Exercise walk-through

Vectors and Motion in Two Dimensions ▪ 1.5

eXercise

You must be able to

- resolve a 2-D vector into perpendicular **components** 分量 with $A_x = A\cos\theta$, $A_y = A\sin\theta$
- add vectors by combining components to find the **resultant** 合矢量 (magnitude and direction)
- explain that a **projectile's** 抛体 horizontal and vertical motions are **independent** 相互独立
- analyse projectile motion by applying the kinematic equations separately to x and y
- find the **range** 射程, maximum height, and time of flight

Method — Resolving a vector

A force $A = 50$ N at 30° above the horizontal:

$$A_x = 50\cos 30^\circ = 43 \text{ N}, \qquad A_y = 50\sin 30^\circ = 25 \text{ N}.$$

Method — Independence of projectile motion

Horizontal motion is constant velocity, $v_x = v_0 \cos\theta$; vertical motion is free fall, $a_y = -g$. The two directions share only the **time**.

Method — A projectile launched horizontally

A ball leaves a 20 m high table at 5.0 m s^{-1} (take $g = 10$ m s^{-2}). Time to fall: $20 = \frac{1}{2}(10)t^2 \Rightarrow t = 2.0$ s. Horizontal range $= 5.0 \times 2.0 = 10$ m.

Practice 2.1 ▪ 2 marks

A force of 80 N acts at 60° above the horizontal. Find its horizontal and vertical components.

Solution 2.1

Method: Resolving a vector

Worked steps

$A_x = 80\cos 60^\circ = 40$ N; $A_y = 80\sin 60^\circ = 69$ N **B1** **B1**.

Practice 2.2 ▪ 2 marks

A ball rolls off a bench 1.25 m high at 3.0 m s^{-1} (take $g = 10$ m s^{-2}). Find the time to land.

Solution 2.2

Method: A projectile launched horizontally

Worked steps

$1.25 = \frac{1}{2}(10)t^2 \Rightarrow t^2 = 0.25,\ t = 0.50$ s M1 A1.

Practice 2.3 ▪ 1 mark

Hence find how far from the base of the bench the ball lands.

Solution 2.3

Worked steps

$x = 3.0 \times 0.50 = 1.5$ m **B1**.

Practice 2.4 ▪ 1 mark

State what the horizontal and vertical motions of a projectile have in common.

Solution 2.4

Method: Independence of projectile motion

Worked steps

the time of flight (both motions last the same time) B1.

Exam-style 3.1 ▪ 1 mark

For a projectile (ignore air resistance), the horizontal component of velocity

A increases

B decreases

C stays constant

D is zero

Solution 3.1

Method: Independence of projectile motion

A increases

B decreases

C stays constant ✓

D is zero

Worked steps

C.

Exam-style 3.2 ▪ 1 mark

At the top of its path, a projectile's vertical velocity is

A maximum

B zero

C equal to its horizontal velocity

D equal to g

Solution 3.2

Method: Independence of projectile motion

A maximum

B zero ✓

C equal to its horizontal velocity

D equal to g

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A stone is thrown at 20 m s^{-1} at 30° above the horizontal (take $g = 10$ m s^{-2}).
(a) Find the initial vertical velocity component.
(b) Find the time to reach the top of the path.
(c) Find the maximum height.

Solution 3.3

Worked steps

(a) $v_{0y} = 20 \sin 30^\circ = 10$ m s^{-1} B1. (b) $t = 10/10 = 1.0$ s B1. (c) $h = \dfrac{v_{0y}^2}{2g} = \dfrac{10^2}{20} = 5.0$ m M1 A1.