Past-paper walk-through

Connecting Linear and Rotational Motion ▪ 5.2

eXercise

Questions in this set

- 2023 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2023 ▪ Q4

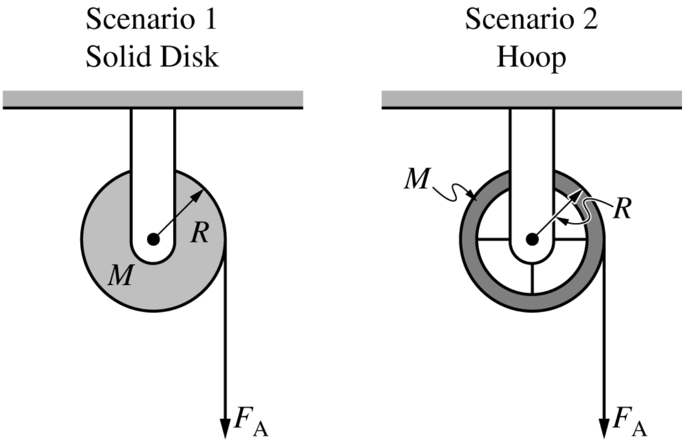
[Figure: A pulley mounted on a horizontal axis through a fixed support ("Axis" labeled with an arrow pointing to the pulley's center). The pulley is a shaded disk of mass M and radius R (with R labeled from center to edge), free to rotate. A string wraps over/around the pulley and hangs down on the right side, connected to a hanging shaded block at the bottom.]

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

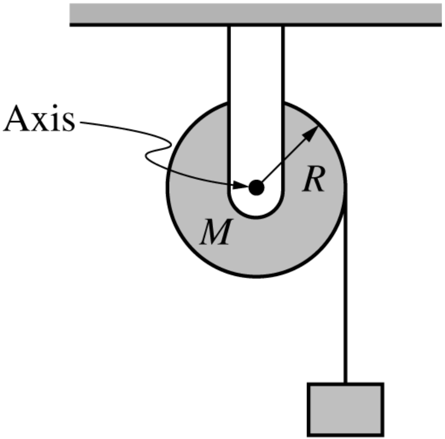
Question 4

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_{T}.
(a) Determine an expression for the magnitude of the angular acceleration α_{D} of the disk as the block travels downward. Express your answer in terms of M, R, F_{T}, and physical constants as appropriate.

Question 4



Question 4



Question 4

2023 ▪ Q4

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Question 4

(b) [Continuation of Question 4.] [Figure: Two side-by-side diagrams. "Scenario 1 — Solid Disk": a shaded disk of mass M and radius R mounted on a horizontal axle, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down). "Scenario 2 — Hoop": a shaded hoop (ring) of mass M and radius R mounted on a horizontal axle in the same way, with a string wrapped around it hanging down and pulled with force F_A (arrow pointing down).]

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without

Question 4

the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.
Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Solution 4

(a)

Mark scheme

- $\alpha_D = \frac{2F_T}{MR}$. Derivation: for the solid disk (pulley), $\tau = I\alpha$ with $\tau = RF_T$ and $I = \frac{1}{2}MR^2$: $RF_T = \frac{1}{2}MR^2\,\alpha_D \Rightarrow \alpha_D = \frac{RF_T}{\frac{1}{2}MR^2} = \frac{2F_T}{MR}$ (either equivalent form is accepted).

Solution 4

(b) Mark scheme

- Paragraph argument comparing a solid disk and a hoop (each mass M, radius R) each pulled by the same force F_A via a string wrapped around the rim: the disk ends up with GREATER rotational kinetic energy than the hoop. Reasoning, worth 6 points total: (1) the torque $\tau = F_A R$ is the same for both pulleys, since the same force is applied at the same radius; (2) since torque and the time interval are the same for both, the angular impulse $\tau\Delta t$ (equivalently the change in angular momentum ΔL) is the same for the disk and the hoop; (3) the rotational inertia I differs between the disk and hoop: $I_{\text{disk}} = \frac{1}{2}MR^2 < I_{\text{hoop}} = MR^2$, since the hoop's mass is concentrated at the rim while the disk's is spread through its area; (4) because $\Delta L = I\Delta\omega$ is the same for both but $I_{\text{hoop}} > I_{\text{disk}}$, the disk must

Solution 4

gain a larger $\Delta\omega$ (and correspondingly larger $\Delta\theta$, α) than the hoop — the kinematic quantities differ because the rotational inertias differ; (5) one of: relating I and ω to show ΔK is greater for the disk (e.g. using $\omega = L/I$ and $K = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller I converts the same angular momentum into more kinetic energy), OR noting $\Delta\theta$ is greater for the disk so the work done on the disk ($\tau\Delta\theta$) is greater; (6) an overall logical, relevant, and internally consistent paragraph following the AP paragraph-response format that ties these steps together to conclude the disk has the greater rotational kinetic energy.