Exercise walk-through

Rotational Kinematics ▪ 5.1

eXercise

You must be able to

- define **angular displacement** 角位移 θ, **angular velocity** 角速度 ω, and **angular acceleration** 角加速度 α
- describe the direction of angular quantities as clockwise or counterclockwise
- apply the **rotational kinematic equations** for constant angular acceleration
- interpret graphs of angular position, velocity, and acceleration

Method — Angular quantities

θ in radians, $\omega = \dfrac{\Delta\theta}{\Delta t}$ in rad s^{-1}, and $\alpha = \dfrac{\Delta\omega}{\Delta t}$ in rad s^{-2}. Counterclockwise is taken as positive.

Method — The rotational kinematic equations (constant α)

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2, \qquad \omega^2 = \omega_0^2 + 2\alpha\theta.$$

These mirror the linear equations, with $x \to \theta$, $v \to \omega$, $a \to \alpha$.

Method — Example

A wheel starts from rest with $\alpha = 2.0$ rad s^{-2}. After 3.0 s: $\omega = 0 + 2.0(3.0) = 6.0$ rad s^{-1}.

Practice 2.1 ▪ 1 mark

A disc accelerates from rest at $\alpha = 4.0$ rad s^{-2} for 5.0 s. Find its final angular velocity.

Solution 2.1

Worked steps

$\omega = 0 + 4.0(5.0) = 20$ rad s^{-1} **B1**.

Practice 2.2 ▪ 2 marks

Find the angle turned by the disc in that time.

Solution 2.2

Worked steps

$\theta = \frac{1}{2}(4.0)(5.0)^2 = 50$ rad M1 A1.

Practice 2.3 ▪ 1 mark

State the rotational quantity that plays the role of velocity in the linear equations.

Solution 2.3

Method: The rotational kinematic equations (constant α)

Worked steps

angular velocity ω B1.

Exam-style 3.1 ▪ 1 mark

Angular acceleration is the rate of change of

A angular displacement

B angular velocity

C torque

D period

Solution 3.1

A angular displacement

B angular velocity ✓

C torque

D period

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A wheel spinning at 10 rad s^{-1} decelerates at 2.0 rad s^{-2}. The time to stop is

A 2.0 s

B 5.0 s

C 8.0 s

D 20 s

Solution 3.2

Method: Example

A 2.0 s

B 5.0 s ✓

C 8.0 s

D 20 s

Worked steps

B — $t = \dfrac{0 - 10}{-2.0} = 5.0$ s.

Exam-style 3.3 ▪ 2 marks

A fan blade slows from 20 rad s^{-1} to 8.0 rad s^{-1} in 4.0 s.
(a) Find the angular acceleration.
(b) Find the angle turned during this time.

Solution 3.3

Method: Angular quantities

Worked steps

(a) $\alpha = \dfrac{8.0 - 20}{4.0} = -3.0$ rad s^{-2} M1 A1. (b) $\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(20 + 8.0)(4.0) = 56$ rad M1 A1.