

Torque & Rotational Dynamics

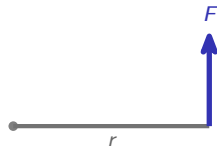
AP Physics 1 ■ Unit 5

AP Physics 1



What we will cover

- 1 Rotational kinematics
- 2 Linking linear and rotational motion
- 3 Torque
- 4 Rotational inertia
- 5 Rotational equilibrium
- 6 Newton's second law for rotation





Rotation mirrors straight-line motion

Spinning is described by angles:

- **angular velocity** ω – rate the angle turns
- **angular acceleration** α – rate ω changes

For constant α , the equations have the **same form** as the linear ones. Rotation rate: $\omega = 2\pi f$.



Rotation links angle, angular speed, and linear speed

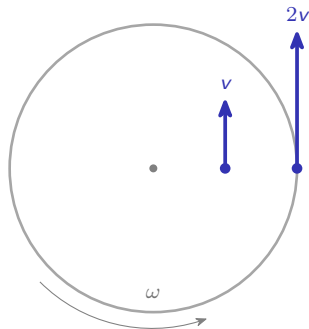
Linking linear and rotational motion

Every point on a spinning body moves along a curve, tied to the **radius** r .

Same ω , different v

$$v = r\omega, \quad a_t = r\alpha, \quad s = r\theta$$

All points share one ω , but points farther out move **faster**.



The angular quantities, and the moment arm

Angular displacement

Angular displacement $\Delta\theta$ in radians, with $s = r\Delta\theta$ linking it to the arc travelled.

Angular velocity

$\omega = d\theta/dt$, and $v = r\omega$ for a point on a rotating body – the further out, the faster.

Moment arm

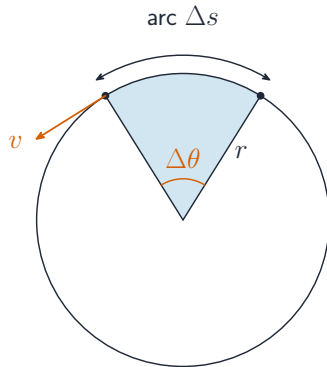
The **moment arm** is the perpendicular distance from the axis to the line of the force: $\tau = r_{\perp}F = rF\sin\theta$.

Why perpendicular

A force pointing at the axis has zero moment arm, so it produces no torque however large it is.

Every rotational quantity mirrors a linear one. Write the linear equation first, then substitute the angular partner.

As the radius turns through an angle



As the radius turns through an angle, a point moves along an arc at speed v



Torque

Torque is the rotational version of force – how strongly a force twists:

Twisting strength

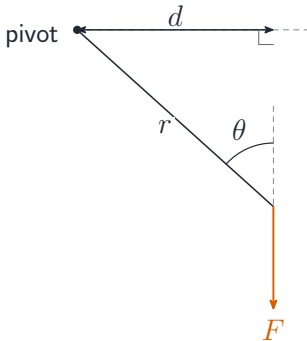
$$\tau = rF \sin \theta$$

The **lever arm** is the perpendicular distance to the force. A bigger lever arm gives **more torque**.



Torque is the rotational analogue of force

The moment of a force depends on



The moment of a force depends on the perpendicular distance from the pivot



Rotational inertia

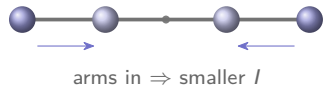
Rotational inertia I resists a change in spin – and depends on **how the mass is spread**:

Distance is squared

$$I = \sum mr^2$$

Parallel-axis theorem : $I = I_{cm} + md^2$.

Move mass out to $2\times$ the radius $\Rightarrow I$ grows $4\times$.

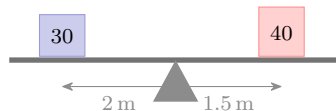




Rotational equilibrium

In **rotational equilibrium** the **net torque** is zero. Full **static equilibrium** : both net force **and** net torque zero.

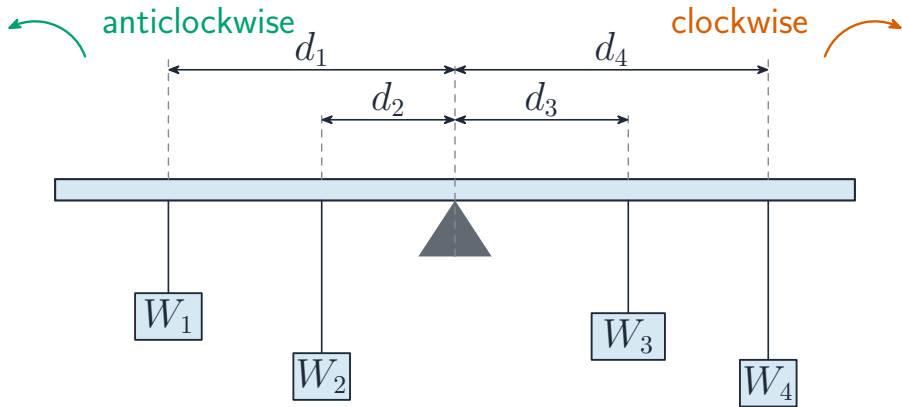
- pick the axis at an unknown force to drop it out
- set clockwise torque = counterclockwise torque



Heavier child

sits **closer** to balance.

At balance the clockwise and anticlockwise moments



At balance the clockwise and anticlockwise moments about the pivot are equal



Newton's second law for rotation

A nonzero net torque gives an angular acceleration – just like $F = ma$:

Rotational form

$$\tau_{net} = I\alpha$$

torque $\leftrightarrow$ force, $I \leftrightarrow$ mass, $\alpha \leftrightarrow$ acceleration.

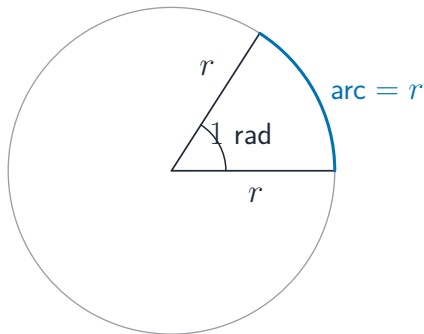
Worked example

$\tau = 12 \text{ N m}$ on $I = 3 \text{ kg m}^2$:

$$\alpha = \frac{\tau}{I} = \frac{12}{3} = 4 \text{ rad/s}^2$$

Double $I \Rightarrow$ half the α – harder to spin up.

Rotational Kinematics



one **radian**: the angle whose arc is exactly one radius long

$\pi \text{ rad} = 180^\circ$, so $1 \text{ rad} \approx 57.3^\circ$

degrees $\rightarrow$ radians: $\times \frac{\pi}{180}$

radians $\rightarrow$ degrees: $\times \frac{180}{\pi}$

One radian is the angle whose arc length equals the radius

Exam tips

- Torque $\tau = Fr_{\perp}$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational analogues: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum \tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount – a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.



Unit 5 – key takeaways

1

Analogy

$$x, v, a, m, F \leftrightarrow \theta, \omega, \alpha, I, \tau; v = r\omega$$

2

Torque

$\tau = rF \sin \theta$; the lever arm sets the twist

3

Inertia

$I = \sum mr^2$; mass farther out counts far more

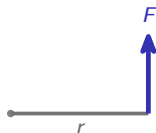
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Dynamics

equilibrium: net $\tau = 0$; otherwise
 $\tau = I\alpha$

Check yourself

- 1 Push a door near the hinge vs at the handle – which gives more torque?
- 2 Move a mass to twice the radius – what happens to I ?
- 3 On a balanced seesaw, who sits closer to the pivot?
- 4 $\tau = 20 \text{ N m}$ on $I = 5 \text{ kg m}^2$ – find α .



Answers 1. the handle (bigger lever arm) 2. it quadruples 3. the heavier one
4. 4 rad/s^2