Exercise walk-through

Rotational Kinetic Energy ▪ 6.1

eXercise

You must be able to

- define **rotational kinetic energy 转动动能** as $K_{rot} = \frac{1}{2} I\omega^2$
- explain that it depends on both the **rotational inertia** and the **angular velocity**
- find the **total kinetic energy** of an object that both translates and rotates
- apply **conservation of energy** to systems that include rotation

Method — Rotational kinetic energy

$K_{rot} = \frac{1}{2} I\omega^2$ — the rotational partner of $\frac{1}{2} mv^2$.

Method — Total kinetic energy of a rolling body

$$K = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \omega^2.$$

Method — A rolling race

Rolling down a ramp, a shape with smaller $I/(mR^2)$ ties up less energy in rotation, so it reaches the bottom **faster**. A solid sphere beats a hoop.

Practice 2.1 ▪ 2 marks

A flywheel of $I = 0.50$ kg m^2 spins at $\omega = 4.0$ rad s^{-1}. Find its rotational kinetic energy.

Solution 2.1

Method: Rotational kinetic energy

Worked steps

$K_{rot} = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J **M1** **A1**.

Practice 2.2 ▪ 1 mark

A rolling ball has 6.0 J of translational and 2.4 J of rotational kinetic energy. Find its total kinetic energy.

Solution 2.2

Method: Total kinetic energy of a rolling body

Worked steps

$K = 6.0 + 2.4 = 8.4$ J **B1**.

Practice 2.3 ▪ 2 marks

State why a solid sphere rolls down a ramp faster than a hoop of the same mass and radius.

Solution 2.3

Method: A rolling race

Worked steps

The sphere has a smaller rotational inertia for its mass, so less energy goes into rotation B1 and more into translation, giving a larger speed B1.

Exam-style 3.1 ▪ 1 mark

Rotational kinetic energy is

A $\frac{1}{2}I\omega$

B $\frac{1}{2}I\omega^2$

C $I\omega$

D $\frac{1}{2}mv^2$

Solution 3.1

Method: Rotational kinetic energy

A $\frac{1}{2}I\omega$

B $\frac{1}{2}I\omega^2$ ✓

C $I\omega$

D $\frac{1}{2}mv^2$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A hoop and a disc of equal mass and radius roll from rest down the same ramp. The one reaching the bottom first is the

A hoop

B disc

C they arrive together

D it depends on the mass

Solution 3.2

Method: A rolling race

A hoop

B disc ✓

C they arrive together

D it depends on the mass

Worked steps

B — the disc has the smaller rotational inertia.

Exam-style 3.3 ▪ 2 marks

A disc of $I = 0.10$ kg m^2 spins at $\omega = 12$ rad s^{-1}.
(a) Find its rotational kinetic energy.
(b) If this energy came from a 0.20 kg mass falling, find the height it fell (take $g = 10$ m s^{-2}).

Solution 3.3

Method: Rotational kinetic energy

Worked steps

(a) $K_{rot} = \frac{1}{2}(0.10)(12)^2 = 7.2$ J M1 A1. (b) $mgh = 7.2 \Rightarrow h = \dfrac{7.2}{0.20 \times 10} = 3.6$ m M1 A1.