Past-paper walk-through

Torque and Work ▪ 6.2

eXercise

Questions in this set

- 2026 Q4
- 2023 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2026 ▪ Q4

Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \dfrac{1}{2} I_X$.

[Figure 1 (Side View): Two spinning-top-style toys shown side by side. Toy X (left): a vertical axle labeled "Axle" at the top with radius r_0 marked at the top of the "Upper Portion" cylindrical section; "String" labels the wrapped string near the top of the upper portion; an arrow labeled F_0 points horizontally to the right from the upper portion; below the upper portion is a wide, ridged, bulbous body tapering to a point at the bottom, with a small curved arrow near the point indicating rotation. Toy Y

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(right): the same axle/radius r_0/"Upper Portion"/"String"/F_0 labeling at the top, but the lower body is a flatter shape with a wide horizontal disk (like a ring) partway down, tapering to a point at the bottom with a rotation arrow, similar to Toy X but with different mass distribution.]

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

[Figure 2 (Overhead View): A large shaded circle (the upper portion of a toy, viewed from above) with a small labeled circle at its center marked r_0, and an arrow labeled F_0 pointing to the right from the center, representing the string being pulled

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tangentially. A curved arrow above and to the left of the circle indicates the direction of rotation (counterclockwise).]

(a) Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

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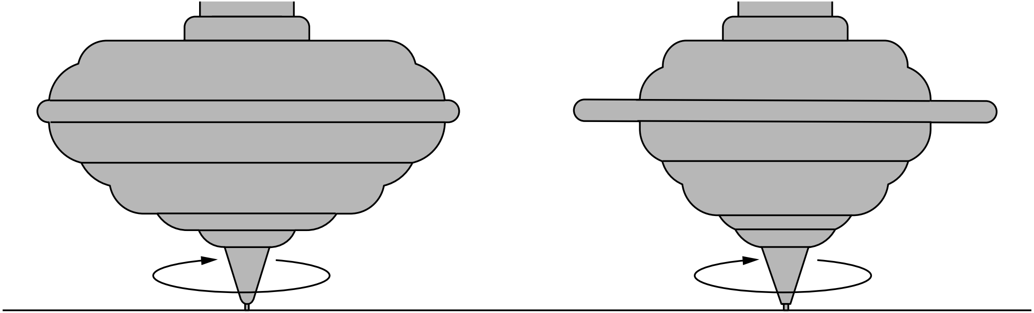

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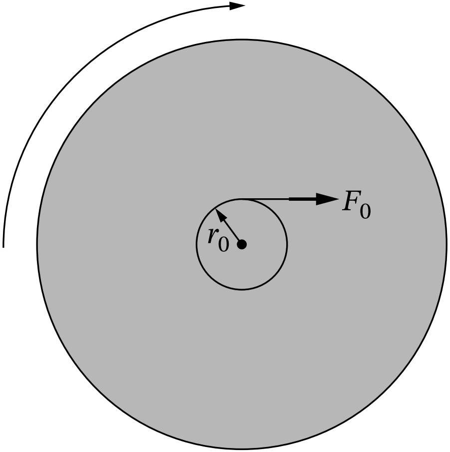

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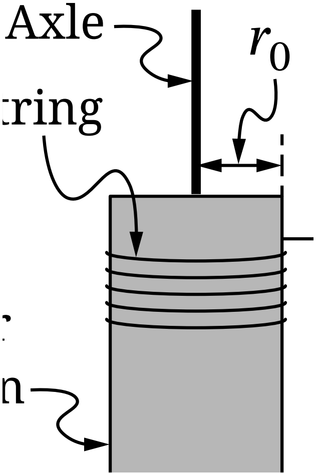

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Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

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When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

[Figure 2 (Overhead View): A large shaded circle (the upper portion of a toy, viewed from above) with a small labeled circle at its center marked r_0, and an arrow labeled F_0 pointing to the right from the center, representing the string being pulled tangentially. A curved arrow above and to the left of the circle indicates the direction of rotation (counterclockwise).]

(b) Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0, I_X, F_0, and physical

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constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

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(c) Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

Question 5

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[Figure 1: A rod mounted horizontally on a fixed axle; the rod extends upward from the axle, with a sphere attached at its top end. The center of mass of the rod-sphere system is marked with a $\otimes$ symbol partway along the rod, between the axle and the sphere. Labels: "Sphere" pointing to the ball at top, "Center of Mass" pointing to the $\otimes$ mark, "Rod" pointing to the connecting bar, "Axle" pointing to the pivot at the bottom.]

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a $\otimes$, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

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A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video. Note: these frames are not equally spaced apart in time.
[Figure 2: Five labeled panels showing the rod-sphere system at different rotation angles, each with the axle shown as a small pivot circle and the center of mass marked with a symbol along the rod: Frame A — rod nearly vertical upward, sphere ($\otimes$) near top, just slightly rotated clockwise from vertical, axle labeled at pivot. Frame B — rod rotated further clockwise, roughly 45° from vertical (upper right), sphere ($\oplus$ symbol) at the end. Frame C — rod horizontal, pointing to the right, sphere ($\otimes$) at the far right end. Frame D — rod rotated further clockwise, roughly 45° below horizontal (lower right), sphere ($\oplus$ symbol) at the end. Frame E — rod nearly vertical downward, sphere ($\otimes$) near bottom, pivot at top.]
(a)(i) Use the frames of the video shown in Figure 2 to answer the following questions.

Question 5

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.
(a)(ii) ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Question 5

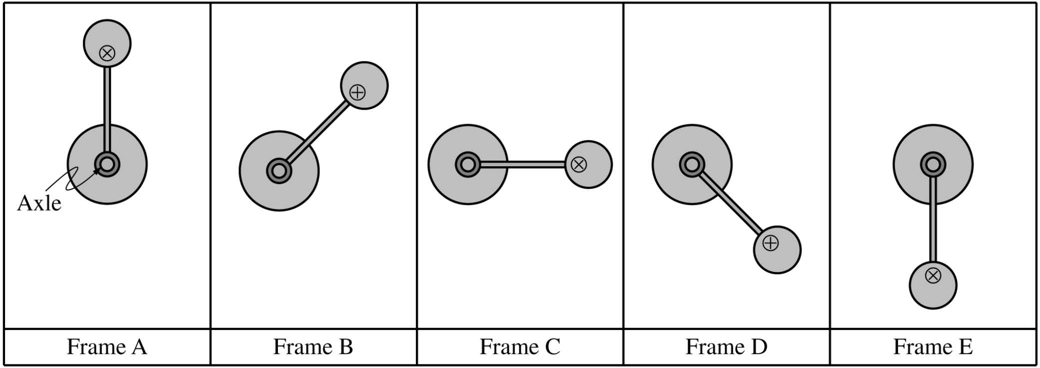

Figure 2

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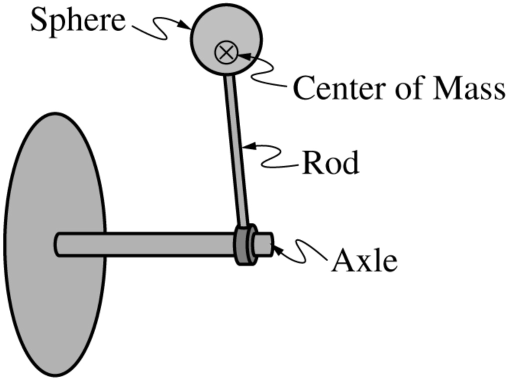

Figure 1

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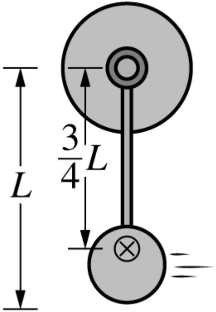

Figure 3

Question 5

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[Figure 1: A rod mounted horizontally on a fixed axle; the rod extends upward from the axle, with a sphere attached at its top end. The center of mass of the rod-sphere system is marked with a ⊗ symbol partway along the rod, between the axle and the sphere. Labels: "Sphere" pointing to the ball at top, "Center of Mass" pointing to the ⊗ mark, "Rod" pointing to the connecting bar, "Axle" pointing to the pivot at the bottom.]

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video. Note: these frames are <u>not</u> equally spaced apart in time.

Question 5

[Figure 2: Five labeled panels showing the rod-sphere system at different rotation angles, each with the axle shown as a small pivot circle and the center of mass marked with a symbol along the rod: Frame A — rod nearly vertical upward, sphere (⊗) near top, just slightly rotated clockwise from vertical, axle labeled at pivot. Frame B — rod rotated further clockwise, roughly 45° from vertical (upper right), sphere (⊕ symbol) at the end. Frame C — rod horizontal, pointing to the right, sphere (⊗) at the far right end. Frame D — rod rotated further clockwise, roughly 45° below horizontal (lower right), sphere (⊕ symbol) at the end. Frame E — rod nearly vertical downward, sphere (⊗) near bottom, pivot at top.]

(b)(i) [Continuation of Question 5.] [Figure 3: A vertical diagram showing the rod-sphere system's axle at top and sphere (⊗) at bottom, with a bracketed measurement on the left showing total length L from axle to sphere, and a separate

Question 5

bracket showing $\frac{3}{4}L$ measured from the axle down to the center-of-mass marker on the rod.]

The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

(b)(ii) ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

Solution 5

(a)(i) Mark scheme

- Frame C — the angular acceleration is greatest in Frame C. Reasoning: angular acceleration is proportional to torque ($\alpha \propto \tau$, from $\tau = I\alpha$), and torque = (lever arm) $\times$ (force). In Frame C the gravitational weight-force vector is directed perpendicular to the rod (the lever arm from the axle to the line of action of gravity is at its maximum there), so the torque due to gravity — and hence the angular acceleration — is greatest in that frame. Points: 1 for identifying 'Frame C' with correct reasoning about the torque being greatest there (e.g. lever arm greatest, or radius-weight angle most perpendicular); 1 for correctly relating torque and angular acceleration ($\alpha \propto \tau$).

Solution 5

(a)(ii)

Mark scheme

- Frame E — the rotational kinetic energy of the rod-sphere system is greatest in Frame E. Accept any one valid justification: (work/energy) this is the frame by which the maximum (cumulative) work has been done on the system by gravity; (angular momentum) the torque due to gravity has acted in the same rotational sense the entire time, so the system's angular momentum — and hence angular speed — has been continuously increasing up to that frame; (kinematics) the rod-sphere system speeds up throughout the motion, so its rotational speed (and KE) is greatest at Frame E.

Solution 5

(b)(i) Mark scheme

- Using conservation of energy (the rod-sphere system starts at rest and swings so the center of mass, located $\frac{3}{4}L$ from the axle, moves through a vertical drop): $E_i = E_f$, i.e. $U_{gi} + K_i = U_{gf} + K_f$ (1 point for beginning a multistep derivation with conservation of energy). The center of mass's change in height is $\Delta y = \frac{3}{4}L + \frac{3}{4}L = \frac{3}{2}L$ (1 point for indicating the height change equals $\frac{3}{2}L$). Then $\Delta K = Mg\Delta y = \frac{3}{2}MgL$, so (starting from rest) $K_f = \frac{3}{2}MgL$ (1 point for a final answer consistent with the height change used — note a bare correct answer of $K_f = \frac{3}{2}MgL$ with no supporting work can earn only this one point, not all three).

Solution 5

(b)(ii)

Mark scheme

- The rod and sphere gain kinetic energy because gravity is an external force doing (positive) work on the rod-sphere system, even though Earth itself is not included as part of the defined system. Since the gravitational force is external to the (rod + sphere) system, it can do net work on the system and increase its total kinetic energy.