Exercise walk-through

Motion of Orbiting Satellites ▪ 6.6

eXercise

You must be able to

- explain how **gravity** provides the centripetal force that keeps a satellite in orbit
- relate the **orbital speed 轨道速度** to the radius with $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$
- describe the energy of a satellite as kinetic plus gravitational potential energy

Method — Gravity is the centripetal force

Setting the gravitational force equal to the required centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}.$$

Method — Closer orbits are faster

Because $v \propto 1/\sqrt{r}$, a satellite in a **lower** orbit moves **faster**.

Method — Energy

A satellite has kinetic energy $\frac{1}{2}mv^2$ and (negative) gravitational potential energy; the two together give its total orbital energy.

Practice 2.1 ▪ 1 mark

State what provides the centripetal force on an orbiting satellite.

Solution 2.1

Method: Gravity is the centripetal force

Worked steps

the gravitational pull of the planet on the satellite **B1**.

Practice 2.2 ▪ 1 mark

State how the orbital speed changes if the orbital radius increases.

Solution 2.2

Worked steps

it decreases ($v \propto 1/\sqrt{r}$) **B1**.

Practice 2.3 ▪ 1 mark

Write the equation obtained by setting the gravitational force equal to the centripetal force.

Solution 2.3

Method: Gravity is the centripetal force

Worked steps

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \boxed{\textbf{B1}}.$$

Exam-style 3.1 ▪ 1 mark

The centripetal force on a satellite in orbit is provided by

A its engine

B gravity

C air resistance

D the normal force

Solution 3.1

A its engine

B gravity ✓

C air resistance

D the normal force

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

As the orbital radius increases, the orbital speed

A increases

B decreases

C stays constant

D becomes zero

Solution 3.2

A increases

B decreases ✓

C stays constant

D becomes zero

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A satellite orbits Earth ($M = 6.0 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}) at radius $r = 7.0 \times 10^{6}$ m.

(a) Write the expression $v = \sqrt{GM/r}$.

(b) Find the orbital speed.

Solution 3.3

Method: Closer orbits are faster

Worked steps

(a) $v = \sqrt{\frac{GM}{r}}$ **B1**. (b) $v = \sqrt{\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{7.0 \times 10^{6}}} = \sqrt{5.7 \times 10^{7}} = 7.6 \times 10^{3}\ \mathrm{m\ s^{-1}}$ **M1** **M1** **A1**.