Past-paper walk-through

Rotational Kinematics ▪ 5.1

eXercise

Questions in this set

- 2024 Q3
- 2021 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2024 ▪ Q3

[Figure 1: A vertical wall is shown on the left. Two points are marked on the wall: Point 1 near the top, and Point 2 lower down (above the hinge). A horizontal beam extends from a hinge at the base of the wall to the right, with length L labeled along its underside. A string connects from Point 1 on the wall diagonally down to the right end of the beam, making angle θ_1 with the beam at the point where the string meets the beam.]

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

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(a) [A rectangle representing the beam, drawn horizontally, blank for the student to draw and label force vectors.]
The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

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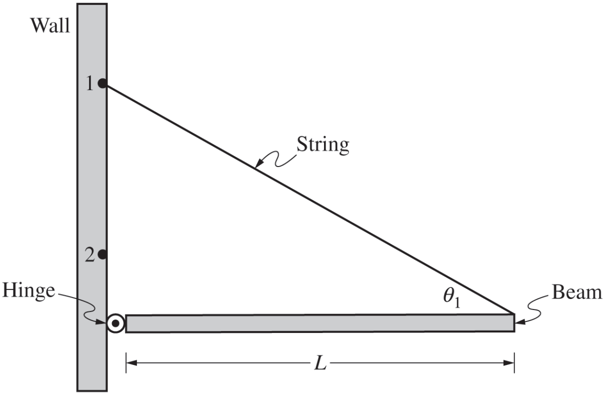

Figure 1

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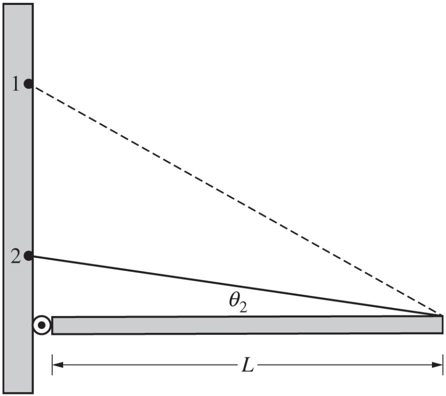

Figure 2

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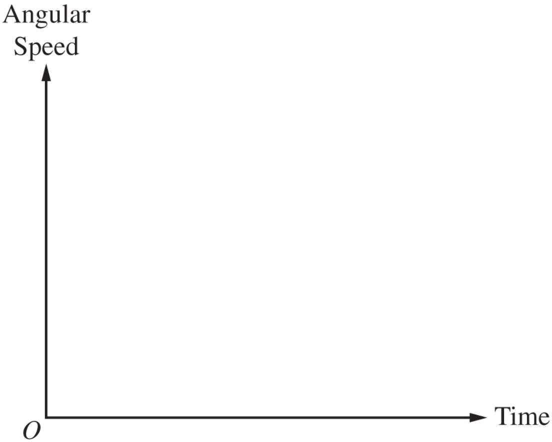

Question 3

2024 ▪ Q3

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(b) [Figure 2: The same wall, hinge, and beam of length L as in Figure 1, but now the string (solid line) connects from Point 2 on the wall (lower than Point 1) to the right

Question 3

end of the beam, making angle θ_2 with the beam. A dashed line is also shown from Point 1 to the right end of the beam, representing the string's position from Figure 1.]
(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.
The magnitude of the tension in the string shown in Figure 1 is F_{T1}. The magnitude of the tension in the string shown in Figure 2 is F_{T2}. **Indicate** which of the following correctly compares F_{T2} with F_{T1}.
$F_{T2} > F_{T1}$ $F_{T2} < F_{T1}$ $F_{T2} = F_{T1}$
Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

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(c) (c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M, θ,

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and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

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(d) (d) Is your derived equation in part (c) consistent with your justification in part (b)? **Explain** your reasoning.

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2024 ▪ Q3

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(e) [A blank set of axes with vertical axis labeled "Angular Speed" and horizontal axis labeled "Time", origin labeled O.]

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(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.

Solution 3

(a) Mark scheme

- Draw three force vectors on the beam, each starting on and pointing away from where the force acts, representing a system in equilibrium: the gravitational force ($F_g/F_G/W/mg/Mg/$"grav force" — not G or g alone) drawn downward from the center of the beam; the tension force ($F_T/F_s/F_{\text{string}}/$"tension force") drawn upward and to the left from the right end of the beam (along the string toward the wall); and the hinge/wall (normal) force ($F_N/F_n/\mathrm{N}/$"normal force"/"wall force") drawn upward and to the right from the left end (hinge). The three vectors must show the beam in equilibrium (net force zero). Points: downward gravitational force at the beam's center (1 point); tension force at the right end

Solution 3

directed up and to the left (1 point); a diagram with all three force vectors consistent with equilibrium (1 point).

Solution 3

(b) Mark scheme

- Select $F_{T2} > F_{T1}$ (the tension is larger when the string attaches lower, at Point 2, making a smaller angle with the beam). Justification: for the beam to remain horizontal and in equilibrium, the torque exerted by the string about the hinge must stay the same regardless of attachment point. As the angle θ between the string and beam decreases, the perpendicular (torque-producing) component of the tension for a given tension magnitude decreases, so a larger tension is needed to supply the same torque — equivalently, the vertical component of the tension must still balance the same weight of the bar, and a smaller angle means more tension is needed to achieve the same vertical component. Points: selecting $F_{T2} > F_{T1}$ with an attempt at a relevant justification (1 point); a justification

Solution 3

that relates either the vertical component of tension to the beam's weight, or the force needed to produce the same torque to the string's angle (1 point).

Solution 3

(c) Mark scheme

- Derive F_T from Newton's second law in rotational form: $\Sigma\tau = I\alpha$ (with $\alpha = 0$ since the beam is in equilibrium), so $\tau_{\text{gravity}} + \tau_{\text{string}} = 0$. Taking torques about the hinge: $(F_g)\frac{L}{2}\sin 90^\circ - F_T(L)\sin\theta = 0 \Rightarrow Mg\frac{L}{2} = (F_T\sin\theta)L \Rightarrow F_T = \dfrac{Mg}{2\sin\theta}$. Points: a multi-step derivation beginning with $\Sigma\tau = I\alpha$ or $\Sigma\tau = 0$ (1 point); indicating that the net torque is zero OR the net force is zero (1 point); indicating one of — the torque from the string is $(F_T\sin\theta)L$, the torque from the beam's weight is $Mg\frac{L}{2}$, or the correct final answer $F_T = \dfrac{Mg}{2\sin\theta}$ (1 point).

Solution 3

(d) Mark scheme

- The derived equation $F_T = \dfrac{Mg}{2\sin\theta}$ is consistent with the part (b) reasoning: tension is inversely proportional to $\sin\theta$, and for $\theta < 90^\circ$, $\sin\theta$ decreases as θ decreases, so the required tension is greater for smaller angles — matching the claim that $F_{T2} > F_{T1}$ when the string is attached lower (smaller angle). Points: an attempt to use the functional dependence of the part (c) equation to relate to the part (b) reasoning, even if not fully correct (1 point); an explanation that correctly relates the two (1 point).

Solution 3

(e)

Mark scheme

- Sketch the angular speed of the beam vs. time (from the moment the string is cut until the beam becomes vertical) as a curve that starts at the origin, is monotonically increasing throughout the interval, and is concave down (the rate of increase in angular speed decreases over time, since the gravitational torque about the hinge shrinks as the beam rotates toward vertical). Points: drawing a graph that is monotonically increasing (1 point); drawing a concave-down curve (1 point).

Question 5

2021 ▪ Q5

[Figure: Two pulleys of different radii are mounted concentrically on a common horizontal axle. The larger pulley (radius $2r_0$) is on the outside, the smaller pulley (radius r_0) is on the inside, both centered on the "Axle at Center of Pulleys." Object 1 (mass m_0) hangs from a light string wrapped around the larger pulley on the left side. Object 2 (mass $1.5m_0$) hangs from a light string wrapped around the smaller pulley on the right side.]

(7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

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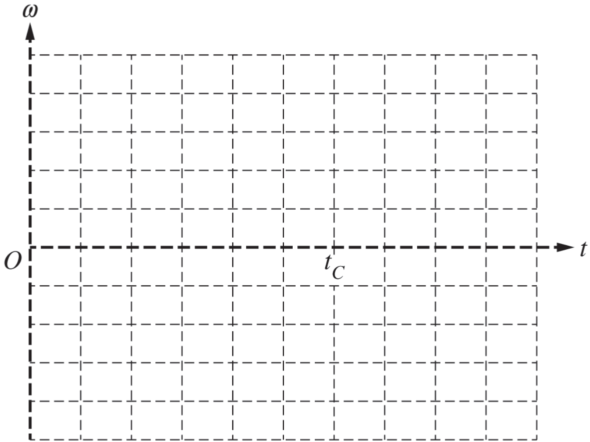
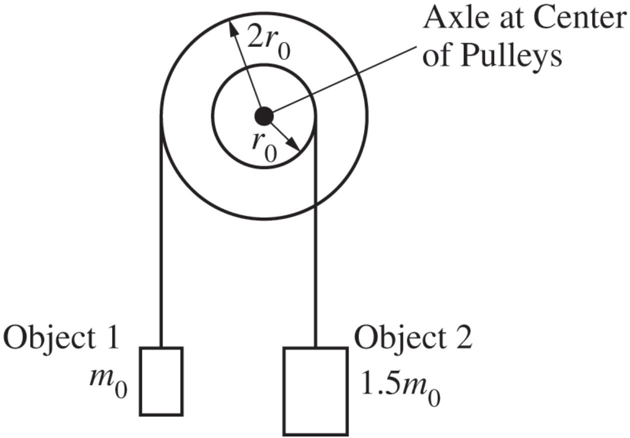
m_0 is the mass of object 1. $1.5m_0$ is the mass of object 2. r_0 is the radius of the smaller pulley. $2r_0$ is the radius of the larger pulley.
At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.
(a)(i) Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.
(a)(ii) Object 1 accelerates downward after the pulleys are released. Briefly explain why.

Question 5



Question 5



Question 5

2021 ▪ Q5

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At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

Question 5

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

______ Same direction ______ Opposite directions

Briefly explain your reasoning.

Question 5

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m_0 is the mass of object 1. $1.5m_0$ is the mass of object 2. r_0 is the radius of the smaller pulley. $2r_0$ is the radius of the larger pulley.

At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

Question 5

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.
[Graph grid: vertical axis labeled "ω" (dashed gridlines, unlabeled numeric scale); horizontal axis labeled "t" with marked points at O and t_C; empty dashed grid for the student to sketch on.]

Solution 5

(a)(i)

Mark scheme

- Torque from object 1's weight: $\tau_1 = m_0 g(2r_0)$ (weight $m_0 g$ at lever arm $2r_0$). Torque from object 2's weight: $\tau_2 = (1.5 m_0 g) r_0$ (weight $1.5 m_0 g$ at lever arm r_0). The two torques act in opposite directions, so the net torque is their difference: $\tau_{net} = \tau_1 - \tau_2 = m_0 g(2r_0) - (1.5 m_0 g) r_0 = 0.5\, m_0 g r_0$. Points: (1) correct torque expressions for each object's weight, (2) indicating the two torques act in opposite directions, (3) the correct derived answer $0.5 m_0 g r_0$.

Solution 5

(a)(ii)

Mark scheme

- Object 1 exerts the larger torque: it is twice as far from the axle ($2r_0$ vs. r_0) while its weight is only m_0g compared to object 2's $1.5m_0g$ - the larger lever arm more than compensates for the smaller weight, so object 1's torque dominates and sets the net-torque direction.

Solution 5

(b)

Mark scheme

- Correct answer: opposite directions. Object 1 (the larger torque) had been determining the net (counterclockwise) torque direction; once its string is cut, the net torque comes only from object 2, which acts in the opposite (clockwise) sense, so the angular acceleration immediately reverses direction. The angular velocity, however, cannot change instantaneously and is still in its original (counterclockwise) direction at that moment - so immediately after the cut, angular velocity and angular acceleration point in opposite directions.

Solution 5

(c) Mark scheme

- Sketch: ω starts at 0 at $t = 0$ and increases linearly (constant nonzero slope, from the constant net angular acceleration while both objects are attached) up to $t = t_C$; at $t = t_C$ the slope abruptly changes sign (angular acceleration reverses, per part (b)) while ω itself remains continuous (no jump); after t_C, ω decreases linearly, eventually crossing to negative values as the pulley's rotation slows and reverses. Points: (1) a linear graph between 0 and t_C with initial angular velocity zero and nonzero slope (sign either way), (2) a sign change in the slope at $t = t_C$ with no discontinuity.