Past-paper walk-through

Newton's Second Law in Rotational Form ▪ 5.6

eXercise

Questions in this set

- 2026 Q4
- 2024 Q3
- 2023 Q4
- 2021 Q5
- 2019 Q2
- 2016 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2026 ▪ Q4

Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

[Figure 1 (Side View): Two spinning-top-style toys shown side by side. Toy X (left): a vertical axle labeled "Axle" at the top with radius r_0 marked at the top of the "Upper Portion" cylindrical section; "String" labels the wrapped string near the top of the upper portion; an arrow labeled F_0 points horizontally to the right from the upper portion; below the upper portion is a wide, ridged, bulbous body tapering to a point at the bottom, with a small curved arrow near the point indicating rotation. Toy Y

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(right): the same axle/radius r_0/"Upper Portion"/"String"/F_0 labeling at the top, but the lower body is a flatter shape with a wide horizontal disk (like a ring) partway down, tapering to a point at the bottom with a rotation arrow, similar to Toy X but with different mass distribution.]

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

[Figure 2 (Overhead View): A large shaded circle (the upper portion of a toy, viewed from above) with a small labeled circle at its center marked r_0, and an arrow labeled F_0 pointing to the right from the center, representing the string being pulled

Question 4

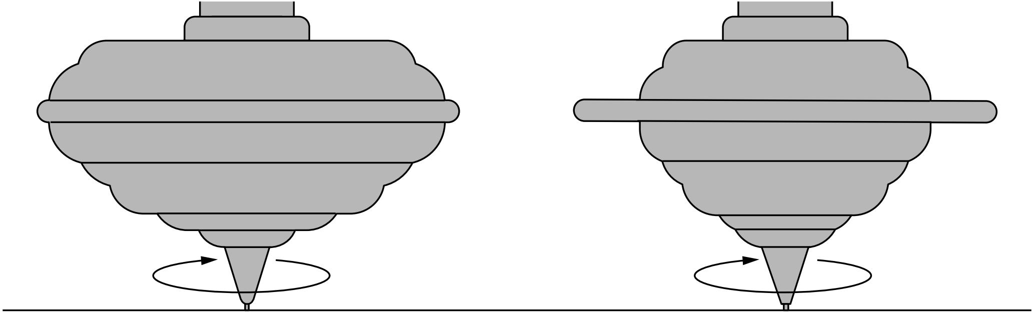
tangentially. A curved arrow above and to the left of the circle indicates the direction of rotation (counterclockwise).]

(a) Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

Question 4



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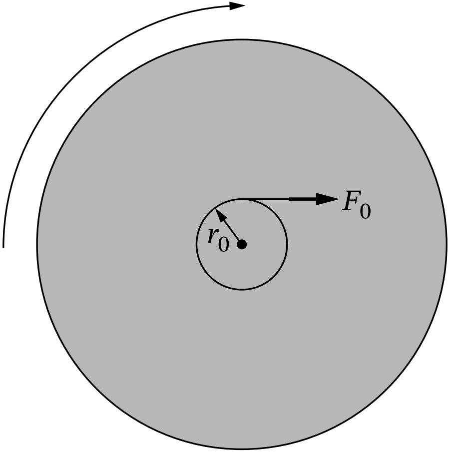

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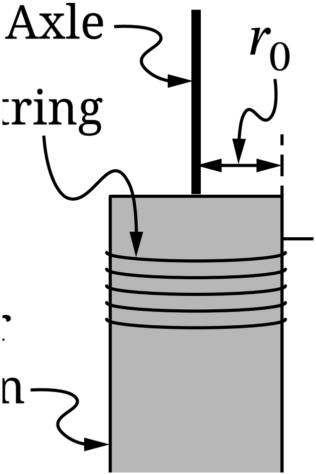

Question 4

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Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

[Figure 1 (Side View): Two spinning-top-style toys shown side by side. Toy X (left): a vertical axle labeled "Axle" at the top with radius r_0 marked at the top of the "Upper Portion" cylindrical section; "String" labels the wrapped string near the top of the upper portion; an arrow labeled F_0 points horizontally to the right from the upper portion; below the upper portion is a wide, ridged, bulbous body tapering to a point at the bottom, with a small curved arrow near the point indicating rotation. Toy Y (right): the same axle/radius r_0/"Upper Portion"/"String"/F_0 labeling at the top, but the lower body is a flatter shape with a wide horizontal disk (like a ring) partway down, tapering to a point at the bottom with a rotation arrow, similar to Toy X but with different mass distribution.]

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Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.
When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.
When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.
[Figure 2 (Overhead View): A large shaded circle (the upper portion of a toy, viewed from above) with a small labeled circle at its center marked r_0, and an arrow labeled F_0 pointing to the right from the center, representing the string being pulled tangentially. A curved arrow above and to the left of the circle indicates the direction of rotation (counterclockwise).]
(b) Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0, I_X, F_0, and physical

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constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

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When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

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(c) Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

Question 3

2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

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(a) [A rectangle representing the beam, drawn horizontally, blank for the student to draw and label force vectors.]
The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

Question 3

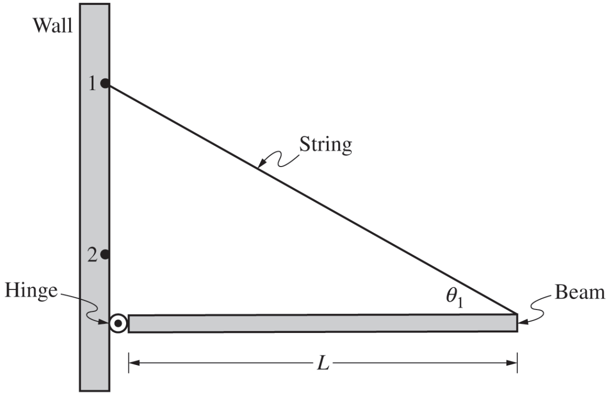

Figure 1

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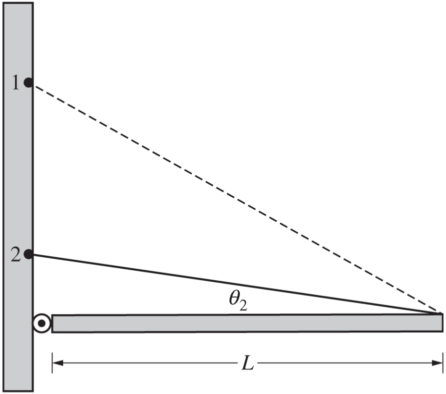

Figure 2

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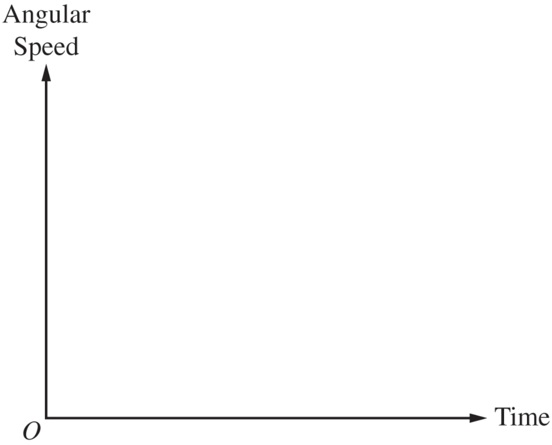

Question 3

2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(b) [Figure 2: The same wall, hinge, and beam of length L as in Figure 1, but now the string (solid line) connects from Point 2 on the wall (lower than Point 1) to the right

Question 3

end of the beam, making angle θ_2 with the beam. A dashed line is also shown from Point 1 to the right end of the beam, representing the string's position from Figure 1.]
(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.
The magnitude of the tension in the string shown in Figure 1 is F_{T1}. The magnitude of the tension in the string shown in Figure 2 is F_{T2}. **Indicate** which of the following correctly compares F_{T2} with F_{T1}.
$F_{T2} > F_{T1}$ $F_{T2} < F_{T1}$ $F_{T2} = F_{T1}$
Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

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2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(c) (c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M, θ,

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and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

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2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

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(d) (d) Is your derived equation in part (c) consistent with your justification in part (b)? **Explain** your reasoning.

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2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(e) [A blank set of axes with vertical axis labeled "Angular Speed" and horizontal axis labeled "Time", origin labeled O.]

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(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.

Solution 3

(a) Mark scheme

- Draw three force vectors on the beam, each starting on and pointing away from where the force acts, representing a system in equilibrium: the gravitational force ($F_g/F_G/W/mg/Mg/$"grav force" — not G or g alone) drawn downward from the center of the beam; the tension force ($F_T/F_s/F_{\text{string}}/$"tension force") drawn upward and to the left from the right end of the beam (along the string toward the wall); and the hinge/wall (normal) force ($F_N/F_n/$N/"normal force"/"wall force") drawn upward and to the right from the left end (hinge). The three vectors must show the beam in equilibrium (net force zero). Points: downward gravitational force at the beam's center (1 point); tension force at the right end

Solution 3

directed up and to the left (1 point); a diagram with all three force vectors consistent with equilibrium (1 point).

Solution 3

(b) Mark scheme

- Select $F_{T2} > F_{T1}$ (the tension is larger when the string attaches lower, at Point 2, making a smaller angle with the beam). Justification: for the beam to remain horizontal and in equilibrium, the torque exerted by the string about the hinge must stay the same regardless of attachment point. As the angle θ between the string and beam decreases, the perpendicular (torque-producing) component of the tension for a given tension magnitude decreases, so a larger tension is needed to supply the same torque — equivalently, the vertical component of the tension must still balance the same weight of the bar, and a smaller angle means more tension is needed to achieve the same vertical component. Points: selecting $F_{T2} > F_{T1}$ with an attempt at a relevant justification (1 point); a justification

Solution 3

that relates either the vertical component of tension to the beam's weight, or the force needed to produce the same torque to the string's angle (1 point).

Solution 3

(c) Mark scheme

- Derive F_T from Newton's second law in rotational form: $\Sigma\tau = I\alpha$ (with $\alpha = 0$ since the beam is in equilibrium), so $\tau_{\text{gravity}} + \tau_{\text{string}} = 0$. Taking torques about the hinge: $(F_g)\frac{L}{2}\sin 90° - F_T(L)\sin\theta = 0 \Rightarrow Mg\frac{L}{2} = (F_T\sin\theta)L \Rightarrow F_T = \dfrac{Mg}{2\sin\theta}$. Points: a multi-step derivation beginning with $\Sigma\tau = I\alpha$ or $\Sigma\tau = 0$ (1 point); indicating that the net torque is zero OR the net force is zero (1 point); indicating one of — the torque from the string is $(F_T\sin\theta)L$, the torque from the beam's weight is $Mg\frac{L}{2}$, or the correct final answer $F_T = \dfrac{Mg}{2\sin\theta}$ (1 point).

Solution 3

(d) Mark scheme

- The derived equation $F_T = \dfrac{Mg}{2\sin\theta}$ is consistent with the part (b) reasoning: tension is inversely proportional to $\sin\theta$, and for $\theta < 90^\circ$, $\sin\theta$ decreases as θ decreases, so the required tension is greater for smaller angles — matching the claim that $F_{T2} > F_{T1}$ when the string is attached lower (smaller angle). Points: an attempt to use the functional dependence of the part (c) equation to relate to the part (b) reasoning, even if not fully correct (1 point); an explanation that correctly relates the two (1 point).

Solution 3

(e)

Mark scheme

- Sketch the angular speed of the beam vs. time (from the moment the string is cut until the beam becomes vertical) as a curve that starts at the origin, is monotonically increasing throughout the interval, and is concave down (the rate of increase in angular speed decreases over time, since the gravitational torque about the hinge shrinks as the beam rotates toward vertical). Points: drawing a graph that is monotonically increasing (1 point); drawing a concave-down curve (1 point).

Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

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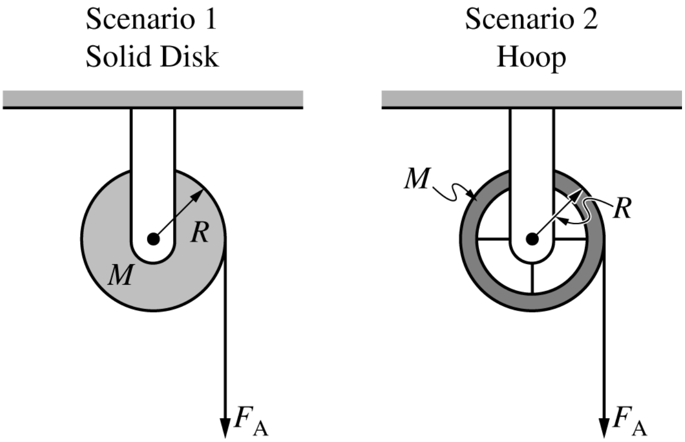
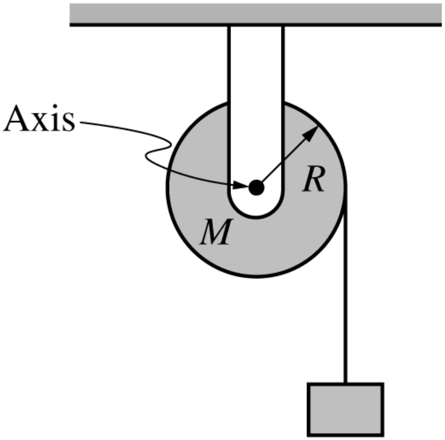
When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_{T}.

(a) Determine an expression for the magnitude of the angular acceleration α_{D} of the disk as the block travels downward. Express your answer in terms of M, R, F_{T}, and physical constants as appropriate.

Question 4



Question 4



Question 4

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[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

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When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

Question 4

(b) [Continuation of Question 4.] [Figure: Two side-by-side diagrams. "Scenario 1 — Solid Disk": a shaded disk of mass M and radius R mounted on a horizontal axle, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down). "Scenario 2 — Hoop": a shaded hoop (ring) of mass M and radius R mounted on a horizontal axle in the same way, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down).]

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without

Question 4

the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Solution 4

(a)

Mark scheme

- $\alpha_D = \dfrac{2F_T}{MR}$. Derivation: for the solid disk (pulley), $\tau = I\alpha$ with $\tau = RF_T$ and $I = \frac{1}{2}MR^2$: $RF_T = \frac{1}{2}MR^2\,\alpha_D \Rightarrow \alpha_D = \dfrac{RF_T}{\frac{1}{2}MR^2} = \dfrac{2F_T}{MR}$ (either equivalent form is accepted).

Solution 4

(b) Mark scheme

- Paragraph argument comparing a solid disk and a hoop (each mass M, radius R) each pulled by the same force F_A via a string wrapped around the rim: the disk ends up with GREATER rotational kinetic energy than the hoop. Reasoning, worth 6 points total: (1) the torque $\tau = F_A R$ is the same for both pulleys, since the same force is applied at the same radius; (2) since torque and the time interval are the same for both, the angular impulse $\tau \Delta t$ (equivalently the change in angular momentum ΔL) is the same for the disk and the hoop; (3) the rotational inertia I differs between the disk and hoop: $I_{\text{disk}} = \frac{1}{2}MR^2 < I_{\text{hoop}} = MR^2$, since the hoop's mass is concentrated at the rim while the disk's is spread through its area; (4) because $\Delta L = I\Delta\omega$ is the same for both but $I_{\text{hoop}} > I_{\text{disk}}$, the disk must

Solution 4

gain a larger $\Delta\omega$ (and correspondingly larger $\Delta\theta$, α) than the hoop — the kinematic quantities differ because the rotational inertias differ; (5) one of: relating I and ω to show ΔK is greater for the disk (e.g. using $\omega = L/I$ and $K = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller I converts the same angular momentum into more kinetic energy), OR noting $\Delta\theta$ is greater for the disk so the work done on the disk ($\tau\Delta\theta$) is greater; (6) an overall logical, relevant, and internally consistent paragraph following the AP paragraph-response format that ties these steps together to conclude the disk has the greater rotational kinetic energy.

Question 5

2021 ▪ Q5

[Figure: Two pulleys of different radii are mounted concentrically on a common horizontal axle. The larger pulley (radius $2r_0$) is on the outside, the smaller pulley (radius r_0) is on the inside, both centered on the "Axle at Center of Pulleys." Object 1 (mass m_0) hangs from a light string wrapped around the larger pulley on the left side. Object 2 (mass $1.5m_0$) hangs from a light string wrapped around the smaller pulley on the right side.]

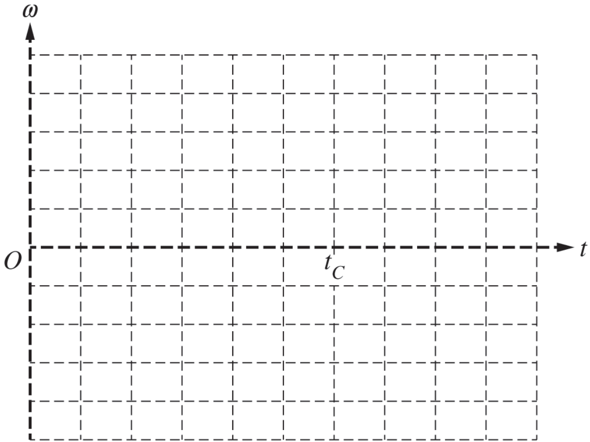
(7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

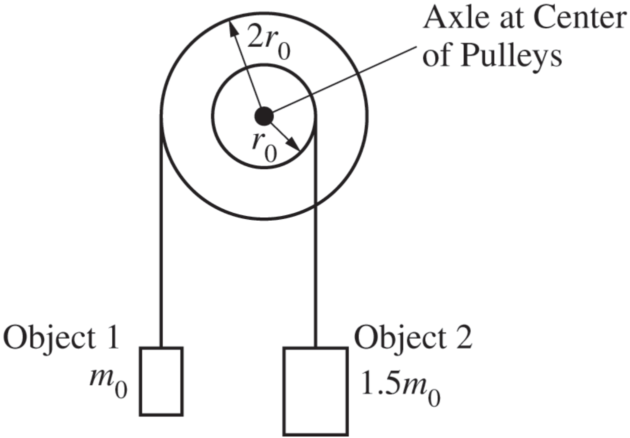
Question 5

m_0 is the mass of object 1. $1.5m_0$ is the mass of object 2. r_0 is the radius of the smaller pulley. $2r_0$ is the radius of the larger pulley.
At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.
(a)(i) Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.
(a)(ii) Object 1 accelerates downward after the pulleys are released. Briefly explain why.

Question 5



Question 5



Question 5

2021 ▪ Q5

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Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.
m_0 is the mass of object 1. $1.5m_0$ is the mass of object 2. r_0 is the radius of the smaller pulley. $2r_0$ is the radius of the larger pulley.

At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

Question 5

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

_____ Same direction _____ Opposite directions

Briefly explain your reasoning.

Question 5

2021 ▪ Q5

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(7 points, suggested time 13 minutes)
Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.
m_0 is the mass of object 1. $1.5m_0$ is the mass of object 2. r_0 is the radius of the smaller pulley. $2r_0$ is the radius of the larger pulley.

At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

Question 5

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.
[Graph grid: vertical axis labeled "ω" (dashed gridlines, unlabeled numeric scale); horizontal axis labeled "t" with marked points at O and t_C; empty dashed grid for the student to sketch on.]

Solution 5

(a)(i)

Mark scheme

- Torque from object 1's weight: $\tau_1 = m_0 g(2r_0)$ (weight $m_0 g$ at lever arm $2r_0$). Torque from object 2's weight: $\tau_2 = (1.5m_0 g)r_0$ (weight $1.5m_0 g$ at lever arm r_0). The two torques act in opposite directions, so the net torque is their difference: $\tau_{net} = \tau_1 - \tau_2 = m_0 g(2r_0) - (1.5m_0 g)r_0 = 0.5\, m_0 g r_0$. Points: (1) correct torque expressions for each object's weight, (2) indicating the two torques act in opposite directions, (3) the correct derived answer $0.5m_0 g r_0$.

Solution 5

(a)(ii)

Mark scheme

- Object 1 exerts the larger torque: it is twice as far from the axle ($2r_0$ vs. r_0) while its weight is only m_0g compared to object 2's $1.5m_0g$ - the larger lever arm more than compensates for the smaller weight, so object 1's torque dominates and sets the net-torque direction.

Solution 5

(b)

Mark scheme

- Correct answer: opposite directions. Object 1 (the larger torque) had been determining the net (counterclockwise) torque direction; once its string is cut, the net torque comes only from object 2, which acts in the opposite (clockwise) sense, so the angular acceleration immediately reverses direction. The angular velocity, however, cannot change instantaneously and is still in its original (counterclockwise) direction at that moment - so immediately after the cut, angular velocity and angular acceleration point in opposite directions.

Solution 5

(c) Mark scheme

- Sketch: ω starts at 0 at $t = 0$ and increases linearly (constant nonzero slope, from the constant net angular acceleration while both objects are attached) up to $t = t_C$; at $t = t_C$ the slope abruptly changes sign (angular acceleration reverses, per part (b)) while ω itself remains continuous (no jump); after t_C, ω decreases linearly, eventually crossing to negative values as the pulley's rotation slows and reverses. Points: (1) a linear graph between 0 and t_C with initial angular velocity zero and nonzero slope (sign either way), (2) a sign change in the slope at $t = t_C$ with no discontinuity.

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)(i) Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

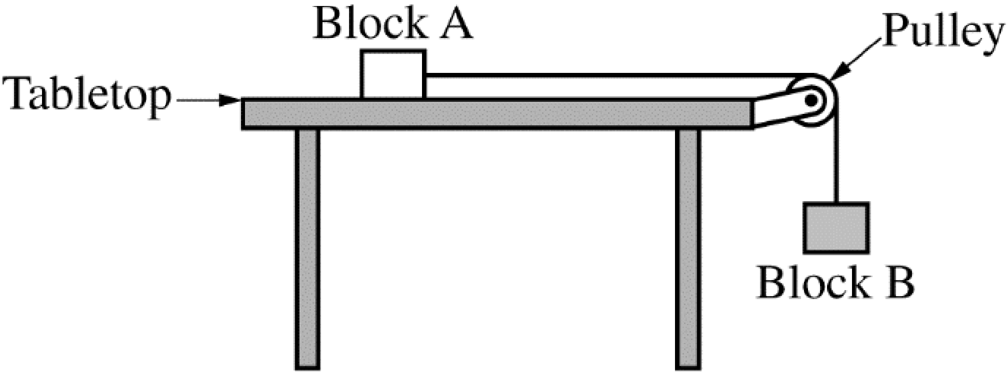
Question 2

Briefly explain your reasoning without deriving or using equations.

(a)(ii) Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

Question 2



Question 2

Question 2

2019 ▪ Q2

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(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on

Question 2

that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.
[Diagram: Two separate large dots side by side on the page, one labeled "Block A" below it and the other labeled "Block B" below it, for the student to draw force vectors on.]

Question 2

2019 ▪ Q2

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(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than

Question 2

what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

Question 2

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

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(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is

Question 2

not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

Solution 2

(a)(i) Mark scheme

- Correct claim: the acceleration is nearly zero / small / negligible / much less than g / '$\ll$g'. Point 1: for a correct qualitative answer with an attempted, consistent justification. Point 2: for correct reasoning. Reasoning: block A has a much greater mass (much greater inertia) than block B, so block A strongly resists acceleration and the light block B cannot meaningfully accelerate it; the acceleration of the connected system is therefore nearly zero. (Example 2-point answers: 'Very small, because block A has a large inertia, it won't speed up much.' / 'Close to zero because block B is so light that it can hardly budge block A.')

Solution 2

(a)(ii)

Mark scheme

- Correct claim: the acceleration is nearly g (acceptable: 'g', '9.8 m/s^2', '10 m/s^2', or just '9.8'/'10'). The single point is earned for a correct answer WITH correct justification. Reasoning: block A has negligible mass compared to block B and negligible friction with the tabletop, so essentially no tension is needed to accelerate block A; block B is therefore nearly in free fall, and the string tension is nearly zero, giving the whole system an acceleration close to g.

Solution 2

(b) Mark scheme

- Free-body diagram, Block A (on the tabletop): three distinct force arrows – normal force N (or F_N) pointing straight UP from the table; gravity F_g (or mg, W, $m_A g$) pointing straight DOWN; tension F_T pointing horizontally toward the pulley (i.e. toward block B's side). Free-body diagram, Block B (hanging): two distinct force arrows – tension F_T pointing straight UP (toward the pulley); gravity F_g (or $m_B g$) pointing straight DOWN. Points: 1 for a correct, correctly labeled normal force on block A only (acceptable labels: N, F_N, 'normal force', F_{table}, 'table force', or any label indicating a normal/table force); 1 for correct, correctly labeled gravitational forces on BOTH diagrams (acceptable: F_g, F_{grav}, W, mg, $m_A g$ – NOT G or g alone – and no extraneous forces on either diagram); 1 for correct,

Solution 2

correctly labeled tension forces on BOTH diagrams (acceptable: 'tension', 'string force', F_T, $F_{tension}$, F_{string}, F_S, T – NOT $m_B g$, F_{m_B}, or 'force from block B'). Forces must be drawn as distinct arrows starting on and pointing away from the dot, not components.

Solution 2

(c) Mark scheme

- Apply Newton's second law separately to each block, then combine. Block B (down positive): $m_B g - T = m_B a$. Block A (horizontal): $T = m_A a$. Since the string is inextensible and the tension magnitude is the same on both blocks, combine to eliminate T: $m_B g - m_A a = m_B a \Rightarrow a = \dfrac{m_B}{m_A + m_B} g$. Alternate whole-system method: treat both blocks + string as one system of total mass $m_A + m_B$ with no net internal force; the only external force along the direction of motion is gravity on block B, so $F_{net} = m_B g = (m_A + m_B)a$, giving the same $a = \dfrac{m_B}{m_A + m_B} g$. Points: 1 for using separate Newton's-second-law equations for each block (or the equivalent whole-system equation containing no internal

Solution 2

forces); 1 for correctly combining the equations with correct notation, using m_A and m_B, correctly indicating that the same tension force acts on both blocks and that they share the same acceleration; 1 for the correct final equation with supporting work: $a = \dfrac{m_B}{m_A + m_B} g$.

Solution 2

(d) Mark scheme

- Correct answer: 'Yes' – the equation from part (c), $a = \dfrac{m_B}{m_A + m_B} g$, agrees with the reasoning in part (a)(ii). The single point is earned for valid reasoning connecting the two: when $m_A \ll m_B$, the m_A term in the denominator can be neglected, so $(m_A + m_B) \approx m_B$ and $a \approx \dfrac{m_B}{m_B} g = g$, matching the claim in part (a)(ii) that the acceleration approaches g. ('No' is acceptable ONLY if it is consistent with an equation from part (c) that does not actually reduce to g in this limit, or with an inconsistent answer given in (a)(ii).)

Solution 2

(e) Mark scheme

- Correct answer: $T_2 > T_1$ (a maximum of 1 point can be earned if the wrong option is selected). Point 1: for reasoning that the acceleration of both blocks is SMALLER in the second (heavier-pulley) scenario, because the pulley's now-nonzero rotational inertia requires torque/energy to spin it up, leaving less net force available to accelerate the blocks. Point 2: for connecting this, via Newton's second law applied to block B ($\vec{a} = \sum \vec{F}/m$, i.e. $T = m_B g - m_B a$ or $m_B g - T = m_B a$), to conclude that a SMALLER acceleration of block B implies a LARGER tension T_2 (closer in magnitude to the gravitational force on block B than T_1 was).

Question 1

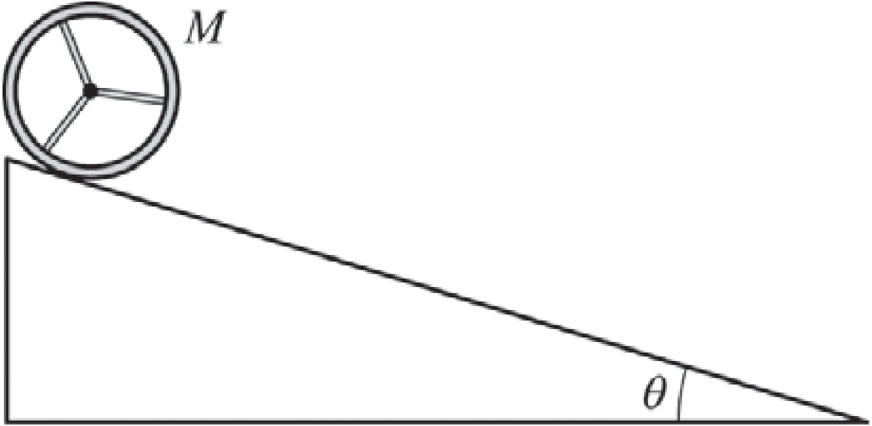
2016 ▪ Q1

A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.
[Figure at top of page: A wheel of mass M sits at the top of an inclined ramp that makes angle θ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]
(a)(i) On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.

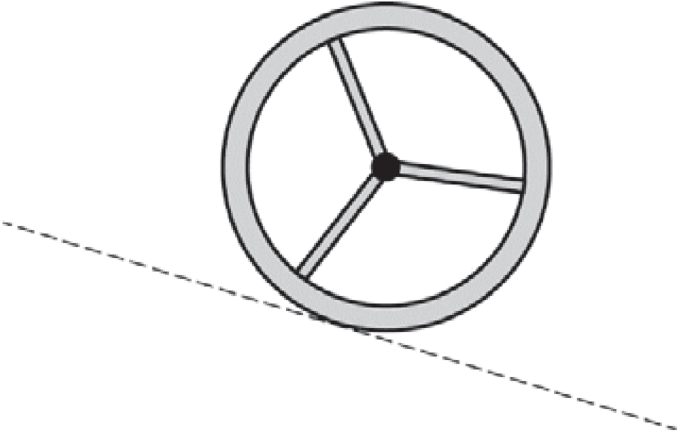
Question 1

[Diagram: a wheel with spokes resting on a dashed line representing the ramp surface, for the student to draw force vectors on.]
(a)(ii) As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?
Briefly explain your reasoning.

Question 1



Question 1



Question 1

2016 ▪ Q1

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[Figure at top of page: A wheel of mass M sits at the top of an inclined ramp that makes angle θ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]

(b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

Question 1

2016 ▪ Q1

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[Figure at top of page: A wheel of mass M sits at the top of an inclined ramp that makes angle θ with the horizontal (a right triangle shown in profile, with the wheel resting at the top of the incline).]

(c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

(c)(i) Which object, if either, reaches the bottom of the ramp with the greatest speed?

______ Wheel ______ Block ______ Neither; both reach the bottom with the same speed.

Question 1

Briefly explain your answer, reasoning in terms of forces.

(c)(ii) Briefly explain your answer again, now reasoning in terms of energy.

Solution 1

(a)(i) Mark scheme

- Force diagram on the wheel: (1) a labeled arrow for the gravitational force mg, starting at the wheel's center and pointing straight down [1 pt]. (2) labeled arrows for the friction force F_f and normal force F_n (or a single arrow for their resultant), with no extraneous forces [1 pt] — F_f starts at the wheel-ramp contact point and points up the ramp (along the ramp surface); F_n starts at the wheel-ramp contact point and is perpendicular to the ramp, directed toward the wheel's center (does not need to pass exactly through the center, just reasonably close). Scoring: 1 pt for the correct mg arrow from the center downward; 1 pt for correct F_f/F_n (or resultant) arrows at the contact point with correct directions and no extra forces.

Solution 1

(a)(ii)

Mark scheme

- Correct answer: the friction force. No credit if the wrong force is named. Explanation: friction is the only one of the three forces (mg, F_n, F_f) that exerts a torque about the wheel's center of mass — both mg and F_n act through/toward the center, so they produce no torque about it — so friction is what changes the wheel's angular velocity. Full credit is also given for a causal-chain answer: gravity leads to a normal force (and acceleration down the ramp), which leads to a frictional force, which exerts a torque that changes the angular velocity.

Solution 1

(b) Mark scheme

- Along the ramp: $\sum F_{\parallel} = Mg\sin\theta - F_f$ [1 pt for recognizing there are two force components parallel to the ramp; the expression need not be correct or consistent with the force diagram in part (a)]. The friction force is 40

Solution 1

(c)

Mark scheme

- Context only, no response scored on this leaf: describes a second experiment on the same ramp in which a block of ice, also of mass M, is released from rest at the same instant and same height as the wheel, sliding down with negligible friction. This setup is used by parts (c)(i) and (c)(ii).

Solution 1

(c)(i)

Mark scheme

- Correct answer: Block. No credit for an answer without explanation. Explanation in terms of forces: the wheel experiences a counteracting frictional force (which reduces its linear acceleration along the ramp), so the block — with negligible friction — has a greater net force along the ramp and therefore greater acceleration, reaching the bottom with greater speed.

Solution 1

(c)(ii)

Mark scheme

- Explanation in terms of energy conservation: both the wheel-Earth system and the block-Earth system lose the same amount of gravitational potential energy (same mass, same height drop), so they gain the same total kinetic energy. With the ice block — but not the wheel — all of that kinetic energy is translational, and none is rotational, so the block's translational (linear) speed at the bottom is greater than the wheel's.