# Past-paper walk-through

## Rotational Kinetic Energy ▪ 6.1

eXercise

# Questions in this set

- 2023 Q4
- 2023 Q5
- 2022 Q3
- 2021 Q4
- 2016 Q1

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass $M$ and radius $R$ (with $R$ labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

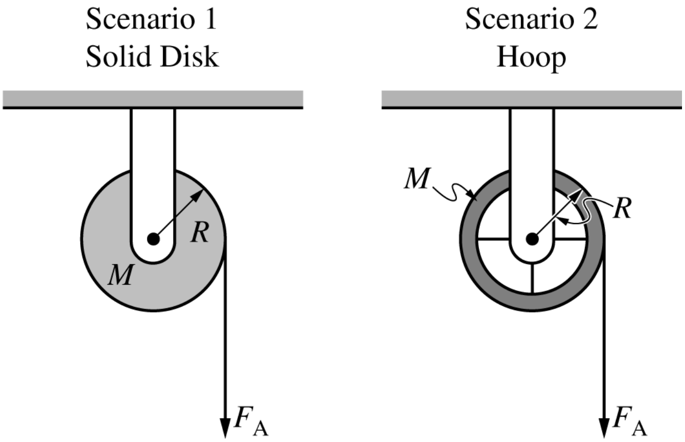
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass $M$ and radius $R$. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

# Question 4

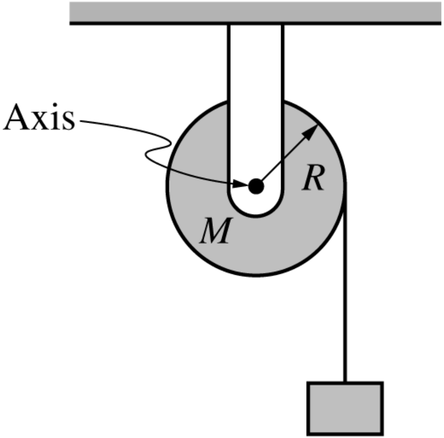
When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is $F_{\mathrm{T}}$.

**(a)** Determine an expression for the magnitude of the angular acceleration $\alpha_{\mathrm{D}}$ of the disk as the block travels downward. Express your answer in terms of $M$, $R$, $F_{\mathrm{T}}$, and physical constants as appropriate.

# Question 4



# Question 4



# Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass $M$ and radius $R$ (with $R$ labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass $M$ and radius $R$. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is $F_T$.

# Question 4

**(b)** [Continuation of Question 4.] [Figure: Two side-by-side diagrams. "Scenario 1 — Solid Disk": a shaded disk of mass $M$ and radius $R$ mounted on a horizontal axle, with a string wrapped around it hanging down and pulled with force $F_{\text{A}}$ (arrow pointing down). "Scenario 2 — Hoop": a shaded hoop (ring) of mass $M$ and radius $R$ mounted on a horizontal axle in the same way, with a string wrapped around it hanging down and pulled with force $F_{\text{A}}$ (arrow pointing down).]

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass $M$ and radius $R$ as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force $F_{\text{A}}$ for the same time interval $\Delta t$, causing the pulleys to rotate without

# Question 4

the string slipping. After time interval $\Delta t$, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.
Consider scenarios 1 and 2 at the end of time interval $\Delta t$. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

# Solution 4

## (a)

**Mark scheme**

- $\alpha_D = \dfrac{2F_T}{MR}$. Derivation: for the solid disk (pulley), $\tau = I\alpha$ with $\tau = RF_T$ and $I = \frac{1}{2}MR^2$: $RF_T = \frac{1}{2}MR^2\,\alpha_D \Rightarrow \alpha_D = \dfrac{RF_T}{\frac{1}{2}MR^2} = \dfrac{2F_T}{MR}$ (either equivalent form is accepted).

# Solution 4

## (b) Mark scheme

- Paragraph argument comparing a solid disk and a hoop (each mass $M$, radius $R$) each pulled by the same force $F_A$ via a string wrapped around the rim: the disk ends up with GREATER rotational kinetic energy than the hoop. Reasoning, worth 6 points total: (1) the torque $\tau = F_A R$ is the same for both pulleys, since the same force is applied at the same radius; (2) since torque and the time interval are the same for both, the angular impulse $\tau\Delta t$ (equivalently the change in angular momentum $\Delta L$) is the same for the disk and the hoop; (3) the rotational inertia $I$ differs between the disk and hoop: $I_{\text{disk}} = \frac{1}{2}MR^2 < I_{\text{hoop}} = MR^2$, since the hoop's mass is concentrated at the rim while the disk's is spread through its area; (4) because $\Delta L = I\Delta\omega$ is the same for both but $I_{\text{hoop}} > I_{\text{disk}}$, the disk must

# Solution 4

gain a larger $\Delta\omega$ (and correspondingly larger $\Delta\theta$, $\alpha$) than the hoop — the kinematic quantities differ because the rotational inertias differ; (5) one of: relating $I$ and $\omega$ to show $\Delta K$ is greater for the disk (e.g. using $\omega = L/I$ and $K = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller $I$ converts the same angular momentum into more kinetic energy), OR noting $\Delta\theta$ is greater for the disk so the work done on the disk ($\tau\Delta\theta$) is greater; (6) an overall logical, relevant, and internally consistent paragraph following the AP paragraph-response format that ties these steps together to conclude the disk has the greater rotational kinetic energy.

# Question 5

2023 ▪ Q5

[Figure 1: A rod mounted horizontally on a fixed axle; the rod extends upward from the axle, with a sphere attached at its top end. The center of mass of the rod-sphere system is marked with a $\otimes$ symbol partway along the rod, between the axle and the sphere. Labels: "Sphere" pointing to the ball at top, "Center of Mass" pointing to the $\otimes$ mark, "Rod" pointing to the connecting bar, "Axle" pointing to the pivot at the bottom.]
A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a $\otimes$, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

# Question 5

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video. Note: these frames are not equally spaced apart in time. [Figure 2: Five labeled panels showing the rod-sphere system at different rotation angles, each with the axle shown as a small pivot circle and the center of mass marked with a symbol along the rod: Frame A — rod nearly vertical upward, sphere ($\otimes$) near top, just slightly rotated clockwise from vertical, axle labeled at pivot. Frame B — rod rotated further clockwise, roughly 45° from vertical (upper right), sphere ($\oplus$ symbol) at the end. Frame C — rod horizontal, pointing to the right, sphere ($\otimes$) at the far right end. Frame D — rod rotated further clockwise, roughly 45° below horizontal (lower right), sphere ($\oplus$ symbol) at the end. Frame E — rod nearly vertical downward, sphere ($\otimes$) near bottom, pivot at top.]

**(a)(i)** Use the frames of the video shown in Figure 2 to answer the following questions.

# Question 5

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.
**(a)(ii)** ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

# Question 5

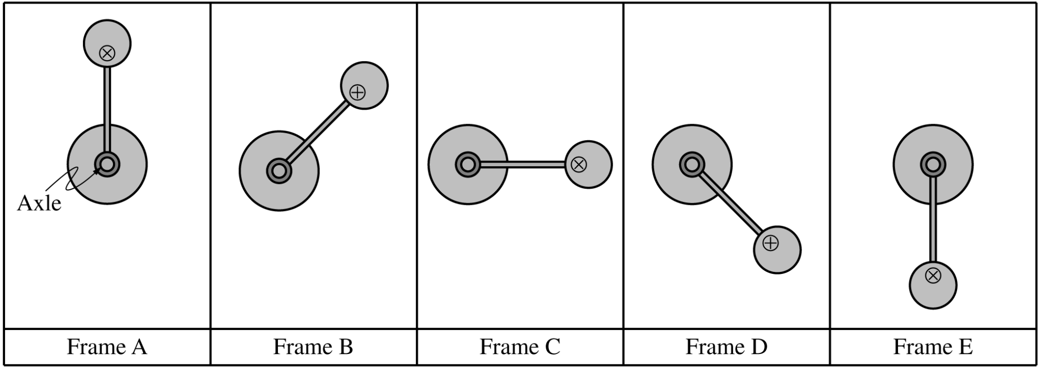



Figure 2

## Question 5

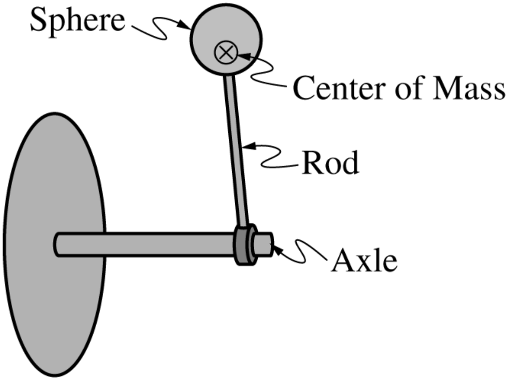



Figure 1

# Question 5

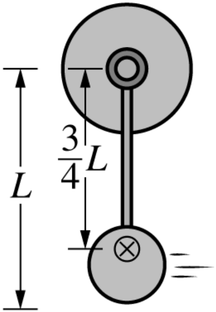



Figure 3

# Question 5

2023 ▪ Q5

[Figure 1: A rod mounted horizontally on a fixed axle; the rod extends upward from the axle, with a sphere attached at its top end. The center of mass of the rod-sphere system is marked with a $\otimes$ symbol partway along the rod, between the axle and the sphere. Labels: "Sphere" pointing to the ball at top, "Center of Mass" pointing to the $\otimes$ mark, "Rod" pointing to the connecting bar, "Axle" pointing to the pivot at the bottom.]

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a $\otimes$, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video. Note: these frames are not equally spaced apart in time.

# Question 5

[Figure 2: Five labeled panels showing the rod-sphere system at different rotation angles, each with the axle shown as a small pivot circle and the center of mass marked with a symbol along the rod: Frame A — rod nearly vertical upward, sphere (⊗) near top, just slightly rotated clockwise from vertical, axle labeled at pivot. Frame B — rod rotated further clockwise, roughly 45° from vertical (upper right), sphere (⊕ symbol) at the end. Frame C — rod horizontal, pointing to the right, sphere (⊗) at the far right end. Frame D — rod rotated further clockwise, roughly 45° below horizontal (lower right), sphere (⊕ symbol) at the end. Frame E — rod nearly vertical downward, sphere (⊗) near bottom, pivot at top.]

**(b)(i)** [Continuation of Question 5.] [Figure 3: A vertical diagram showing the rod-sphere system's axle at top and sphere (⊗) at bottom, with a bracketed measurement on the left showing total length $L$ from axle to sphere, and a separate

# Question 5

bracket showing $\frac{3}{4}L$ measured from the axle down to the center-of-mass marker on the rod.]

The rod-sphere system has mass $M$ and length $L$, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of $M$, $L$, and fundamental constants, as appropriate.

**(b)(ii)** ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

# Solution 5

## (a)(i) Mark scheme

- Frame C — the angular acceleration is greatest in Frame C. Reasoning: angular acceleration is proportional to torque ($\alpha \propto \tau$, from $\tau = I\alpha$), and torque = (lever arm) $\times$ (force). In Frame C the gravitational weight-force vector is directed perpendicular to the rod (the lever arm from the axle to the line of action of gravity is at its maximum there), so the torque due to gravity — and hence the angular acceleration — is greatest in that frame. Points: 1 for identifying 'Frame C' with correct reasoning about the torque being greatest there (e.g. lever arm greatest, or radius-weight angle most perpendicular); 1 for correctly relating torque and angular acceleration ($\alpha \propto \tau$).

# Solution 5

## (a)(ii)

**Mark scheme**

- Frame E — the rotational kinetic energy of the rod-sphere system is greatest in Frame E. Accept any one valid justification: (work/energy) this is the frame by which the maximum (cumulative) work has been done on the system by gravity; (angular momentum) the torque due to gravity has acted in the same rotational sense the entire time, so the system's angular momentum — and hence angular speed — has been continuously increasing up to that frame; (kinematics) the rod-sphere system speeds up throughout the motion, so its rotational speed (and KE) is greatest at Frame E.

# Solution 5

## (b)(i) Mark scheme

- Using conservation of energy (the rod-sphere system starts at rest and swings so the center of mass, located $\frac{3}{4}L$ from the axle, moves through a vertical drop): $E_i = E_f$, i.e. $U_{gi} + K_i = U_{gf} + K_f$ (1 point for beginning a multistep derivation with conservation of energy). The center of mass's change in height is $\Delta y = \frac{3}{4}L + \frac{3}{4}L = \frac{3}{2}L$ (1 point for indicating the height change equals $\frac{3}{2}L$). Then $\Delta K = Mg\Delta y = \frac{3}{2}MgL$, so (starting from rest) $K_f = \frac{3}{2}MgL$ (1 point for a final answer consistent with the height change used — note a bare correct answer of $K_f = \frac{3}{2}MgL$ with no supporting work can earn only this one point, not all three).

# Solution 5

## (b)(ii)

**Mark scheme**

- The rod and sphere gain kinetic energy because gravity is an external force doing (positive) work on the rod-sphere system, even though Earth itself is not included as part of the defined system. Since the gravitational force is external to the (rod + sphere) system, it can do net work on the system and increase its total kinetic energy.

# Question 3

2022 ▪ Q3

[Figure: A wheel mounted on a horizontal axle, standing on a support post above a floor. A string is wrapped around the wheel's rim and hangs down to a small hanging block. Labels: "Wheel" (top, pointing to the wheel), "Axle" (pointing to the horizontal axle at the wheel's center), "Hanging Block" (pointing to the small block hanging from the string), "Floor" (the ground below).]

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

# Question 3

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

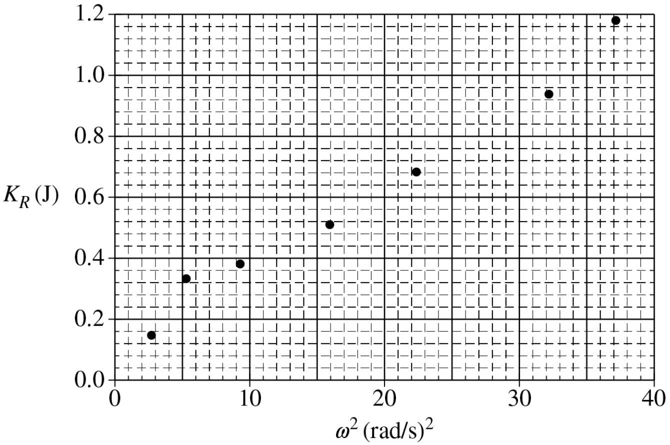
**(a)** Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

# Question 3

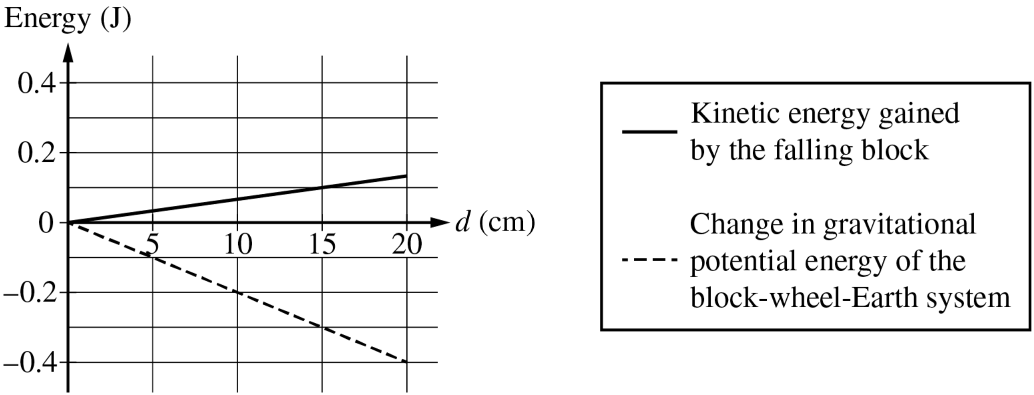
In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them in the space just below the table.
In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.
If needed, you may include a simple diagram of the setup with your procedure.

| **Quantity to Be Measured** | **Symbol for Quantity** | **Equipment for Measurement** | **Procedure (and diagram, if needed)** |
|---|---|---|---|
| | | | |

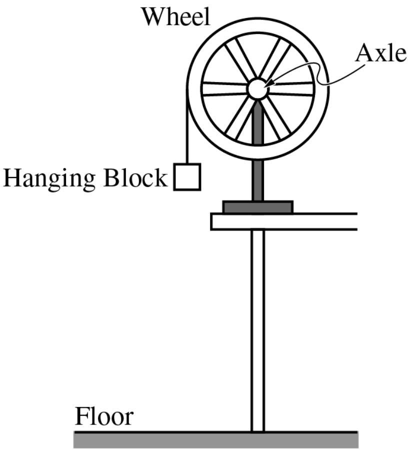
# Question 3



# Question 3


Energy (J)
0.4
0.2
0
–0.2
–0.4
5
10
15
20
d (cm)
Kinetic energy gained by the falling block
Change in gravitational potential energy of the block-wheel-Earth system

# Question 3



# Question 3

2022 ▪ Q3

[Figure: A wheel mounted on a horizontal axle, standing on a support post above a floor. A string is wrapped around the wheel's rim and hangs down to a small hanging block. Labels: "Wheel" (top, pointing to the wheel), "Axle" (pointing to the horizontal axle at the wheel's center), "Hanging Block" (pointing to the small block hanging from the string), "Floor" (the ground below).]

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in

# Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

**(b)** Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

# Question 3

2022 ▪ Q3

[Figure: A wheel mounted on a horizontal axle, standing on a support post above a floor. A string is wrapped around the wheel's rim and hangs down to a small hanging block. Labels: "Wheel" (top, pointing to the wheel), "Axle" (pointing to the horizontal axle at the wheel's center), "Hanging Block" (pointing to the small block hanging from the string), "Floor" (the ground below).]

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in

# Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

**(c)** [Graph: "Energy (J)" on the y-axis from $-0.4$ to $0.4$ J (gridlines at $-0.4, -0.2, 0, 0.2, 0.4$), "*d* (cm)" on the x-axis from 0 to 20 cm (gridlines at 5, 10, 15, 20). A solid line labeled "Kinetic energy gained by the falling block" starts near $(0, 0)$ and rises gently to about $(20, 0.15)$. A dashed line labeled "Change in gravitational potential energy of the block-wheel-Earth system" starts near $(0, 0)$ and falls to about $(20, -0.35)$.]

The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance *d*. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance *d*.

# Question 3

2022 ▪ Q3

[Figure: A wheel mounted on a horizontal axle, standing on a support post above a floor. A string is wrapped around the wheel's rim and hangs down to a small hanging block. Labels: "Wheel" (top, pointing to the wheel), "Axle" (pointing to the horizontal axle at the wheel's center), "Hanging Block" (pointing to the small block hanging from the string), "Floor" (the ground below).]

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in

# Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

**(d)(i)** The students also measure the angular velocity $\omega$ of the wheel as the block falls and determine the rotational kinetic energy $K_R$ of the wheel. The students then make a graph of $K_R$ as a function of $\omega^2$, as shown.

[Graph: "$K_R$ (J)" on the y-axis from 0.0 to 1.2 J (gridlines at 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2), "$\omega^2$ (rad/s)$^2$" on the x-axis from 0 to 40 (gridlines at 0, 10, 20, 30, 40). Data points (approximate): $(5, 0.15)$, $(8, 0.28)$, $(10, 0.35)$, $(17, 0.5)$, $(24, 0.65)$, $(32, 0.95)$, $(35, 1.15)$.]

On the above graph, draw a straight line that best represents the data.

**(d)(ii)** Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

# Solution 3

## (a) Mark scheme

- Design: measure the mass m_B of the block with a mass balance; hold the block with the string taut and measure the fall distance d (initial height above the floor) with a meterstick; release the block and use a stopwatch to measure the fall time t_B (start at release, stop when the block hits the floor); repeat 3 trials at the same d and repeat for several different values of d to reduce uncertainty (e.g. by averaging). Table: Mass of block m_B – mass balance; Distance block falls d – meterstick; Time for block to fall t_B – stopwatch. Scoring (1 point each): listing relevant equipment matching the measured quantities; listing measurements sufficient to determine the block's kinetic energy; listing measurements sufficient to determine the gravitational PE of the block-Earth system; a plausible procedure

## Solution 3

(can be done in a typical school physics lab); attempting to reduce uncertainty (e.g. repeated trials/averaging).

# Solution 3

## (b)

### Mark scheme

- Compute the block's average speed as d/t$_B$; $sinceitfallsfromrestwithconstantacceleration, thef$
$2d/t_B. ThenthefinalkineticenergyisK = \frac{1}{2}m_B v_F^2 = \frac{1}{2}m_B(2d/t_B)^2.Scoring :$
$1pointforindicatingthatmassandvelocityareneeded(neednotbethefinalvelocity); 1pointforavalic$

# Solution 3

## (c) Mark scheme

- On the Energy vs. d graph, draw a straight line through the origin with positive slope representing the kinetic energy gained by the falling block, and a line representing the change in gravitational PE such that the total energy sum is (approximately) zero at all values of d – the correct line passes through the origin and through (15 cm, 0.2 J), so that KE gained plus the (negative) change in PE sums to zero, showing energy conservation. Scoring: 1 point for drawing a straight line that passes through the origin and has a positive slope; 1 point for drawing a line that yields a total energy sum of zero at all values of d.

# Solution 3

**(d)(i)**

**Mark scheme**

- On the K$_R vs. \omega^2 graph, draw a reasonable best-fit straight line through the plotted data points. Scor$
$1 point for drawing a reasonable best-fit line.$

# Solution 3

## (d)(ii)

**Mark scheme**

- From the best-fit line, slope $= \dfrac{1.04 - 0.20}{35 - 2.5} = 0.0258$ kg · m$^2 . Since K_R = \dfrac{1}{2} I\omega^2, the slope of K_R vs. \omega^2 equals \dfrac{1}{2} I, so I = 2 \times$ slope $= 0.0517$ kg · m$^2 . Scoring : 1 point for correctly calculating a value for the slope using points on the drawn line (or stating a calculato$

# Question 4

2021 ▪ Q4

[Figure: A cylinder of mass $m_0$ sits at the top of an incline of length $L_0$ and height $H_0$, which descends to a horizontal surface. The horizontal distance from the base of the incline is also marked $L_0$.]

(7 points, suggested time 13 minutes)

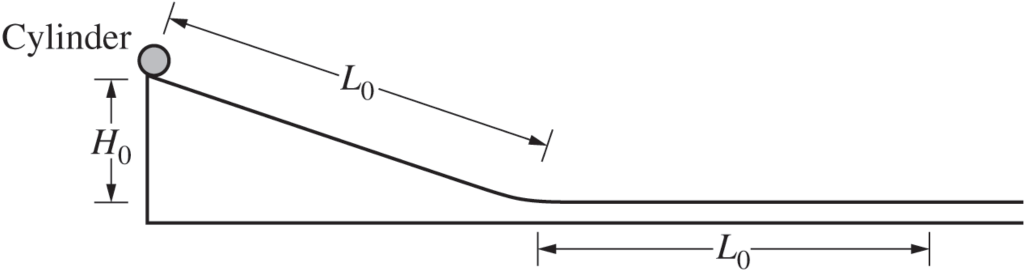
A cylinder of mass $m_0$ is placed at the top of an incline of length $L_0$ and height $H_0$, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

**(a)** On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.

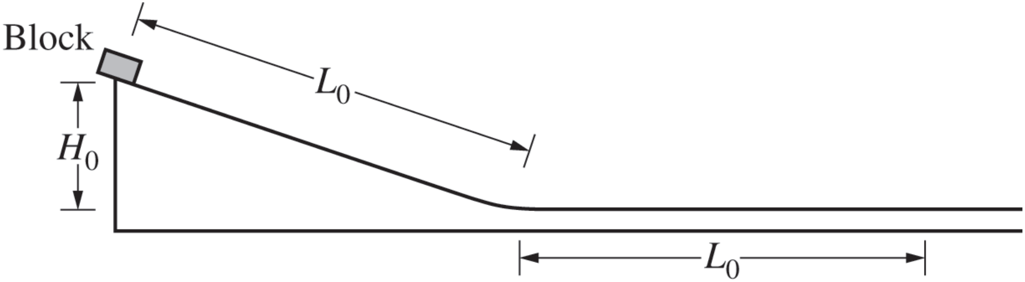
# Question 4

[Graph grid: vertical axis labeled "Total Kinetic Energy" (dashed gridlines, unlabeled numeric scale); horizontal axis labeled "Distance Traveled" with marked points at $O$, $L_0$, and $2L_0$; empty dashed grid for the student to sketch on.]

# Question 4



# Question 4



# Question 4

2021 ▪ Q4

[Figure: A cylinder of mass $m_0$ sits at the top of an incline of length $L_0$ and height $H_0$, which descends to a horizontal surface. The horizontal distance from the base of the incline is also marked $L_0$.]

(7 points, suggested time 13 minutes)

A cylinder of mass $m_0$ is placed at the top of an incline of length $L_0$ and height $H_0$, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

**(b)** [Figure: A block of mass $m_0$ sits at the top of a separate incline of length $L_0$ and height $H_0$, which descends to a horizontal surface. The horizontal distance from the base of the incline is also marked $L_0$.]

The cylinder is again placed at the top of the incline. A block, also of mass $m_0$, is placed at the top of a separate rough incline of length $L_0$ and height $H_0$, as shown

# Question 4

above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

# Solution 4

## (a)

**Mark scheme**

- Sketch: total kinetic energy rises along a straight line of positive slope from the origin as the cylinder accelerates down the incline, reaching its maximum value at distance $L_0$ (the bottom of the incline); then it stays constant (a nonzero horizontal line) from $L_0$ to $2L_0$, since the cylinder rolls without slipping (no energy loss) across the horizontal surface. Points: (1) a straight positive-slope line from the origin reaching a maximum at $L_0$, (2) a nonzero horizontal line between $L_0$ and $2L_0$.

# Solution 4

## (b) Mark scheme

- Both the cylinder and the block start with the same gravitational potential energy in their respective object-Earth systems (same mass $m_0$, same height $H_0$). As the block slides down its rough ramp, the block-Earth system's mechanical energy converts to kinetic energy, with some dissipated as heat by friction. As the cylinder rolls without slipping down its incline, the cylinder-Earth system's mechanical energy converts entirely into kinetic energy, split between translational and rotational kinetic energy (no dissipation, since rolling without slipping does no work against friction). Because both start with equal PE and the cylinder loses none of it to dissipation while the block does, the cylinder's final rotational kinetic energy equals the amount of the block-Earth system's initial mechanical energy

## Solution 4

that was dissipated by friction. Points: (1) both objects start with the same gravitational PE, (2) a correct statement of the cylinder's energy transformations, (3) a correct statement of the block's energy transformations, (4) indicating the cylinder's final rotational KE equals the block-Earth system's energy dissipated by friction, (5) a logical, internally consistent paragraph-length argument.

# Question 1

2016 ▪ Q1

A wooden wheel of mass $M$, consisting of a rim with spokes, rolls down a ramp that makes an angle $\theta$ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.
[Figure at top of page: A wheel of mass $M$ sits at the top of an inclined ramp that makes angle $\theta$ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]
**(a)(i)** On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.

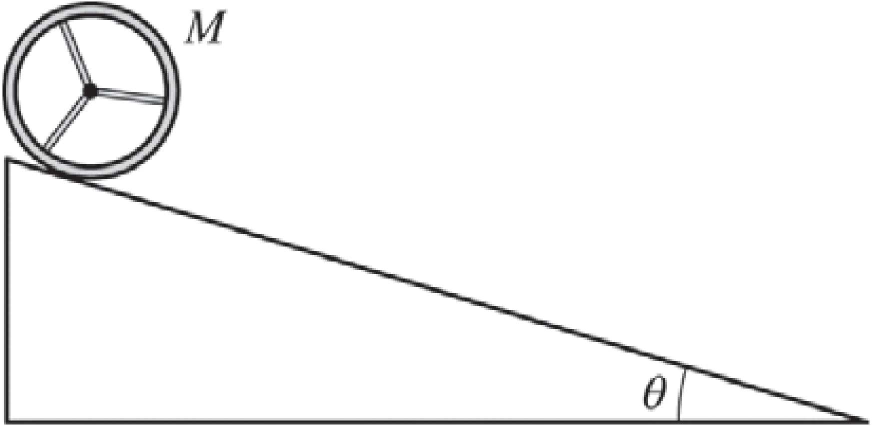
# Question 1

[Diagram: a wheel with spokes resting on a dashed line representing the ramp surface, for the student to draw force vectors on.]

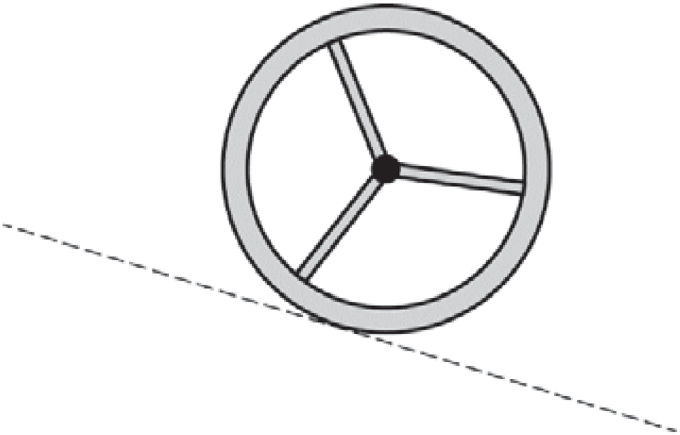
**(a)(ii)** As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

# Question 1



# Question 1



# Question 1

2016 ▪ Q1

A wooden wheel of mass $M$, consisting of a rim with spokes, rolls down a ramp that makes an angle $\theta$ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

[Figure at top of page: A wheel of mass $M$ sits at the top of an inclined ramp that makes angle $\theta$ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]

**(b)** For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of $M$, $\theta$, and physical constants, as appropriate.

# Question 1

2016 ▪ Q1

A wooden wheel of mass $M$, consisting of a rim with spokes, rolls down a ramp that makes an angle $\theta$ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

[Figure at top of page: A wheel of mass $M$ sits at the top of an inclined ramp that makes angle $\theta$ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]

**(c)** In a second experiment on the same ramp, a block of ice, also with mass $M$, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

**(c)(i)** Which object, if either, reaches the bottom of the ramp with the greatest speed?

_____ Wheel _____ Block _____ Neither; both reach the bottom with the same speed.

# Question 1

Briefly explain your answer, reasoning in terms of forces.

**(c)(ii)** Briefly explain your answer again, now reasoning in terms of energy.

# Solution 1

## (a)(i) Mark scheme

- Force diagram on the wheel: (1) a labeled arrow for the gravitational force $mg$, starting at the wheel's center and pointing straight down [1 pt]. (2) labeled arrows for the friction force $F_f$ and normal force $F_n$ (or a single arrow for their resultant), with no extraneous forces [1 pt] — $F_f$ starts at the wheel-ramp contact point and points up the ramp (along the ramp surface); $F_n$ starts at the wheel-ramp contact point and is perpendicular to the ramp, directed toward the wheel's center (does not need to pass exactly through the center, just reasonably close). Scoring: 1 pt for the correct $mg$ arrow from the center downward; 1 pt for correct $F_f/F_n$ (or resultant) arrows at the contact point with correct directions and no extra forces.

# Solution 1

## (a)(ii)

**Mark scheme**

- Correct answer: the friction force. No credit if the wrong force is named. Explanation: friction is the only one of the three forces ($mg$, $F_n$, $F_f$) that exerts a torque about the wheel's center of mass — both $mg$ and $F_n$ act through/toward the center, so they produce no torque about it — so friction is what changes the wheel's angular velocity. Full credit is also given for a causal-chain answer: gravity leads to a normal force (and acceleration down the ramp), which leads to a frictional force, which exerts a torque that changes the angular velocity.

# Solution 1

## (b) Mark scheme

- Along the ramp: $\sum F_{\|} = Mg\sin\theta - F_f$ [1 pt for recognizing there are two force components parallel to the ramp; the expression need not be correct or consistent with the force diagram in part (a)]. The friction force is 40

# Solution 1

## (c)

**Mark scheme**

- Context only, no response scored on this leaf: describes a second experiment on the same ramp in which a block of ice, also of mass $M$, is released from rest at the same instant and same height as the wheel, sliding down with negligible friction. This setup is used by parts (c)(i) and (c)(ii).

# Solution 1

## (c)(i)

**Mark scheme**

- Correct answer: Block. No credit for an answer without explanation. Explanation in terms of forces: the wheel experiences a counteracting frictional force (which reduces its linear acceleration along the ramp), so the block — with negligible friction — has a greater net force along the ramp and therefore greater acceleration, reaching the bottom with greater speed.

# Solution 1

## (c)(ii)

**Mark scheme**

- Explanation in terms of energy conservation: both the wheel-Earth system and the block-Earth system lose the same amount of gravitational potential energy (same mass, same height drop), so they gain the same total kinetic energy. With the ice block — but not the wheel — all of that kinetic energy is translational, and none is rotational, so the block's translational (linear) speed at the bottom is greater than the wheel's.