Past-paper walk-through

Rotational Inertia ▪ 5.4

eXercise

Questions in this set

- 2026 Q4
- 2023 Q4
- 2022 Q3
- 2019 Q2
- 2017 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 4

2026 ▪ Q4

Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

[Figure 1 (Side View): Two spinning-top-style toys shown side by side. Toy X (left): a vertical axle labeled "Axle" at the top with radius r_0 marked at the top of the "Upper Portion" cylindrical section; "String" labels the wrapped string near the top of the upper portion; an arrow labeled F_0 points horizontally to the right from the upper portion; below the upper portion is a wide, ridged, bulbous body tapering to a point at the bottom, with a small curved arrow near the point indicating rotation. Toy Y

Question 4

(right): the same axle/radius r_0/"Upper Portion"/"String"/F_0 labeling at the top, but the lower body is a flatter shape with a wide horizontal disk (like a ring) partway down, tapering to a point at the bottom with a rotation arrow, similar to Toy X but with different mass distribution.]

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

[Figure 2 (Overhead View): A large shaded circle (the upper portion of a toy, viewed from above) with a small labeled circle at its center marked r_0, and an arrow labeled F_0 pointing to the right from the center, representing the string being pulled

Question 4

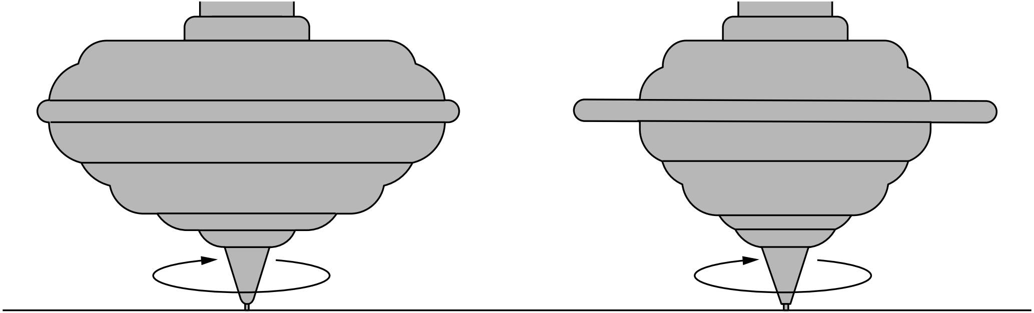
tangentially. A curved arrow above and to the left of the circle indicates the direction of rotation (counterclockwise).]

(a) Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

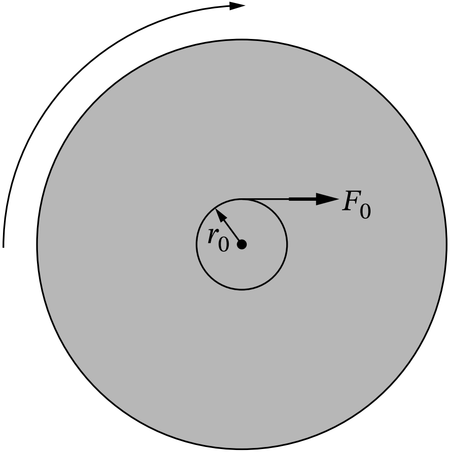
- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

Question 4



Question 4



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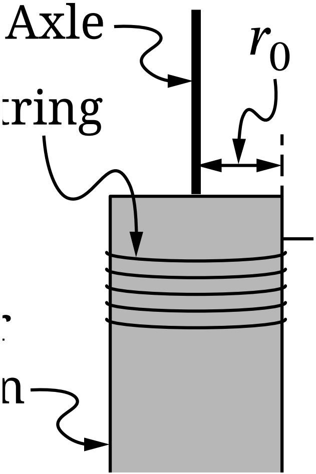

Question 4

2026 ▪ Q4

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Question 4

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(b) Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0, I_X, F_0, and physical

Question 4

constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

2026 ▪ Q4

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Question 4

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(c) Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

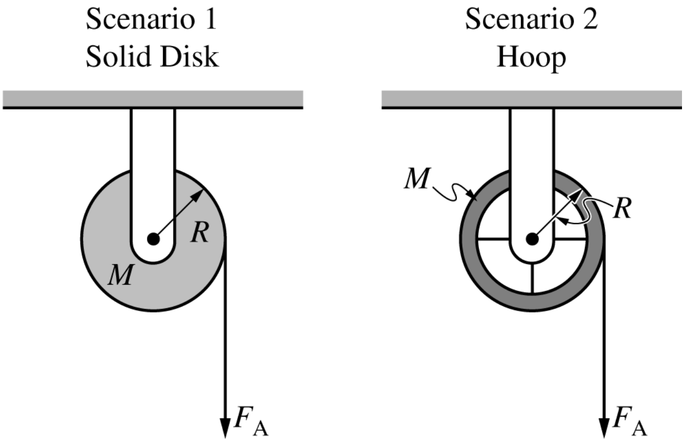
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

Question 4

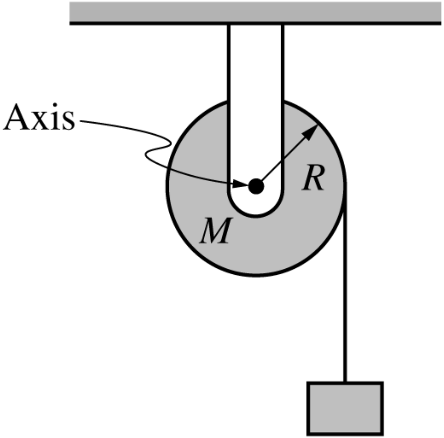
When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_{T}.

(a) Determine an expression for the magnitude of the angular acceleration α_{D} of the disk as the block travels downward. Express your answer in terms of M, R, F_{T}, and physical constants as appropriate.

Question 4



Question 4



Question 4

2023 ▪ Q4

[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

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When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

Question 4

(b) [Continuation of Question 4.] [Figure: Two side-by-side diagrams. "Scenario 1 — Solid Disk": a shaded disk of mass M and radius R mounted on a horizontal axle, with a string wrapped around it hanging down and pulled with force F_{A} (arrow pointing down). "Scenario 2 — Hoop": a shaded hoop (ring) of mass M and radius R mounted on a horizontal axle in the same way, with a string wrapped around it hanging down and pulled with force F_{A} (arrow pointing down).]
Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.
In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_{A} for the same time interval Δt, causing the pulleys to rotate without

Question 4

the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Solution 4

(a)

Mark scheme

- $\alpha_D = \dfrac{2F_T}{MR}$. Derivation: for the solid disk (pulley), $\tau = I\alpha$ with $\tau = RF_T$ and $I = \frac{1}{2}MR^2$: $RF_T = \frac{1}{2}MR^2\,\alpha_D \Rightarrow \alpha_D = \dfrac{RF_T}{\frac{1}{2}MR^2} = \dfrac{2F_T}{MR}$ (either equivalent form is accepted).

Solution 4

(b) Mark scheme

- Paragraph argument comparing a solid disk and a hoop (each mass M, radius R) each pulled by the same force F_A via a string wrapped around the rim: the disk ends up with GREATER rotational kinetic energy than the hoop. Reasoning, worth 6 points total: (1) the torque $\tau = F_A R$ is the same for both pulleys, since the same force is applied at the same radius; (2) since torque and the time interval are the same for both, the angular impulse $\tau \Delta t$ (equivalently the change in angular momentum ΔL) is the same for the disk and the hoop; (3) the rotational inertia I differs between the disk and hoop: $I_{\text{disk}} = \frac{1}{2}MR^2 < I_{\text{hoop}} = MR^2$, since the hoop's mass is concentrated at the rim while the disk's is spread through its area; (4) because $\Delta L = I\Delta\omega$ is the same for both but $I_{\text{hoop}} > I_{\text{disk}}$, the disk must

Solution 4

gain a larger $\Delta\omega$ (and correspondingly larger $\Delta\theta$, α) than the hoop — the kinematic quantities differ because the rotational inertias differ; (5) one of: relating I and ω to show ΔK is greater for the disk (e.g. using $\omega = L/I$ and $K = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller I converts the same angular momentum into more kinetic energy), OR noting $\Delta\theta$ is greater for the disk so the work done on the disk ($\tau\Delta\theta$) is greater; (6) an overall logical, relevant, and internally consistent paragraph following the AP paragraph-response format that ties these steps together to conclude the disk has the greater rotational kinetic energy.

Question 3

2022 ▪ Q3

[Figure: A wheel mounted on a horizontal axle, standing on a support post above a floor. A string is wrapped around the wheel's rim and hangs down to a small hanging block. Labels: "Wheel" (top, pointing to the wheel), "Axle" (pointing to the horizontal axle at the wheel's center), "Hanging Block" (pointing to the small block hanging from the string), "Floor" (the ground below).]

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Question 3

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

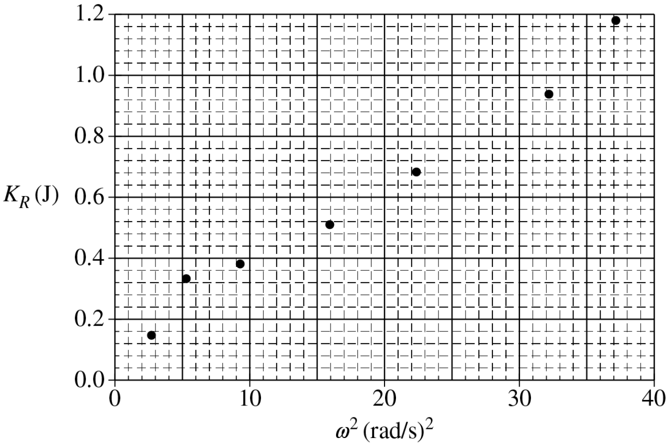
(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

Question 3

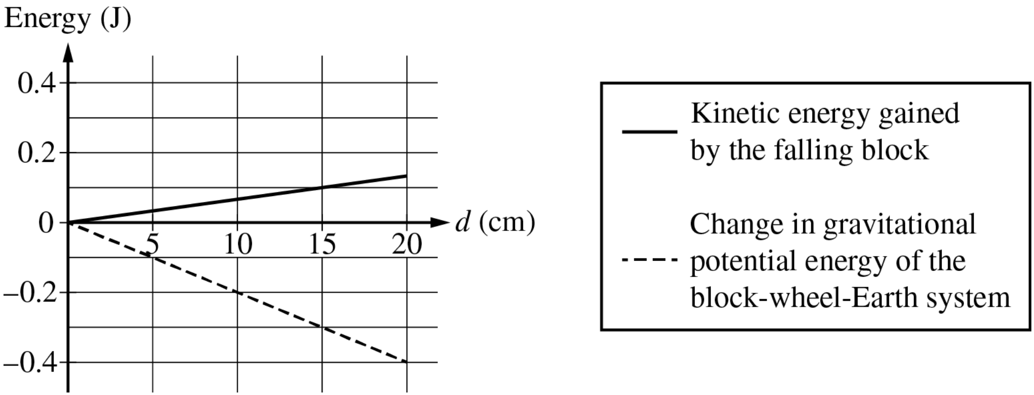
In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them in the space just below the table.
In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.
If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	**Symbol for Quantity**	**Equipment for Measurement**	**Procedure (and diagram, if needed)**

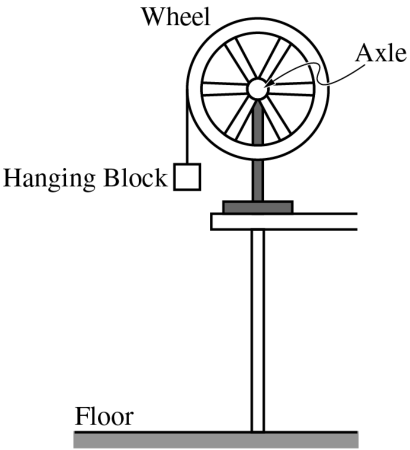
Question 3



Question 3


Energy (J)
0.4
0.2
0
–0.2
–0.4
5
10
15
20
d (cm)
Kinetic energy gained by the falling block
Change in gravitational potential energy of the block-wheel-Earth system

Question 3



Question 3

2022 ▪ Q3

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A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in

Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

Question 3

2022 ▪ Q3

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Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(c) [Graph: "Energy (J)" on the y-axis from -0.4 to 0.4 J (gridlines at $-0.4, -0.2, 0, 0.2, 0.4$), "*d* (cm)" on the x-axis from 0 to 20 cm (gridlines at 5, 10, 15, 20). A solid line labeled "Kinetic energy gained by the falling block" starts near $(0, 0)$ and rises gently to about $(20, 0.15)$. A dashed line labeled "Change in gravitational potential energy of the block-wheel-Earth system" starts near $(0, 0)$ and falls to about $(20, -0.35)$.]

The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance *d*. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance *d*.

Question 3

2022 ▪ Q3

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Question 3

the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(d)(i) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.

[Graph: "K_R (J)" on the y-axis from 0.0 to 1.2 J (gridlines at 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2), "ω^2 (rad/s)^2" on the x-axis from 0 to 40 (gridlines at 0, 10, 20, 30, 40). Data points (approximate): $(5, 0.15)$, $(8, 0.28)$, $(10, 0.35)$, $(17, 0.5)$, $(24, 0.65)$, $(32, 0.95)$, $(35, 1.15)$.]

On the above graph, draw a straight line that best represents the data.

(d)(ii) Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

Solution 3

(a) Mark scheme

- Design: measure the mass m_B of the block with a mass balance; hold the block with the string taut and measure the fall distance d (initial height above the floor) with a meterstick; release the block and use a stopwatch to measure the fall time t_B (start at release, stop when the block hits the floor); repeat 3 trials at the same d and repeat for several different values of d to reduce uncertainty (e.g. by averaging). Table: Mass of block m_B – mass balance; Distance block falls d – meterstick; Time for block to fall t_B – stopwatch. Scoring (1 point each): listing relevant equipment matching the measured quantities; listing measurements sufficient to determine the block's kinetic energy; listing measurements sufficient to determine the gravitational PE of the block-Earth system; a plausible procedure

Solution 3

(can be done in a typical school physics lab); attempting to reduce uncertainty (e.g. repeated trials/averaging).

Solution 3

(b)

Mark scheme

- Compute the block's average speed as d$/t_B; sinceitfallsfromrestwithconstantacceleration, thef$
$2d/t_B. ThenthefinalkineticenergyisK = \frac{1}{2}m_B v_F^2 = \frac{1}{2}m_B(2d/t_B)^2. Scoring :$
$1 pointforindicatingthatmassandvelocityareneeded(neednotbethefinalvelocity); 1 pointforavalic$

Solution 3

(c) Mark scheme

- On the Energy vs. d graph, draw a straight line through the origin with positive slope representing the kinetic energy gained by the falling block, and a line representing the change in gravitational PE such that the total energy sum is (approximately) zero at all values of d – the correct line passes through the origin and through (15 cm, 0.2 J), so that KE gained plus the (negative) change in PE sums to zero, showing energy conservation. Scoring: 1 point for drawing a straight line that passes through the origin and has a positive slope; 1 point for drawing a line that yields a total energy sum of zero at all values of d.

Solution 3

(d)(i)

Mark scheme

- On the K$_R vs. \omega^2 graph, drawareasonablebest-fitstraightlinethroughtheplotteddatapoints.Scor$
$1 pointfordrawingareasonablebest - fitline.$

Solution 3

(d)(ii)

Mark scheme

- From the best-fit line, slope $= \dfrac{1.04 - 0.20}{35 - 2.5} = 0.0258$ kg $\cdot$ m$^2 . Since K_R = \dfrac{1}{2} I\omega^2, the slope of K_R vs. \omega^2 equals \dfrac{1}{2} I, so I = 2 \times$ slope $= 0.0517$ kg $\cdot$ m$^2 . Scoring : 1 point for correctly calculating a value for the slope using points on the drawn line (or stating a calculat$

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

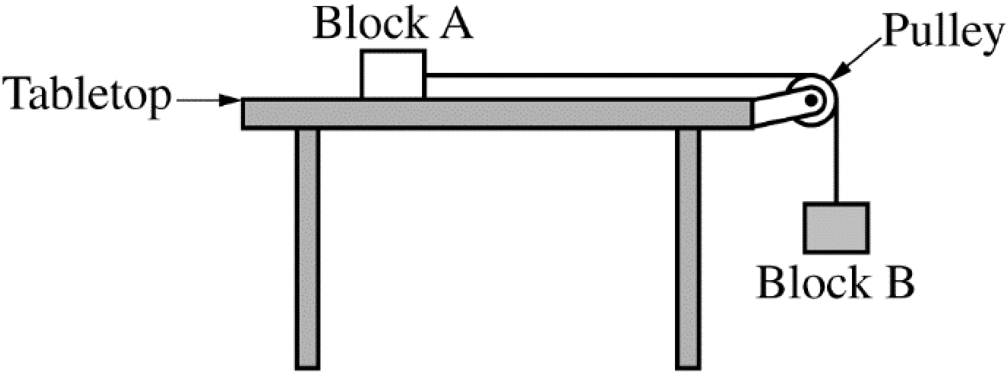
(a)(i) Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Question 2

Briefly explain your reasoning without deriving or using equations.

(a)(ii) Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release. Briefly explain your reasoning without deriving or using equations.

Question 2



Question 2

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

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(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on

Question 2

that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.
[Diagram: Two separate large dots side by side on the page, one labeled “Block A” below it and the other labeled “Block B” below it, for the student to draw force vectors on.]

Question 2

2019 ▪ Q2

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(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than

Question 2

what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

Question 2

2019 ▪ Q2

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(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

Question 2

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

Question 2

2019 ▪ Q2

[Diagram: A horizontal tabletop with Block A resting on top of it near the right edge. A pulley is mounted at the right edge of the tabletop. A light string runs from Block A horizontally to the pulley, then vertically down over the pulley to Block B, which hangs below the tabletop's edge.]

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is

Question 2

not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

Solution 2

(a)(i) Mark scheme

- Correct claim: the acceleration is nearly zero / small / negligible / much less than g / '$\ll$g'. Point 1: for a correct qualitative answer with an attempted, consistent justification. Point 2: for correct reasoning. Reasoning: block A has a much greater mass (much greater inertia) than block B, so block A strongly resists acceleration and the light block B cannot meaningfully accelerate it; the acceleration of the connected system is therefore nearly zero. (Example 2-point answers: 'Very small, because block A has a large inertia, it won't speed up much.' / 'Close to zero because block B is so light that it can hardly budge block A.')

Solution 2

(a)(ii)

Mark scheme

- Correct claim: the acceleration is nearly g (acceptable: 'g', '9.8 m/s^2', '10 m/s^2', or just '9.8'/'10'). The single point is earned for a correct answer WITH correct justification. Reasoning: block A has negligible mass compared to block B and negligible friction with the tabletop, so essentially no tension is needed to accelerate block A; block B is therefore nearly in free fall, and the string tension is nearly zero, giving the whole system an acceleration close to g.

Solution 2

(b) Mark scheme

- Free-body diagram, Block A (on the tabletop): three distinct force arrows – normal force N (or F_N) pointing straight UP from the table; gravity F_g (or mg, W, $m_A g$) pointing straight DOWN; tension F_T pointing horizontally toward the pulley (i.e. toward block B's side). Free-body diagram, Block B (hanging): two distinct force arrows – tension F_T pointing straight UP (toward the pulley); gravity F_g (or $m_B g$) pointing straight DOWN. Points: 1 for a correct, correctly labeled normal force on block A only (acceptable labels: N, F_N, 'normal force', F_{table}, 'table force', or any label indicating a normal/table force); 1 for correct, correctly labeled gravitational forces on BOTH diagrams (acceptable: F_g, F_{grav}, W, mg, $m_A g$ – NOT G or g alone – and no extraneous forces on either diagram); 1 for correct,

Solution 2

correctly labeled tension forces on BOTH diagrams (acceptable: 'tension', 'string force', F_T, $F_{tension}$, F_{string}, F_S, T – NOT $m_B g$, F_{m_B}, or 'force from block B'). Forces must be drawn as distinct arrows starting on and pointing away from the dot, not components.

Solution 2

(c) Mark scheme

- Apply Newton's second law separately to each block, then combine. Block B (down positive): $m_B g - T = m_B a$. Block A (horizontal): $T = m_A a$. Since the string is inextensible and the tension magnitude is the same on both blocks, combine to eliminate T: $m_B g - m_A a = m_B a \Rightarrow a = \dfrac{m_B}{m_A + m_B} g$. Alternate whole-system method: treat both blocks + string as one system of total mass $m_A + m_B$ with no net internal force; the only external force along the direction of motion is gravity on block B, so $F_{net} = m_B g = (m_A + m_B)a$, giving the same $a = \dfrac{m_B}{m_A + m_B} g$. Points: 1 for using separate Newton's-second-law equations for each block (or the equivalent whole-system equation containing no internal

Solution 2

forces); 1 for correctly combining the equations with correct notation, using m_A and m_B, correctly indicating that the same tension force acts on both blocks and that they share the same acceleration; 1 for the correct final equation with supporting work: $a = \dfrac{m_B}{m_A + m_B} g$.

Solution 2

(d) Mark scheme

- Correct answer: 'Yes' – the equation from part (c), $a = \dfrac{m_B}{m_A + m_B} g$, agrees with the reasoning in part (a)(ii). The single point is earned for valid reasoning connecting the two: when $m_A \ll m_B$, the m_A term in the denominator can be neglected, so $(m_A + m_B) \approx m_B$ and $a \approx \dfrac{m_B}{m_B} g = g$, matching the claim in part (a)(ii) that the acceleration approaches g. ('No' is acceptable ONLY if it is consistent with an equation from part (c) that does not actually reduce to g in this limit, or with an inconsistent answer given in (a)(ii).)

Solution 2

(e) Mark scheme

- Correct answer: $T_2 > T_1$ (a maximum of 1 point can be earned if the wrong option is selected). Point 1: for reasoning that the acceleration of both blocks is SMALLER in the second (heavier-pulley) scenario, because the pulley's now-nonzero rotational inertia requires torque/energy to spin it up, leaving less net force available to accelerate the blocks. Point 2: for connecting this, via Newton's second law applied to block B ($\vec{a} = \sum \vec{F}/m$, i.e. $T = m_B g - m_B a$ or $m_B g - T = m_B a$), to conclude that a SMALLER acceleration of block B implies a LARGER tension T_2 (closer in magnitude to the gravitational force on block B than T_1 was).

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]
The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0

Question 3

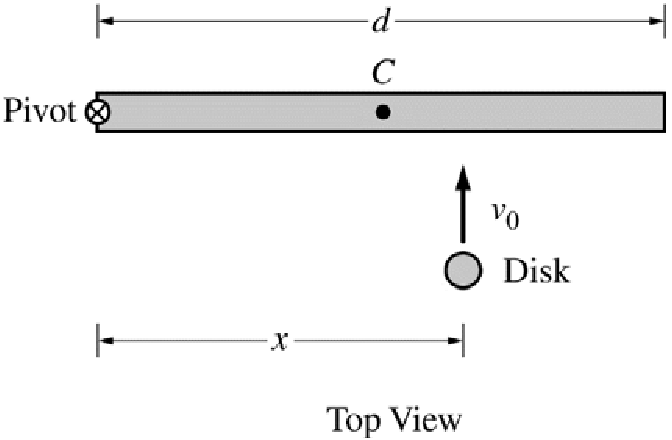
perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?

_____ To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

Question 3



Question 3

2017 ▪ Q3

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The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will

Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \dfrac{m_{\text{disk}}\, x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

$Y^{es}{}_{N^o}$

Briefly explain your reasoning without deriving an equation for ω.

Question 3

2017 ▪ Q3

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{Ixv_0}{m_{\text{disk}}\, d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible — in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

Question 3

2017 ▪ Q3

[Figure, "Top View": a horizontal rod of length d lies along a line, with its left end marked "Pivot" (shown as a circled X, indicating an axis perpendicular to the page) and its right end free. Point C is marked at the center (midpoint) of the rod, at distance d from the pivot (the full length d is indicated by a double-headed arrow spanning from the pivot to the rod's right end). Below the rod, a disk is shown approaching with velocity v_0 directed upward (perpendicular to the rod), at a horizontal distance x from the pivot (indicated by a double-headed arrow from the pivot to the point below the disk).]

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Question 3

stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

Solution 3

(a) Mark scheme

- Correct answer: To the right of C (reasoning only earns credit if this option is selected). The farther from the pivot the disk strikes the rod, the greater the torque the disk exerts on the rod during the collision (equivalently, the disk's angular momentum about the pivot, $m_{disk}v_0x$, is larger for larger x, and nearly all of it transfers to the rod). A greater torque/angular momentum imparted to the rod produces a greater post-collision angular speed, so striking to the right of C (farther from the pivot) gives the rod the largest possible angular speed. Points: 1 pt for an explanation that the torque exerted by the disk on the rod, or the angular momentum of the disk about the pivot, is greater when the disk strikes farther from the pivot.

Solution 3

(b) Mark scheme

- Correct answer: Yes (consistent with the answer selected in part (a)). The proposed equation $\omega = \dfrac{m_{disk}\, x\, v_0}{I}$ shows ω increasing as x increases, since x is in the numerator. This matches the qualitative reasoning of part (a): striking farther from the pivot (larger x) should give the rod a larger post-collision angular speed. So the equation's dependence on x agrees with the part (a) reasoning. Points: 1 pt for a selection consistent with the answer selected in part (a); 1 pt for indicating that the equation shows ω increases with increasing x. (If "No" is selected but the explanation is consistent with an incorrect part (a) selection, full credit may still be earned.)

Solution 3

(c) Mark scheme

- The proposed equation $\omega = \frac{I\, x\, v_0}{m_{disk}\, d^4}$ is not physically plausible because of its dependence on m_{disk} and I. Physically, a larger m_{disk} should transfer more angular momentum to the rod, so ω should increase with m_{disk} — but the equation puts m_{disk} in the denominator, so it predicts ω decreasing as m_{disk} increases, which is backwards. Likewise, a larger rotational inertia I of the rod should make the rod harder to spin up, so ω should decrease with I — but the equation has I in the numerator, predicting ω increasing with I, also backwards. This wrong functional (not dimensional) dependence shows the equation cannot be correct. Points: 1 pt for focusing on functional dependence rather than, e.g., a

Solution 3

units/dimensional-analysis argument; 1 pt for addressing m_{disk}, I, or both; 1 pt for correctly concluding the equation is wrong because of its (backwards) dependence on m_{disk}, I, or both.

Solution 3

(d) Mark scheme

- Apply conservation of angular momentum about the pivot for the disk+rod system during the collision. Before collision: $L_i = m_{disk} v_0 x$ (the disk's angular momentum about the pivot). After the collision (disk sticks to rod at distance x): $L_f = (I + m_{disk} x^2)\,\omega$, where I is the rod's rotational inertia about the pivot and $m_{disk} x^2$ is the disk's own rotational inertia about the pivot once stuck to the rod at distance x. Setting $L_i = L_f$: $m_{disk} v_0 x = (I + m_{disk} x^2)\,\omega$. Solving for ω: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$. Points: 1 pt for using an expression of conservation of angular momentum for the disk-and-rod system (not awarded for equating angular and linear momentum); 1 pt for indicating the initial angular momentum of the system

Solution 3

equals $m_{disk} v_0 x$; 1 pt for a dimensionally correct expression of the post-collision angular momentum that includes $I\omega$; 1 pt for indicating the correct rotational inertia of the system after the collision, $I + m_{disk} x^2$. Final answer: $\omega = \dfrac{m_{disk} v_0 x}{I + m_{disk} x^2}$.

Solution 3

(e) Mark scheme

- Correct answer: Greater than. When the disk bounces backward off the rod instead of sticking, the disk's velocity (and hence its angular momentum about the pivot) changes by a larger amount than in the sticking case — going from $+m_{disk}v_0x$ to some negative value, rather than just to the smaller value it has once moving together with the rod. By conservation of angular momentum for the system, whatever angular momentum the disk loses, the rod gains; since the disk's angular-momentum change (equivalently, the impulse delivered to the rod) is larger in the bouncing case, the rod gains more angular momentum — and hence acquires a greater post-collision angular speed — than in the sticking case. Example: after the bouncy collision the disk has angular momentum in the

Solution 3

clockwise direction; to keep the system's total angular momentum constant, the rod's counterclockwise angular momentum must be greater than before. Points: 1 pt for indicating, directly or by analogy to the linear (elastic vs. inelastic) case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario, OR an equivalent impulse-based argument (describes what happens to the disk); 1 pt for using conservation of angular momentum or momentum-impulse reasoning to conclude the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario (describes what happens to the rod).