Exercise walk-through

Rolling ▪ 6.5

eXercise

You must be able to

- describe **rolling without slipping 无滑滚动** using $v = \omega r$
- split the total kinetic energy of a rolling body into **translational** and **rotational** parts
- compare how objects of different rotational inertia accelerate down an incline

Method — Rolling without slipping

The contact point is momentarily at rest, so $v = \omega r$ links the linear speed of the centre and the angular speed.

Method — Total kinetic energy

$$K = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \omega^2.$$

Method — Down an incline

A smaller $I/(mR^2)$ ties up less energy in spin, so the object accelerates faster: **sphere** $>$ **disc** $>$ **hoop**.

Practice 2.1 ▪ 1 mark

A wheel of radius 0.25 m rolls without slipping at $v = 5.0$ m s^{-1}. Find its angular speed.

Solution 2.1

Method: Rolling without slipping

Worked steps

$$\omega = \frac{v}{r} = \frac{5.0}{0.25} = 20 \text{ rad s}^{-1}$$ B1.

Practice 2.2 ▪ 1 mark

State the rolling-without-slipping condition.

Solution 2.2

Method: Rolling without slipping

Worked steps

$v = \omega r$ (the centre's speed equals angular speed times radius) B1.

Practice 2.3 ▪ 1 mark

A hoop and a solid cylinder roll from rest down the same ramp. State which reaches the bottom first.

Solution 2.3

Method: Down an incline

Worked steps

the solid cylinder (smaller rotational inertia) B1.

Exam-style 3.1 ▪ 1 mark

For an object rolling without slipping,

A $v = \omega r$

B $v = \omega / r$

C $\omega = vr$

D $v = \frac{1}{2}\omega r$

Solution 3.1

Method: Rolling without slipping

A $v = \omega r$ ✓

B $v = \omega / r$

C $\omega = vr$

D $v = \frac{1}{2}\omega r$

Worked steps

A.

Exam-style 3.2 ▪ 1 mark

The total kinetic energy of a rolling object is

A translational only

B rotational only

C the sum of translational and rotational

D neither

Solution 3.2

Method: Total kinetic energy

A translational only

B rotational only

C the sum of translational and rotational ✓

D neither

Worked steps

C.

Exam-style 3.3 ▪ 1 mark

A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0$ m s^{-1}.
(a) Find its angular speed.
(b) Find its total kinetic energy.

Solution 3.3

Method: Total kinetic energy

Worked steps

(a) $\omega = \dfrac{3.0}{0.10} = 30$ rad s^{-1} **B1**. (b) $I = \frac{1}{2}(2.0)(0.10)^2 = 0.010$ kg m^2; $K = \frac{1}{2}(2.0)(3.0)^2 + \frac{1}{2}(0.010)(30)^2 = 9.0 + 4.5 = 13.5$ J **M1** **M1** **A1**.