Exercise walk-through

Connecting Linear and Rotational Motion ▪ 5.2

eXercise

You must be able to

- relate **tangential velocity** 切向速度 to angular velocity, $v = r\omega$
- relate **tangential acceleration** 切向加速度 to angular acceleration, $a_t = r\alpha$
- distinguish tangential from **centripetal acceleration** 向心加速度 for a point in a circle
- calculate **arc length** 弧长 with $s = r\theta$

Method — Linear from angular

$v = r\omega$ and $a_t = r\alpha$. On a rigid body every point has the **same** ω, but a point farther from the axis has a **larger** linear speed.

Method — Two accelerations

Tangential $a_t = r\alpha$ changes the speed; **centripetal** $a_c = \dfrac{v^2}{r} = \omega^2 r$ changes only the direction.

Method — Example

A wheel of radius 0.30 m spins at $\omega = 10$ rad s^{-1}: a rim point moves at $v = 0.30 \times 10 = 3.0$ m s^{-1}.

Practice 2.1 ▪ 1 mark

A point 0.50 m from the axis rotates at $\omega = 6.0$ rad s^{-1}. Find its linear speed.

Solution 2.1

Method: Linear from angular

Worked steps

$v = r\omega = 0.50 \times 6.0 = 3.0$ m s^{-1} **B1**.

Practice 2.2 ▪ 1 mark

A wheel of radius 0.20 m has angular acceleration 5.0 rad s^{-2}. Find the tangential acceleration of its rim.

Solution 2.2

Method: Example

Worked steps

$a_t = r\alpha = 0.20 \times 5.0 = 1.0$ m s^{-2} B1.

Practice 2.3 ▪ 2 marks

Two points on a disc are 0.10 m and 0.30 m from the axis. State which has the greater (a) angular speed and (b) linear speed.

Solution 2.3

Method: Linear from angular

Worked steps

(a) equal — same angular speed for both B1. (b) the point at 0.30 m, since $v = r\omega$ B1.

Practice 2.4 ▪ 1 mark

Find the arc length swept by a point 2.0 m from the axis when it turns through 3.0 rad.

Solution 2.4

Worked steps

$s = r\theta = 2.0 \times 3.0 = 6.0$ m B1.

Exam-style 3.1 ▪ 1 mark

On a rigid rotating disc, every point has the same

A linear speed

B angular speed

C tangential acceleration

D arc length

Solution 3.1

Method: Linear from angular

A linear speed

B angular speed ✓

C tangential acceleration

D arc length

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The tangential acceleration of a point at radius r is

A $r\omega$

B $r\alpha$

C v^2/r

D $\omega^2 r$

Solution 3.2

Method: Example

A $r\omega$

B $r\alpha$ ✓

C v^2/r

D $\omega^2 r$

Worked steps

B.

Exam-style 3.3 ▪ 2 marks

A car tyre of radius 0.35 m rotates at $\omega = 20$ rad s^{-1} while rolling without slipping.
(a) Find the linear speed of a point on the tread.
(b) State the speed of the car.

Solution 3.3

Worked steps

(a) $v = r\omega = 0.35 \times 20 = 7.0$ m s^{-1} M1 A1. (b) 7.0 m s^{-1} (rolling without slipping) B1.