

The Experimental Design and Analysis question is one of four free-response questions, and the one most candidates under-answer. It is not about apparatus: it is about what you measure, how you linearize it, and what the slope of your graph means.

### The Experimental Design and Analysis question

One of the four free-response questions is a lab question. It asks you to design a procedure, say how you would graph the data so the graph is a straight line, and then work with data – plot it, find a slope, and say what the slope tells you. It is not about naming fancy apparatus; it is about what you measure and what you do with the numbers.

Read the question for what is GIVEN and what is to be DETERMINED. The quantity to be determined is almost always the slope (or the intercept) of the graph you are about to design.

Answer every part, even if an earlier part went wrong. The graph and slope parts are marked on what you plotted, not on whether your design matched the reference.

### Writing the procedure

Name each quantity you will measure AND the instrument that measures it: ‘a meterstick to measure the height  $h$  of the ramp’ earns the point; ‘a meterstick’ does not.

Vary the independent variable over a RANGE – at least five values, spread out. One measurement cannot give a slope.

Say what you hold constant and how. ‘Use the same cart, released from rest from the same mark each time’.

Repeat each trial and average. Say that this reduces RANDOM error; it does nothing for a systematic one.

Remove a specific systematic error by name: zero the force sensor, measure to the center of mass, start timing from a fixed release, use a motion detector instead of a hand-held stopwatch.

Write the steps so someone else could follow them: numbered, in order, with each measurement said once. A paragraph of good intentions loses points that a numbered list keeps.

### Linearizing the data

Almost every design question wants a straight-line graph. Start from the equation, put the quantity you control on one side and the one you measure on the other, and match it to  $y = mx + b$ .

| Relationship          | Plot $y$ | against $x$   | Slope                   |
|-----------------------|----------|---------------|-------------------------|
| $a = g \sin \theta$   | $a$      | $\sin \theta$ | $g$                     |
| $T = 2\pi \sqrt{L/g}$ | $T^2$    | $L$           | $4\pi^2/g$              |
| $T = 2\pi \sqrt{m/k}$ | $T^2$    | $m$           | $4\pi^2/k$              |
| $v^2 = 2gh$           | $v^2$    | $h$           | $2g$                    |
| $F = kq_1q_2/r^2$     | $F$      | $1/r^2$       | $kq_1q_2$               |
| $1/s_o + 1/s_i = 1/f$ | $1/s_i$  | $1/s_o$       | $-1$ (intercept $1/f$ ) |
| $V = V_0e^{-t/RC}$    | $\ln V$  | $t$           | $-1/RC$                 |

Write down what the slope equals, in symbols, BEFORE you take data. That sentence is usually a point on its own.

Units of the slope are the y-unit over the x-unit. For  $T^2$  against  $L$  that is  $\text{s}^2/\text{m}$  – and checking it against  $4\pi^2/g$  (units  $\text{s}^2/\text{m}$ ) is a quick way to catch a wrong plot.

A constant you multiply by does not stop a line being straight; a power or a square root does. If the equation has one, plot that power.

### The graph and its slope

Label both axes with the quantity and its unit, and pick scales that spread the data over most of the grid.

Plot the points; do not connect the dots. Draw ONE straight best-fit line with points scattered on both sides.

Find the slope from two points ON THE LINE, far apart, and show the calculation. Two data points are not the line unless the line happens to pass through them.

Say what the slope means and calculate the quantity from it, with units: ‘slope =  $4\pi^2/g$ , so  $g = 4\pi^2/\text{slope} = 9.7 \text{ m/s}^2$ ’.

If the theory says the line passes through the origin and yours does not, the intercept points to a systematic error. Name a plausible one: friction, a reaction time, a zero offset, the mass of a hanger left out.

### Error and agreement

**Percent difference** –  $|\text{experimental} - \text{accepted}| \div \text{accepted} \times 100\%$ . A few percent is good agreement for a school lab

**Random error** – scatter about the line; reduced by repeating trials and by a best-fit line through many points

**Systematic error** – shifts every point the same way; it shows up as a wrong slope or an unexpected intercept, and repeating does not remove it

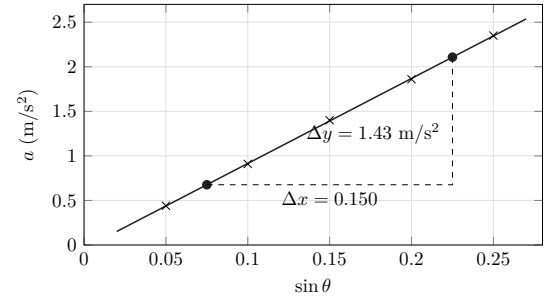
When asked how an error affects the result, push it through the equation and the graph: ‘measuring  $L$  to the top of the bob makes every  $L$  too short by the bob’s radius; that shifts the line sideways, so it shows in the intercept and leaves the slope – and  $g$  – unchanged’. Guessing a direction without the reasoning earns nothing.

‘Human error’ and ‘measure more carefully’ are not sources of error. Name the measurement and what goes wrong with it.

### Worked analysis: g from a cart on a ramp :

A low-friction cart is released from rest on a 1.00 m track. One end is raised to height  $h$ , so  $\sin \theta = h \div 1.00$  m. A motion detector at the top measures the cart's acceleration  $a$ . Prediction:  $a = g \sin \theta$ , so a graph of  $a$  against  $\sin \theta$  is a line through the origin with slope  $g$ .

| $h$ (m)                            | 0.050 | 0.100 | 0.150 | 0.200 | 0.250 |
|------------------------------------|-------|-------|-------|-------|-------|
| $\sin \theta$                      | 0.050 | 0.100 | 0.150 | 0.200 | 0.250 |
| $a$ (m/s <sup>2</sup> ), mean of 3 | 0.44  | 0.91  | 1.40  | 1.86  | 2.35  |



The slope is read from two points on the LINE (filled dots), not from two data points.

Slope =  $1.43 \div 0.150 = 9.5 \text{ m/s}^2$ , so  $g = 9.5 \text{ m/s}^2$ . Percent difference from  $9.8 \text{ m/s}^2$ :  $0.3 \div 9.8 = 3\%$ .

The line meets the  $a$ -axis slightly below zero (about  $-0.04 \text{ m/s}^2$ ). Friction takes a roughly constant amount off every acceleration, which shifts the whole line down without changing its slope much.

The slope itself is 3% low. Some of the cart's energy goes into spinning its wheels, so  $a$  is a fixed fraction less than  $g \sin \theta$  at every angle, which lowers the slope. Naming each error, the direction it pushes, and whether it shows in the slope or the intercept is the analysis point.

### Labs that keep coming back

**Newton's second law** – a cart pulled by a hanging mass over a pulley. Move masses from the cart to the hanger so the total mass stays constant; a against hanging weight is a line of slope  $1/(\text{total mass})$

**Pendulum** – time 10 oscillations for each length  $L$ , measured to the center of the bob;  $T^2$  against  $L$  has slope  $4\pi^2/g$ . The period does not depend on the mass, at small angles

**Spring constant** – static: force against stretch, slope  $k$ . Dynamic:  $T^2$  against  $m$ , slope  $4\pi^2/k$ . Two methods for one constant are a natural 'compare' question

**Energy conservation** – release from height  $h$  and measure the speed at the bottom with a photogate;  $v^2$  against  $h$  has slope  $2g$  if no energy is lost. A smaller slope measures the loss

**Momentum in collisions** – carts on a track with motion detectors or photogates; compare total momentum before and after. Define the system as both carts – the track's friction is the external force to minimize

**Rotational motion** – a hanging mass unwinding string from a pulley gives a known torque; the angular acceleration from a rotary sensor or from  $a$  and the radius. More hanging weight, larger  $\alpha$  – but not proportionally, because the tension is less than  $mg$

### Reasoning and justification

When asked to justify, refer to a physical principle by name (conservation of momentum, Newton's third law) and then apply it to the specific objects in the question.

For a system question, define the system first. Whether momentum or energy is conserved depends entirely on where you draw the boundary and what forces cross it.

A comparison answer needs both sides: 'greater than in case A because ...', not 'it is large'.

Do not derive a formula the question did not ask for; answer the question that was asked, in the form it was asked.