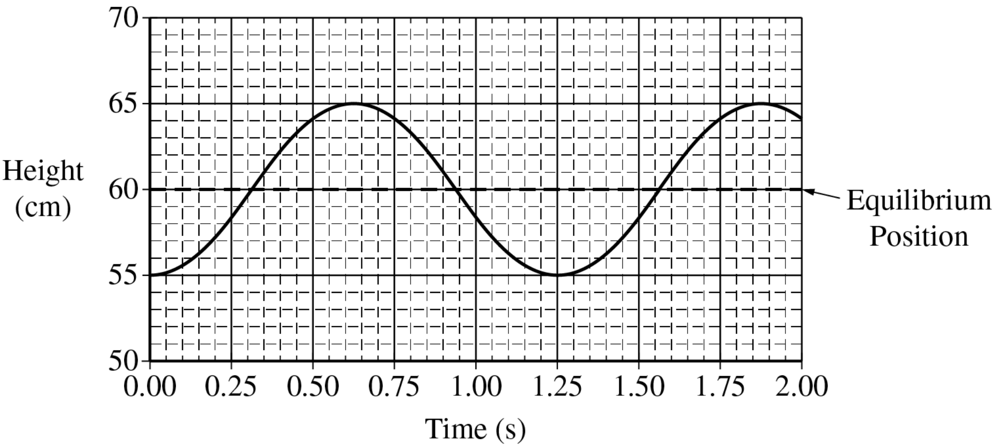

70
65
60
55
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Height
(cm)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Time (s)
Equilibrium
Position

Question 5

2022 ▪ Q5

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A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

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[Graph: "Height (cm)" on the y-axis from 50 to 70 cm (gridlines at 50, 55, 60, 65, 70), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). A smooth sinusoidal curve oscillates between about 55 cm and 65 cm, starting near a minimum around $t = 0$, with "Equilibrium Position" labeled at the horizontal midline around 60 cm. The curve appears to have a period of about 1.00 s.]

(b)(i) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

(b)(ii) Briefly explain why the total potential energy of the system is the same at these two times.

Question 5

2022 ▪ Q5

[Figure: A spring hangs vertically from a ceiling (shown as a hatched surface at top). A lightweight hanger is attached to the lower end of the spring. Below the hanger, a vertical distance of "1.00 m" is marked down to a "Motion Detector" resting on the floor, facing upward directly under the hanger.]

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Question 5

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(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

(c)(i) Determine the new equilibrium position above the motion detector.

(c)(ii) The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.

Question 5

[Graph: "Height (cm)" on the y-axis (unlabeled scale, to be filled in by student), "Time (s)" on the x-axis from 0.00 to 2.00 s (gridlines at 0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00). "Equilibrium Position" labeled at the horizontal midline. Blank grid for the student to draw the curve.]
The following graph is provided for scratch work only and will not be graded.
[Two "Not Graded / Scratch Work" grids follow, each labeled "Height (cm)" vs. "Time (s)" from 0.00 to 2.00 s, with "Equilibrium Position" marked at the midline — for student scratch work, not graded.]

Solution 5

(a) Mark scheme

- From the graph, the period is T = 1.25 s. Using T = $2\pi\sqrt{m/k}, solvefork: k_0 = \frac{m}{(T/2\pi)^2} = \frac{0.50\ \mathsf{kg}}{(1.25\ \mathsf{s}/2\pi)^2} \approx 12.6\ \mathsf{N/m}.(Alternatesolution:$ $thespringstretches 0.40 mtoreachequilibrium -$ $-sincetheequilibriumheightis 60 cmandtheoriginalunstretchedheightwas 100 cm -$ $-andatequilibriummg = kx, sok = mg/x = 5\ \mathsf{N}/0.40\ \mathsf{m} = 12.5\ \mathsf{N/m}.)Scoring:$ $1 pointforobtainingaperiodof 1.25 sfromthegraph(or, inthealternatemethod, 1 pointforcorrectlyfinc$

Solution 5

(b)(i)

Mark scheme

- The magnitude of the slope of the height-vs-time graph (the object's speed) is the same at $t=0.75$ s and $t=1.13$ s, so the kinetic energy (which depends only on speed) is the same at both times. (Equivalently: the object is the same distance from the equilibrium position at both times, so its speed – and hence KE – must be the same.) Scoring: 1 point for a valid reason why the kinetic energy is the same at both times.

Solution 5

(b)(ii)

Mark scheme

- The total mechanical energy of the object-spring-Earth system is constant, so since the kinetic energy K_0 is the same at both times, the potential energy U_0 ($=$ total energy minus K_0) must also be the same at both times. (Equivalently: equal energy is transferred between gravitational and spring potential energy as the object oscillates.) Scoring: 1 point for a valid reason why the total potential energy is the same at both times.

Solution 5

(c)

Mark scheme

- This is the introductory stem describing the repeated experiment with a new spring of constant 4k_0 (same natural length) – it poses no scorable question itself; the two scored sub-parts are (c)(i) and (c)(ii), which together carry all 3 points allocated to part (c) in the scoring guidelines.

Solution 5

(c)(i)

Mark scheme

- The new equilibrium position is 90 cm (0.90 m) above the motion detector – with a spring 4 times stiffer, the same weight stretches it only 1/4 as far (0.10 m instead of 0.40 m), so the equilibrium position rises from 60 cm to 90 cm. Scoring: 1 point for writing 90 cm or 0.90 m.

Solution 5

(c)(ii) Mark scheme

- Draw a sinusoidal curve centered on the new equilibrium height of 90 cm, with the same amplitude as the original graph (the object is pulled down the same distance $\mathrm{d}_0 fromequilibrium), andwithhalftheperiodoftheoriginalmotion(since T = 2\pi\sqrt{m/k} andkincreasedbyafactorof 4, T_new = T_old/2). Labeltheverticalaxiswithanappropriatenumericalscalereflectingthe 90 cmcenterandsamea$
$1 pointforboth(agraphwiththesameamplitudeastheoriginalANDcenteredonthenew 90 cmequilibriu$

Question 5

2019 ▪ Q5

[Diagram: A tuning fork labeled "Tuning Fork" is shown to the left of a cylindrical tube labeled "Tube", open at both ends (shown with dashed lines at the right end indicating the opening). The length of the tube is labeled L, marked by a double-headed arrow spanning its full length.]

A tuning fork vibrating at 512 Hz is held near one end of a tube of length L that is open at both ends, as shown above. The column of air in the tube resonates at its fundamental frequency. The speed of sound in air is 340 m/s.

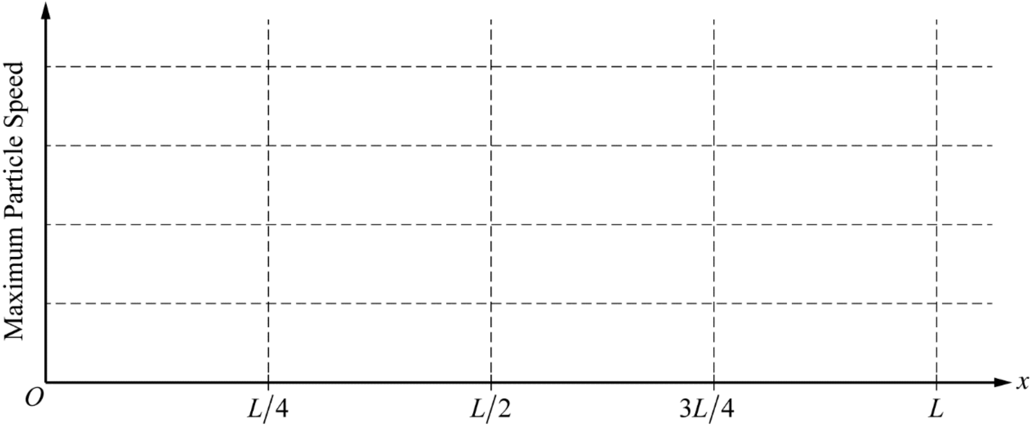
(a) Calculate the length L of the tube.

(b) The column of air in the tube is still resonating at its fundamental frequency. On the axes below, sketch a graph of the maximum speed of air molecules as they oscillate in the tube, as a function of position x, from $x = 0$ (left end of tube) to $x = L$ (right end of tube). (Ignore random thermal motion of the air molecules.)

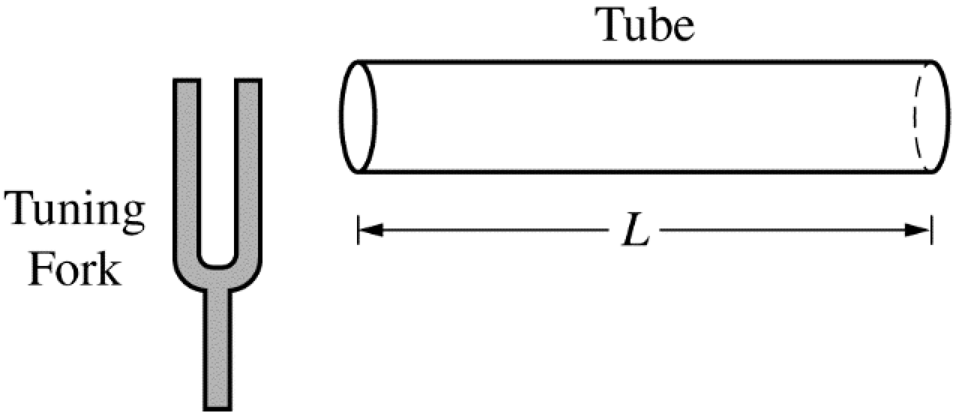
Question 5

[Graph: y-axis "Maximum Particle Speed", no numeric scale shown, with four evenly spaced horizontal dashed gridlines above 0; x-axis labeled x with tick marks at O (origin), $L/4$, $L/2$, $3L/4$, L; axes otherwise blank dashed for the student to sketch on.]
(c) The right end of the tube is now capped shut, and the tube is placed in a chamber that is filled with another gas in which the speed of sound is 1005 m/s. Calculate the new fundamental frequency of the tube.

Question 5



Question 5



Solution 5

(a)

Mark scheme

- For a tube open at both ends resonating at its fundamental frequency, $L = \lambda/2$. $\lambda = v/f = (340\ \text{m/s})/(512\ \text{Hz}) = 0.66$ m, so $L = \lambda/2 = 0.33$ m. Points: 1 for using $\lambda = v/f$; 1 for correctly taking L as half of the calculated wavelength with correct units, $L = 0.33$ m.

Solution 5

(b) Mark scheme

- Sketch of maximum air-molecule speed vs. position x (tube open at both ends, fundamental mode): a symmetric curve with MAXIMA (velocity antinodes) at both ends $x = 0$ and $x = L$, decreasing smoothly and symmetrically to a single NODE (zero) exactly at the midpoint $x = L/2$, and nonnegative everywhere – reflecting that both open ends are displacement/velocity antinodes and the fundamental standing wave has exactly one node, at the center. Points: 1 for a curve with a node (value zero) at $x = L/2$; 1 for a curve with maxima at $x = 0$ and $x = L$ and no other local maxima; 1 for a nonhorizontal curve that is symmetric about $x = L/2$ and nonnegative everywhere.

Solution 5

(c)

Mark scheme

- With the right end now capped (closed), the tube is closed-open, so the fundamental wavelength becomes $\lambda = 4L = 4(0.33 \text{ m}) = 1.32 \text{ m}$ (rather than $2L$). Using the new speed of sound, $f = v/\lambda = (1005 \text{ m/s})/(1.32 \text{ m}) \approx 757$ Hz. Correct final answer: 757 Hz. Points: 1 for an indication that the fundamental wavelength is now $4L$; 1 for substituting the new sound speed into $v = \lambda f$ to obtain the numeric answer.

Question 5

2015 ▪ Q5

[Figure: Side view of a string with one end attached to an oscillator and the other end attached to a block, with the string passing over a pulley mounted at the edge of a table. The distance between the oscillator and the pulley is labeled L. The block (mass M) hangs below the pulley.]
The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency f. The distance between each oscillator and pulley L is the same, and the mass M of each block is the same. However, the fundamental frequency of each string is different.

Question 5

[Figure: Top view of the four parallel string setups, each spanning the same distance L from its oscillator to its pulley, each with a hanging block of mass M. Labeled fundamental frequencies: $f_A = 400$ Hz, $f_B = 382$ Hz, $f_C = 364$ Hz, $f_D = 350$ Hz.]

The equation for the velocity v of a wave on a string is $v = \sqrt{\frac{F_T}{m/L}}$, where F_T is the tension of the string and m/L is the mass per unit length (linear mass density) of the string.

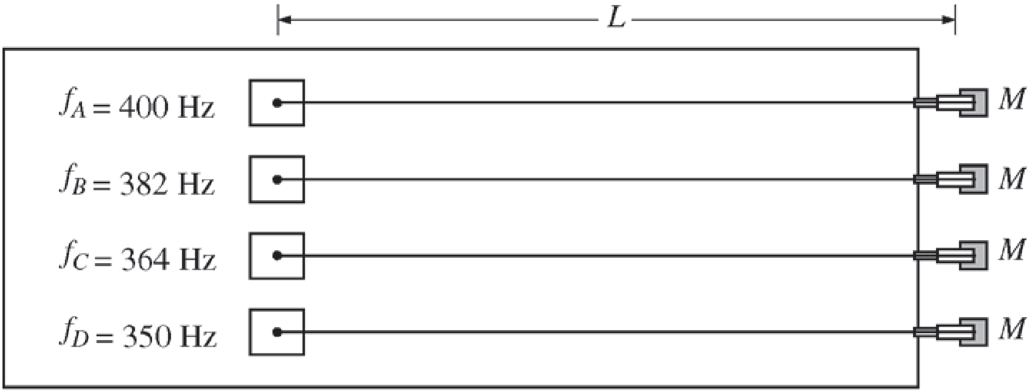
(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

Question 5

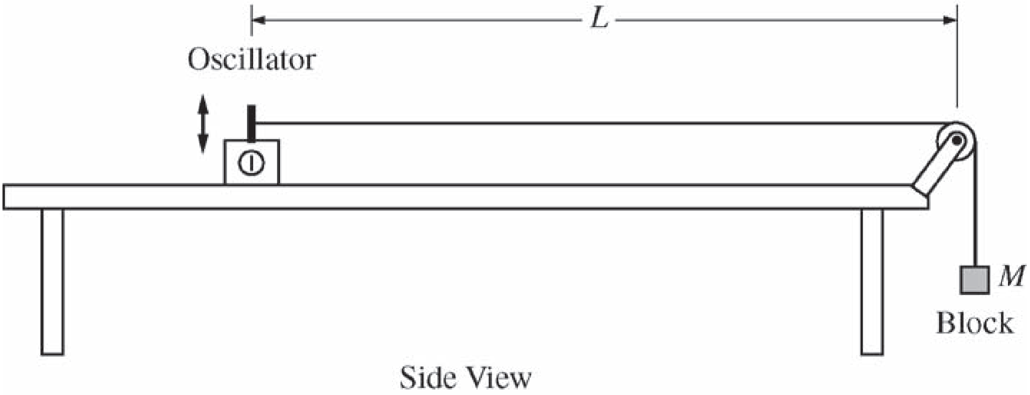
(c) The frequency of the oscillator connected to string D is changed so that the string vibrates in its second harmonic. On the side view of string D below, mark and label the points on the string that have the greatest average vertical speed.
[Figure: Side view of string D, now labeled $f_D = 700$ Hz, drawn taut between the oscillator and the pulley with a dashed pattern of evenly spaced vertical tick marks along its length, for the student to mark the antinode points on. Block hangs below the pulley as before.]

Question 5

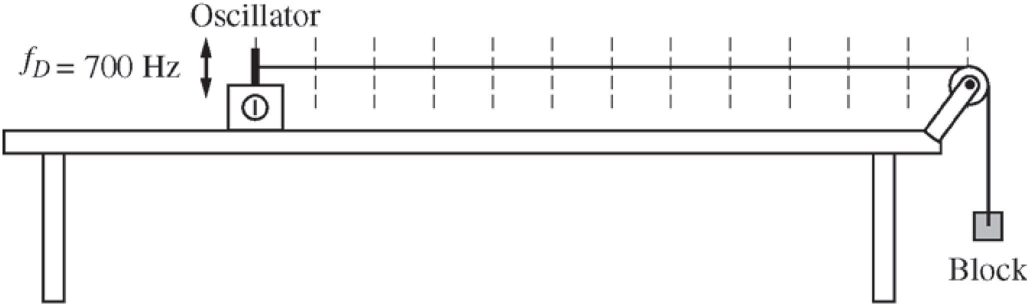


Top View

Question 5



Question 5



Side View

Solution 5

(a) Mark scheme

- Four strings of equal length L, each vibrating at a different fundamental frequency, each under tension from an identical hanging mass M. Since all strings have the same length, and the fundamental wavelength depends only on length ($\lambda_1 = 2L$), all four waves have the SAME wavelength. Since the wavelengths are equal but the frequencies differ, and $v = f\lambda$, the wave SPEEDS must differ between strings. Since the string tensions are equal (same mass M on each) and $v = \sqrt{F_T/(m/L)}$, the LINEAR MASS DENSITIES m/L of the four strings must differ to produce the different wave speeds (equivalently: different linear mass densities give different vertical accelerations of a string element under the same tension, producing different frequencies). Scoring (3 points, 1 each): (1) reasoning that

Solution 5

equal string lengths give equal fundamental wavelengths ($\lambda_1 = 2L$); (2) reasoning that equal wavelengths with different frequencies imply different wave velocities; (3) reasoning that equal tensions (same M) require different linear mass densities to give different velocities, via $v = \sqrt{F_T/(m/L)}$. Note: responses may describe the physical difference between strings in various equivalent ways (different linear mass density / total mass / thickness of the same material).

Solution 5

(b) Mark scheme

- Combine $v = f\lambda$ with $v = \sqrt{F_T/(m/L)}$ to get $f = \dfrac{1}{\lambda}\sqrt{\dfrac{F_T}{m/L}}$, i.e. $f \propto \sqrt{1/(m/L)}$ – frequency is proportional to the SQUARE ROOT of the inverse linear mass density, not directly proportional to it. So a graph of frequency vs. $1/(m/L)$ would NOT be linear (it would curve like a square root). Scoring (2 points, 1 each): (1) combining $v = f\lambda$ with $v = \sqrt{F_T/(m/L)}$ (or referencing an equivalent combined equation from part (a)); (2) indicating how the combined equation shows frequency is not proportional to (not linear in) the inverse of the linear mass density.

Solution 5

(c) Mark scheme

- String D ($f_D = 700$ Hz) is driven at its SECOND harmonic, so its wavelength is $\lambda_2 = L$ (the string shows 2 antinodes / loops, with nodes at both ends and at the midpoint). The points of greatest average vertical speed are the ANTINODES of this standing wave, located at $\frac{1}{4}L$ and $\frac{3}{4}L$ from the oscillator. Scoring (2 points, 1 each): (1) any indication of the second harmonic on the string (a wave shape such that $\lambda_2 = L$); (2) marking points at the antinodes of the second harmonic (or of whatever standing wave was drawn). Full credit requires two points located at one-fourth and three-fourths of the string's length from the oscillator; one point is deducted for each incorrectly marked point on the figure.