Past-paper walk-through

Angular Momentum and Angular Impulse ▪ 6.3

eXercise

Questions in this set

- 2023 Q4
- 2019 Q1
- 2017 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

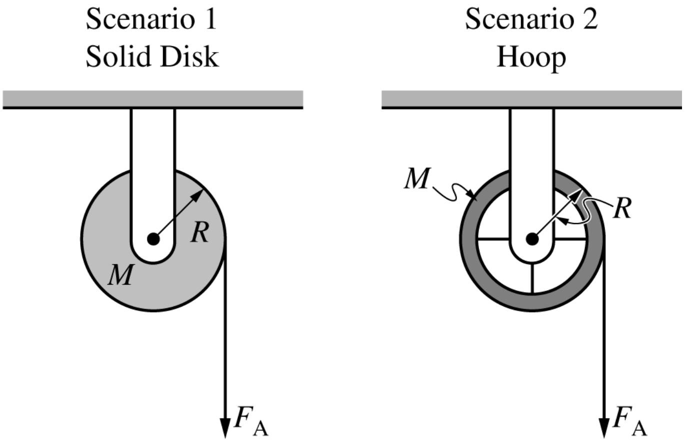
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

Question 4

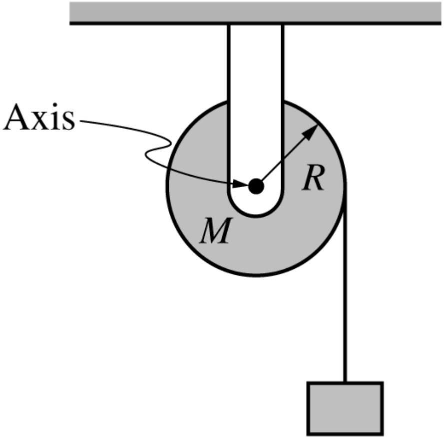
When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_{T}.

(a) Determine an expression for the magnitude of the angular acceleration α_{D} of the disk as the block travels downward. Express your answer in terms of M, R, F_{T}, and physical constants as appropriate.

Question 4



Question 4



Question 4

2023 ▪ Q4

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Question 4

(b) [Continuation of Question 4.] [Figure: Two side-by-side diagrams. "Scenario 1 — Solid Disk": a shaded disk of mass M and radius R mounted on a horizontal axle, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down). "Scenario 2 — Hoop": a shaded hoop (ring) of mass M and radius R mounted on a horizontal axle in the same way, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down).]

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without

Question 4

the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.
Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Solution 4

(a)

Mark scheme

- $\alpha_D = \dfrac{2F_T}{MR}$. Derivation: for the solid disk (pulley), $\tau = I\alpha$ with $\tau = RF_T$ and $I = \frac{1}{2}MR^2$: $RF_T = \frac{1}{2}MR^2\,\alpha_D \Rightarrow \alpha_D = \dfrac{RF_T}{\frac{1}{2}MR^2} = \dfrac{2F_T}{MR}$ (either equivalent form is accepted).

Solution 4

(b) Mark scheme

- Paragraph argument comparing a solid disk and a hoop (each mass M, radius R) each pulled by the same force F_A via a string wrapped around the rim: the disk ends up with GREATER rotational kinetic energy than the hoop. Reasoning, worth 6 points total: (1) the torque $\tau = F_A R$ is the same for both pulleys, since the same force is applied at the same radius; (2) since torque and the time interval are the same for both, the angular impulse $\tau \Delta t$ (equivalently the change in angular momentum ΔL) is the same for the disk and the hoop; (3) the rotational inertia I differs between the disk and hoop: $I_{\text{disk}} = \frac{1}{2}MR^2 < I_{\text{hoop}} = MR^2$, since the hoop's mass is concentrated at the rim while the disk's is spread through its area; (4) because $\Delta L = I\Delta\omega$ is the same for both but $I_{\text{hoop}} > I_{\text{disk}}$, the disk must

Solution 4

gain a larger $\Delta\omega$ (and correspondingly larger $\Delta\theta$, α) than the hoop — the kinematic quantities differ because the rotational inertias differ; (5) one of: relating I and ω to show ΔK is greater for the disk (e.g. using $\omega = L/I$ and $K = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller I converts the same angular momentum into more kinetic energy), OR noting $\Delta\theta$ is greater for the disk so the work done on the disk ($\tau\Delta\theta$) is greater; (6) an overall logical, relevant, and internally consistent paragraph following the AP paragraph-response format that ties these steps together to conclude the disk has the greater rotational kinetic energy.

Question 1

2019 ▪ Q1

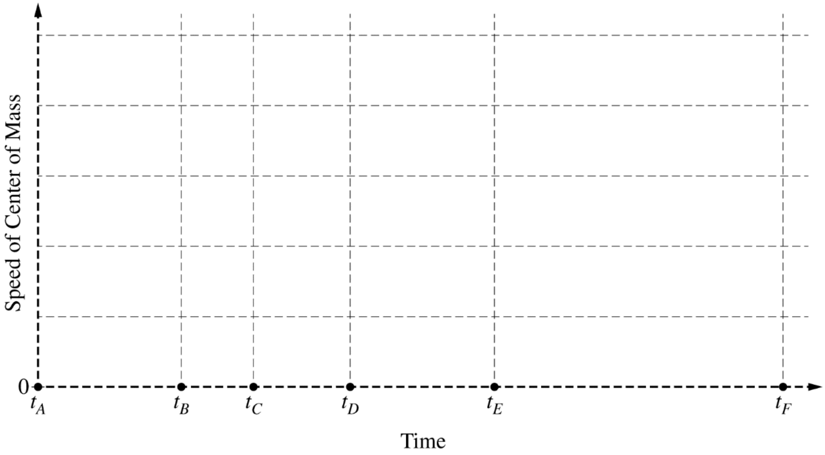
Identical blocks 1 and 2 are placed on a horizontal surface at points A and E, respectively, as shown. The surface is frictionless except for the region between points C and D, where the surface is rough. Beginning at time t_A, block 1 is pushed with a constant horizontal force from point A to point B by a mechanical plunger. Upon reaching point B, block 1 loses contact with the plunger and continues moving to the right along the horizontal surface toward block 2. Block 1 collides with and sticks to block 2 at point E, after which the two-block system continues moving across the surface, eventually passing point F.

[Diagram: A plunger (with block 1 loaded against it) sits at the left, followed by a horizontal line marked with points A, B, C, D, E, F from left to right. Block 2 sits at point E. The region between C and D is hatched, indicating it is rough (roughened surface). The plunger pushes block 1 from A to B.]

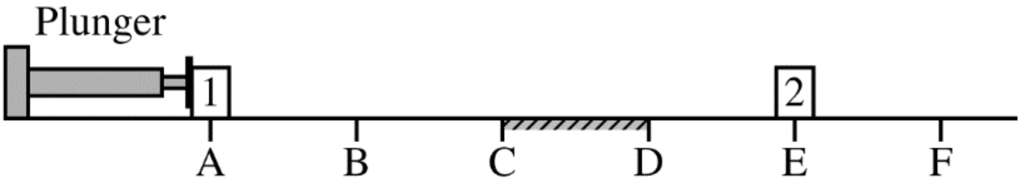
Question 1

(a) On the axes below, sketch the speed of the center of mass of the two-block system as a function of time, from time t_A until the blocks pass point F at time t_F. The times at which block 1 reaches points A through F are indicated on the time axis.
[Graph: y-axis "Speed of Center of Mass" starting at 0, no numeric scale shown; x-axis "Time" with labeled tick marks at t_A, t_B, t_C, t_D, t_E, t_F (unevenly spaced, t_E noticeably farther from t_D than the others); axes are otherwise blank dashed gridlines for the student to sketch on.]

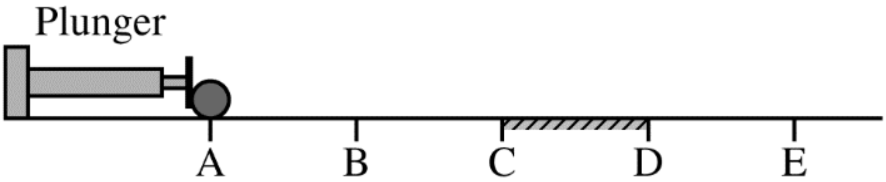
Question 1



Question 1



Question 1



Question 1

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Question 1

(b) The plunger is returned to its original position, and both blocks are removed. A uniform solid sphere is placed at point A, as shown. The sphere is pushed by the plunger from point A to point B with a constant horizontal force that is directed toward the sphere's center of mass. The sphere loses contact with the plunger at point B and continues moving across the horizontal surface toward point E. In which interval(s), if any, does the sphere's angular momentum about its center of mass change? Check all that apply.
[Diagram: A plunger with a sphere loaded against it sits at the left, followed by a horizontal line marked with points A, B, C, D, E from left to right. The region between C and D is hatched, indicating it is rough.]
_____ A to B _____ B to C _____ C to D _____ D to E _____ None
Briefly explain your reasoning.

Solution 1

(a) Mark scheme

- Correct graph of center-of-mass speed vs time has 5 required features (1 pt each): (1) a straight line beginning at 0 at t_A and increasing to t_B (block 1 accelerates under the plunger's constant external force, block 2 stationary, so $v_{cm} = v_1/2$ rises linearly); (2) a horizontal, nonzero segment from t_B to t_C (frictionless surface, block 1 moves at constant velocity, so v_{cm} constant); (3) a segment decreasing linearly from t_C to t_D (friction between C and D is an external force decelerating block 1, so v_{cm} decreases); (4) a horizontal, nonzero, constant segment from t_D through t_F at a DIFFERENT (lower) value than the t_B-t_C plateau, with NO change at t_E; (5) a curve continuous from t_A through t_F, with the only possible exception being a jump at t_E. Note: if the speed changes at t_E, point (4) is not

Solution 1

earned but point (5) may still be earned. No credit for a flat line along the time axis. Physical reasoning: the collision between block 1 and block 2 at E is an INTERNAL interaction of the two-block system, so it does not change the system's center-of-mass velocity (momentum conservation) – the correct graph shows the D-to-F plateau passing smoothly through t_E with no jump, at a lower speed than the B-to-C plateau because the mass tracked (2 blocks) is now effectively combined.

Solution 1

(b) Mark scheme

- Correct answer: the sphere's angular momentum about its center of mass changes ONLY in the interval C to D (not A-B, not B-C, not D-E). Point 1: for reasoning that a change in angular momentum is caused by a net external torque. Point 2: for correctly identifying that friction acting between C and D is the only force producing an external torque over the entire interval A to E. Full reasoning: the plunger's force (A to B) is directed toward the sphere's center of mass, so it produces zero torque about the center of mass; the surface is frictionless everywhere except C to D, so no other external force can exert a torque about the center of mass; therefore only the friction force acting in the rough region C to D changes the sphere's angular momentum about its center of mass (this point is

Solution 1

NOT earned if the response claims angular momentum/angular speed decreases between C and D or that the sphere stops rotating at D – friction here INcreases/changes the spin from zero, since the sphere arrives at C already translating without spin).

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0

Question 3

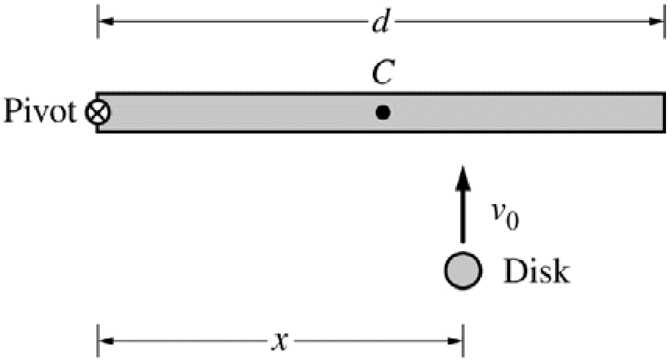
perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?

_____ To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

Question 3



Top View

Question 3

2017 ▪ Q3

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

Yes No

Briefly explain your reasoning without deriving an equation for ω.

Question 3

2017 ▪ Q3

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{lxv_0}{m_{\text{disk}}\, d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible — in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

Question 3

2017 ▪ Q3

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

Question 3

2017 ▪ Q3

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

______ Greater than ______ Less than ______ Equal to

Briefly explain your reasoning.

Solution 3

(a) Mark scheme

- Correct answer: To the right of C (reasoning only earns credit if this option is selected). The farther from the pivot the disk strikes the rod, the greater the torque the disk exerts on the rod during the collision (equivalently, the disk's angular momentum about the pivot, $m_{disk}v_0x$, is larger for larger x, and nearly all of it transfers to the rod). A greater torque/angular momentum imparted to the rod produces a greater post-collision angular speed, so striking to the right of C (farther from the pivot) gives the rod the largest possible angular speed. Points: 1 pt for an explanation that the torque exerted by the disk on the rod, or the angular momentum of the disk about the pivot, is greater when the disk strikes farther from the pivot.

Solution 3

(b) Mark scheme

- Correct answer: Yes (consistent with the answer selected in part (a)). The proposed equation $\omega = \frac{m_{disk} x v_0}{I}$ shows ω increasing as x increases, since x is in the numerator. This matches the qualitative reasoning of part (a): striking farther from the pivot (larger x) should give the rod a larger post-collision angular speed. So the equation's dependence on x agrees with the part (a) reasoning. Points: 1 pt for a selection consistent with the answer selected in part (a); 1 pt for indicating that the equation shows ω increases with increasing x. (If "No" is selected but the explanation is consistent with an incorrect part (a) selection, full credit may still be earned.)

Solution 3

(c) Mark scheme

- The proposed equation $\omega = \dfrac{I\,x\,v_0}{m_{disk}\,d^4}$ is not physically plausible because of its dependence on m_{disk} and I. Physically, a larger m_{disk} should transfer more angular momentum to the rod, so ω should increase with m_{disk} — but the equation puts m_{disk} in the denominator, so it predicts ω decreasing as m_{disk} increases, which is backwards. Likewise, a larger rotational inertia I of the rod should make the rod harder to spin up, so ω should decrease with I — but the equation has I in the numerator, predicting ω increasing with I, also backwards. This wrong functional (not dimensional) dependence shows the equation cannot be correct. Points: 1 pt for focusing on functional dependence rather than, e.g., a

Solution 3

units/dimensional-analysis argument; 1 pt for addressing m_{disk}, l, or both; 1 pt for correctly concluding the equation is wrong because of its (backwards) dependence on m_{disk}, l, or both.

Solution 3

(d) Mark scheme

- Apply conservation of angular momentum about the pivot for the disk+rod system during the collision. Before collision: $L_i = m_{disk} v_0 x$ (the disk's angular momentum about the pivot). After the collision (disk sticks to rod at distance x): $L_f = (I + m_{disk} x^2)\,\omega$, where I is the rod's rotational inertia about the pivot and $m_{disk} x^2$ is the disk's own rotational inertia about the pivot once stuck to the rod at distance x. Setting $L_i = L_f$: $m_{disk} v_0 x = (I + m_{disk} x^2)\,\omega$. Solving for ω: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$. Points: 1 pt for using an expression of conservation of angular momentum for the disk-and-rod system (not awarded for equating angular and linear momentum); 1 pt for indicating the initial angular momentum of the system

Solution 3

equals $m_{disk} v_0 x$; 1 pt for a dimensionally correct expression of the post-collision angular momentum that includes $I\omega$; 1 pt for indicating the correct rotational inertia of the system after the collision, $I + m_{disk} x^2$. Final answer: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$.

Solution 3

(e) Mark scheme

- Correct answer: Greater than. When the disk bounces backward off the rod instead of sticking, the disk's velocity (and hence its angular momentum about the pivot) changes by a larger amount than in the sticking case — going from $+m_{disk}v_0x$ to some negative value, rather than just to the smaller value it has once moving together with the rod. By conservation of angular momentum for the system, whatever angular momentum the disk loses, the rod gains; since the disk's angular-momentum change (equivalently, the impulse delivered to the rod) is larger in the bouncing case, the rod gains more angular momentum — and hence acquires a greater post-collision angular speed — than in the sticking case. Example: after the bouncy collision the disk has angular momentum in the

Solution 3

clockwise direction; to keep the system's total angular momentum constant, the rod's counterclockwise angular momentum must be greater than before. Points: 1 pt for indicating, directly or by analogy to the linear (elastic vs. inelastic) case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario, OR an equivalent impulse-based argument (describes what happens to the disk); 1 pt for using conservation of angular momentum or momentum-impulse reasoning to conclude the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario (describes what happens to the rod).