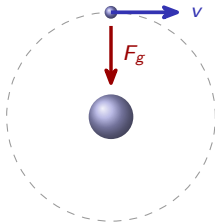


Energy & Momentum of Rotating Systems

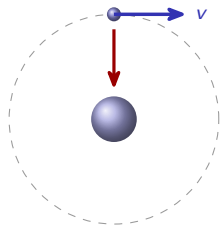
AP Physics 1 ■ Unit 6

AP Physics 1



What we will cover

- 1 Rotational kinetic energy
- 2 Work and power of a torque
- 3 Angular momentum and impulse
- 4 Conservation of angular momentum
- 5 Rolling without slipping
- 6 Orbiting satellites





Rotational kinetic energy

A spinning object stores energy even if its centre stays put:

Energy of spin

$$K_{rot} = \frac{1}{2}I\omega^2$$

Mass $\rightarrow I$, speed $\rightarrow \omega$. Double the spin, **quadruple** the energy.

A rolling object has **both**: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

Down a ramp

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

A shape with mass near the rim (a hoop) puts more energy into spin, so it moves *slower* at the bottom.



Work and power of a torque

A torque does work when it turns an object through an angle:

Rotational work and power

$$W = \tau \Delta\theta, \quad P = \tau\omega$$

$W_{\text{net}} = \Delta K_{\text{rot}}$. A changing torque: work = area under a τ - θ graph.

Worked example

Motor: $\tau = 15 \text{ N m}$ over $\Delta\theta = 8 \text{ rad}$:

$$W = 15 \times 8 = 120 \text{ J}$$

At $\omega = 20 \text{ rad/s}$:

$$P = 15 \times 20 = 300 \text{ W}$$



Angular momentum and impulse

Angular momentum is the rotational twin of momentum:

Spin that resists change

$$L = I\omega$$

$$\Delta L = \tau \Delta t$$

Angular impulse = net torque $\times$ time.

Worked example

$L_i = 30$; a torque $\tau = 5 \text{ N m}$ acts for 4 s:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50 \text{ kg m}^2/\text{s}$$



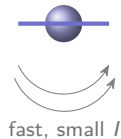
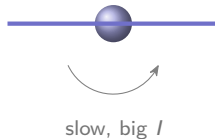
Conservation of angular momentum

With **zero net external torque**, angular momentum is conserved – even as the body changes shape:

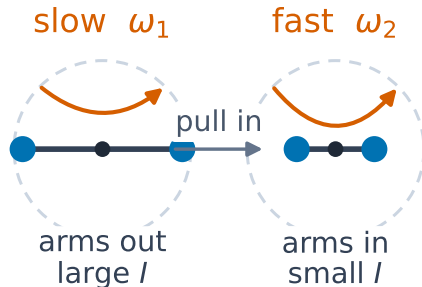
Before = after

$$I_i \omega_i = I_f \omega_f$$

A skater pulls their arms in: I drops, so ω rises – they spin **faster**.



Pulling mass inward lowers I



smaller $I \Rightarrow$ larger ω (angular momentum conserved)

$$L = I\omega \text{ conserved (no external torque): } I_1\omega_1 = I_2\omega_2$$

Pulling mass inward lowers I , so ω rises to conserve $L = I\omega$



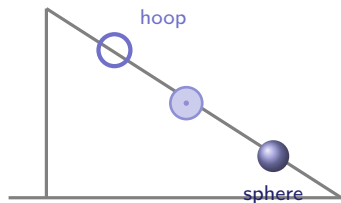
Rolling without slipping

Rolling without slipping locks forward speed to spin:

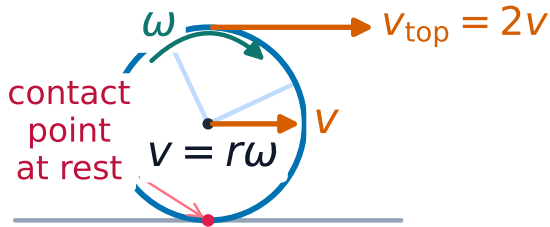
The rolling condition

$$v = \omega r, \quad K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Race down a ramp: the **solid sphere wins**, the hoop comes last – more I means more energy lost to spin.



Rolling without slipping, in detail



$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

$$\text{rolling no slip: } v = r\omega \quad \cdot \quad a = r\alpha$$

In rolling without slipping the contact point is instantaneously at rest, so $v = r\omega$ and the top of the wheel moves at $2v$.



Orbiting satellites

Gravity supplies the centripetal force. Set them equal:

Orbital speed

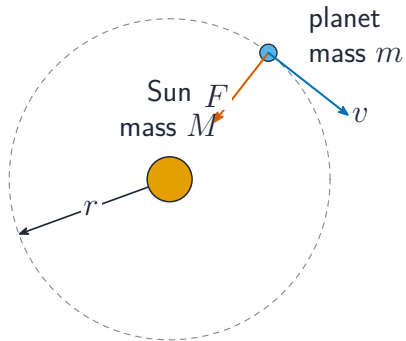
$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

- speed is independent of the satellite's mass
- a **bigger** orbit means a **slower** speed



Satellites are continuous free-fall around Earth

Motion of Orbiting Satellites



Gravity provides the centripetal force that keeps a satellite in orbit

Escape velocity

the definition

the launch speed that just lets an object leave for good

the condition

its total mechanical energy is exactly **zero** – it arrives at infinity with no speed left

so

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0, \text{ giving } v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

the relation

$v_{\text{esc}} = \sqrt{2} \times$ the circular orbital speed at the same radius

no mass in it

the escaping object's own mass cancels, so it is the same for a pebble and a rocket

Escape speed is set by **where you start**, not by what you are. Launching from a higher orbit needs less of it, which is why staged launches work.

Exam tips

- Conserve **angular momentum** $L = I\omega$ when no external torque acts: a smaller I (arms pulled in) gives a larger ω .
- A rolling object splits its energy between $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$, linked by $v = r\omega$ – so it accelerates down a ramp **more slowly** than a sliding one.
- For a circular orbit set gravity equal to the centripetal requirement: $v = \sqrt{GM/r}$, so a larger orbit is **slower** and the satellite's mass cancels.
- Pulling in raises the spin **and** the kinetic energy – the extra energy comes from the work done pulling inward; L is unchanged.
- Watch which rotational quantity is conserved: L (no torque) versus energy (no friction) are different conditions.



Unit 6 – key takeaways

1

Rotational energy

$K_{rot} = \frac{1}{2} I \omega^2$; rolling splits energy in two

2

Angular momentum

$L = I \omega$; angular impulse $\tau \Delta t = \Delta L$

3

Conservation

no external torque $\Rightarrow I_i \omega_i = I_f \omega_f$
(skater)

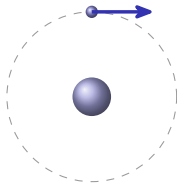
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Orbits

gravity is the centripetal force;
 $v = \sqrt{GM/r}$

Check yourself

- 1 A skater pulls their arms in – what happens to their spin rate?
- 2 Down a ramp: hoop, disk, or sphere – which wins?
- 3 Does orbital speed depend on the satellite's mass?
- 4 A bigger orbit radius means a faster or slower speed?



Answers 1. it speeds up 2. the sphere 3. no 4. slower