

On this exam the number is rarely the point. Each solution below is written the way a scored response is written: the principle named, the condition that lets it apply stated, then the algebra, then the answer with its unit and a sentence saying what it means.

Kinematics and dynamics

A 2.0 kg block is pulled along a horizontal surface by a 12 N force directed 30 degrees above the horizontal. The coefficient of kinetic friction is 0.25. Find the acceleration.

2.4 m s^{-2} , horizontally, in the direction of motion.

Commentary: Resolve the applied force: horizontal component $12 \cos 30 = 10.4 \text{ N}$, vertical component $12 \sin 30 = 6.0 \text{ N}$ upward. The normal force is NOT mg here, because the pull lifts part of the weight: $N = mg - 6.0 = (2.0)(9.8) - 6.0 = 13.6 \text{ N}$. Friction opposes motion, so $f = 0.25 \times 13.6 = 3.4 \text{ N}$. Net horizontal force $= 10.4 - 3.4 = 7.0 \text{ N}$, and $a = F/m = 7.0/2.0 = 3.5 \text{ m s}^{-2}$. Working to two significant figures with $g = 9.8$ gives 3.5 m s^{-2} . The step that carries the most credit is recognising that the vertical component changes the normal force; a solution using $N = mg$ loses it immediately.

A ball is thrown horizontally at 15 m/s from a 20 m cliff. How far from the base does it land, and what is its speed on impact?

30 m from the base; impact speed 25 m s^{-1} .

Commentary: Treat the two directions independently. Vertically the initial velocity is zero, so $20 = \frac{1}{2}(9.8)t^2$, giving $t = 2.02 \text{ s}$. Horizontally the velocity is constant, so $x = 15 \times 2.02 = 30 \text{ m}$. For the impact speed, the vertical velocity is $v = 9.8 \times 2.02 = 19.8 \text{ m s}^{-1}$ and the horizontal is still 15 m s^{-1} , so the resultant is $\sqrt{(15^2 + 19.8^2)} = 24.8$, i.e. 25 m s^{-1} . Note that the horizontal speed never appears in the time calculation – a common error is to use the resultant velocity in the vertical equation.

Energy and momentum

A 0.50 kg cart moving at 4.0 m/s collides with a stationary 1.5 kg cart and they stick together. They then slide up a frictionless ramp. How high do they rise?

0.051 m, about 5.1 cm.

Commentary: This needs both principles, in order. Through the collision, momentum is conserved but kinetic energy is not, because the carts stick – say so. $(0.50)(4.0) = (2.0)v$, so $v = 1.0 \text{ m s}^{-1}$. After the collision the ramp is frictionless, so mechanical energy IS conserved: $\frac{1}{2}(2.0)(1.0)^2 = (2.0)(9.8)h$, giving $h = 0.051 \text{ m}$. Using energy conservation through the collision would give 0.82 m – sixteen times too large – and is the single most common error on this question type.

A 60 kg student runs up a 4.0 m staircase in 5.0 s. Calculate the average power developed against gravity.

470 W.

Commentary: The work done against gravity is the gain in gravitational potential energy: $W = mgh = (60)(9.8)(4.0) = 2352 \text{ J}$. Average power is work over time: $P = 2352/5.0 = 470 \text{ W}$. The phrase “against gravity” matters: the student’s total power output is larger, because some energy goes to kinetic energy and to internal losses. Saying that explicitly is what distinguishes a full answer.

Circular motion and rotation

I A 0.30 kg ball on a 0.80 m string is swung in a vertical circle. What is the minimum speed at the top for the string to stay taut, and what is the tension at the bottom if the speed there is 6.0 m/s?

Minimum speed at the top 2.8 m s^{-1} ; tension at the bottom 16 N.

Commentary: At the top, both gravity and tension point towards the centre, so $T + mg = mv^2/r$. The minimum condition is $T = 0$, which gives $v = \sqrt{gr} = \sqrt{(9.8 \times 0.80)} = 2.8 \text{ m s}^{-1}$. At the bottom, tension points towards the centre and gravity away, so $T - mg = mv^2/r$, giving $T = m(v^2/r + g) = 0.30(36/0.80 + 9.8) = 0.30(45 + 9.8) = 16 \text{ N}$. The reasoning to state is that the NET force must be centripetal at every point; “centripetal force” is not a separate force to be added to the diagram.

I A uniform 3.0 m beam of mass 20 kg is supported at both ends. A 50 kg person stands 1.0 m from the left end. Find the force from the left support.

About 260 N.

Commentary: Take torques about the RIGHT support, which eliminates its unknown force from the equation – that choice is itself worth stating. The beam’s weight acts at its centre, 1.5 m from either end; the person is 2.0 m from the right support. So $F_{\text{left}}(3.0) = (20)(9.8)(1.5) + (50)(9.8)(2.0) = 294 + 980 = 1274$, giving $F_{\text{left}} = 425 \text{ N}$. Checking with vertical equilibrium: total weight is $(70)(9.8) = 686 \text{ N}$, so the right support carries 261 N. Always name the pivot you chose and why.

Waves and circuits

I A string fixed at both ends is 1.2 m long and vibrates in its third harmonic at 180 Hz. Find the wave speed.

144 m s^{-1} .

Commentary: In the third harmonic a string fixed at both ends holds three half-wavelengths, so $1.2 = 3(\lambda/2)$ and $\lambda = 0.80 \text{ m}$. Then $v = f\lambda = 180 \times 0.80 = 144 \text{ m s}^{-1}$. Sketching the standing wave with its nodes and antinodes before writing anything is the reliable way to get the harmonic relationship right; from memory it is easy to use $\lambda = 2L/n$ with the wrong n .

I Two resistors, 4.0Ω and 12Ω , are connected in parallel across a 24 V battery of negligible internal resistance. Find the current from the battery.

8.0 A.

Commentary: In parallel the voltage is common, so compute each branch separately: $24/4.0 = 6.0 \text{ A}$ and $24/12 = 2.0 \text{ A}$, giving 8.0 A total. Equivalently the combined resistance is $1/R = 1/4 + 1/12 = 1/3$, so $R = 3.0 \Omega$ and $I = 24/3.0 = 8.0 \text{ A}$. Doing it both ways is a free check, and the branch method makes it obvious why the combined resistance is smaller than either resistor.