Past-paper walk-through

Conservation of Angular Momentum ▪ 6.4

eXercise

Questions in this set

- 2017 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0

Question 3

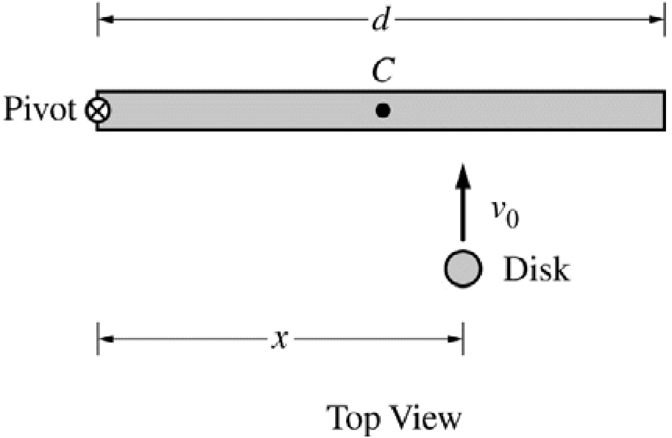
perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?

_____ To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

Question 3



Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will

Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

Yes No

Briefly explain your reasoning without deriving an equation for ω.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will

Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{lxv_0}{m_{\text{disk}}\, d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible — in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will

Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will

Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

______ Greater than ______ Less than ______ Equal to

Briefly explain your reasoning.

Solution 3

(a) Mark scheme

- Correct answer: To the right of C (reasoning only earns credit if this option is selected). The farther from the pivot the disk strikes the rod, the greater the torque the disk exerts on the rod during the collision (equivalently, the disk's angular momentum about the pivot, $m_{disk}v_0x$, is larger for larger x, and nearly all of it transfers to the rod). A greater torque/angular momentum imparted to the rod produces a greater post-collision angular speed, so striking to the right of C (farther from the pivot) gives the rod the largest possible angular speed. Points: 1 pt for an explanation that the torque exerted by the disk on the rod, or the angular momentum of the disk about the pivot, is greater when the disk strikes farther from the pivot.

Solution 3

(b) Mark scheme

- Correct answer: Yes (consistent with the answer selected in part (a)). The proposed equation $\omega = \frac{m_{disk} x v_0}{I}$ shows ω increasing as x increases, since x is in the numerator. This matches the qualitative reasoning of part (a): striking farther from the pivot (larger x) should give the rod a larger post-collision angular speed. So the equation's dependence on x agrees with the part (a) reasoning. Points: 1 pt for a selection consistent with the answer selected in part (a); 1 pt for indicating that the equation shows ω increases with increasing x. (If "No" is selected but the explanation is consistent with an incorrect part (a) selection, full credit may still be earned.)

Solution 3

(c) Mark scheme

- The proposed equation $\omega = \dfrac{I\,x\,v_0}{m_{disk}\,d^4}$ is not physically plausible because of its dependence on m_{disk} and I. Physically, a larger m_{disk} should transfer more angular momentum to the rod, so ω should increase with m_{disk} — but the equation puts m_{disk} in the denominator, so it predicts ω decreasing as m_{disk} increases, which is backwards. Likewise, a larger rotational inertia I of the rod should make the rod harder to spin up, so ω should decrease with I — but the equation has I in the numerator, predicting ω increasing with I, also backwards. This wrong functional (not dimensional) dependence shows the equation cannot be correct. Points: 1 pt for focusing on functional dependence rather than, e.g., a

Solution 3

units/dimensional-analysis argument; 1 pt for addressing m_{disk}, I, or both; 1 pt for correctly concluding the equation is wrong because of its (backwards) dependence on m_{disk}, I, or both.

Solution 3

(d) Mark scheme

- Apply conservation of angular momentum about the pivot for the disk+rod system during the collision. Before collision: $L_i = m_{disk} v_0 x$ (the disk's angular momentum about the pivot). After the collision (disk sticks to rod at distance x): $L_f = (I + m_{disk} x^2)\,\omega$, where I is the rod's rotational inertia about the pivot and $m_{disk} x^2$ is the disk's own rotational inertia about the pivot once stuck to the rod at distance x. Setting $L_i = L_f$: $m_{disk} v_0 x = (I + m_{disk} x^2)\,\omega$. Solving for ω: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$. Points: 1 pt for using an expression of conservation of angular momentum for the disk-and-rod system (not awarded for equating angular and linear momentum); 1 pt for indicating the initial angular momentum of the system

Solution 3

equals $m_{disk} v_0 x$; 1 pt for a dimensionally correct expression of the post-collision angular momentum that includes $I\omega$; 1 pt for indicating the correct rotational inertia of the system after the collision, $I + m_{disk} x^2$. Final answer: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$.

Solution 3

(e) Mark scheme

- Correct answer: Greater than. When the disk bounces backward off the rod instead of sticking, the disk's velocity (and hence its angular momentum about the pivot) changes by a larger amount than in the sticking case — going from $+m_{disk}v_0x$ to some negative value, rather than just to the smaller value it has once moving together with the rod. By conservation of angular momentum for the system, whatever angular momentum the disk loses, the rod gains; since the disk's angular-momentum change (equivalently, the impulse delivered to the rod) is larger in the bouncing case, the rod gains more angular momentum — and hence acquires a greater post-collision angular speed — than in the sticking case. Example: after the bouncy collision the disk has angular momentum in the

Solution 3

clockwise direction; to keep the system's total angular momentum constant, the rod's counterclockwise angular momentum must be greater than before. Points: 1 pt for indicating, directly or by analogy to the linear (elastic vs. inelastic) case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario, OR an equivalent impulse-based argument (describes what happens to the disk); 1 pt for using conservation of angular momentum or momentum-impulse reasoning to conclude the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario (describes what happens to the rod).