

Exercise walk-through

6.5

Rolling

AP Physics 1

Rolling without slipping

Rolling without slipping

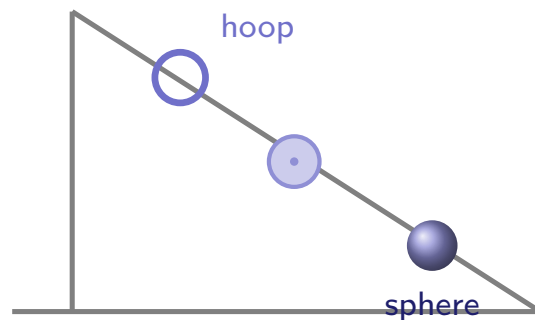
locks

forward speed to spin:

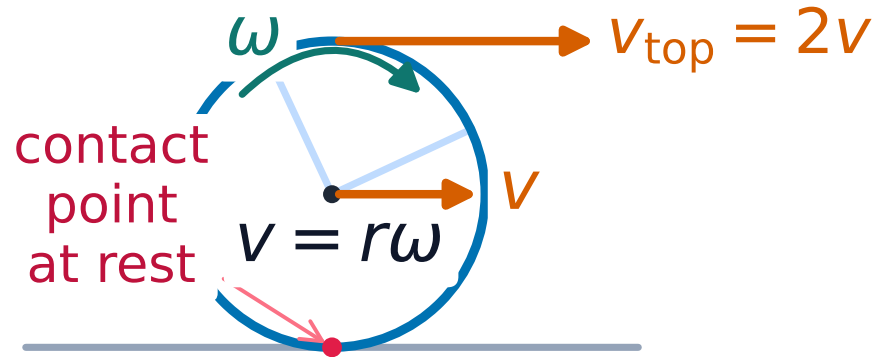
The rolling condition

$$v = \omega r, \quad K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Race down a ramp: the **solid sphere wins**, the hoop comes last – more I means more energy lost to spin.



Rolling without slipping, in detail



$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

$$\text{rolling no slip: } v = r\omega \quad \cdot \quad a = r\alpha$$

In rolling without slipping the contact point is instantaneously at rest, so $v = r\omega$ and the top of the wheel moves at $2v$.

Exercises 2.1-2.3

2.1 A wheel of radius 0.25 m rolls without slipping at $v = 5.0 \text{ m s}^{-1}$. Find its angular speed. [1]

2.2 State the rolling-without-slipping condition. [1]

2.3 A hoop and a solid cylinder roll from rest down the same ramp. State which reaches the bottom first. [1]

Exercises 2.1-2.3

2.1 A wheel of radius 0.25 m rolls without slipping at $v = 5.0 \text{ m s}^{-1}$. Find its angular speed. [1]

2.1

- $\omega = \frac{v}{r} = \frac{5.0}{0.25} = 20 \text{ rad s}^{-1}$

2.2 State the rolling-without-slipping condition. [1]

2.2

- $v = \omega r$ (the centre's speed equals angular speed times radius)

2.3 A hoop and a solid cylinder roll from rest down the same ramp. State which reaches the bottom first. [1]

2.3

- the solid cylinder (smaller rotational inertia)

Exercises 3.1-3.2

3.1 For an object rolling without slipping,

[1]

☐

A $v = \omega r$

B $v = \omega / r$

C $\omega = vr$

D $v = \frac{1}{2}\omega r$

3.2 The total kinetic energy of a rolling object is

[1]

☐

A translational only

B rotational only

C the sum of translational and rotational

D neither

Exercises 3.1·3.2

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[1]

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3.1 A

- $v = \omega r$.

3.2 The total kinetic energy of a rolling object is

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A translational only

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D neither

3.2 C

- the sum of translational and rotational.

Exercise 3.3 (a)

3.3 A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0 \text{ m s}^{-1}$.

(a) Find its angular speed.

[1]

Exercise 3.3 (a)

3.3 A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0 \text{ m s}^{-1}$.

(a) Find its angular speed. [1]

(a)

- $\omega = \frac{3.0}{0.10} = 30 \text{ rad s}^{-1}$

Exercise 3.3 (b)

3.3 A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0 \text{ m s}^{-1}$.

(b) Find its total kinetic energy. [3]

Exercise 3.3 (b)

3.3 A solid cylinder ($I = \frac{1}{2}mR^2$) of mass 2.0 kg and radius 0.10 m rolls at $v = 3.0 \text{ m s}^{-1}$.

(b) Find its total kinetic energy. [3]

(b)

- $I = \frac{1}{2}(2.0)(0.10)^2 = 0.010 \text{ kg m}^2$; $K = \frac{1}{2}(2.0)(3.0)^2 + \frac{1}{2}(0.010)(30)^2 = 9.0 + 4.5 = 13.5 \text{ J}$

Exercise 3.4

3.4 A uniform solid disc rolls without slipping from rest through a vertical drop of 0.60 m. Use $g = 9.8 \text{ m s}^{-2}$. (a) Derive an expression for its final speed and calculate it. [2]

(b) Explain why using $mgh = \frac{1}{2}mv^2$ alone overestimates that speed. [2]

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(b) Explain why using $mgh = \frac{1}{2}mv^2$ alone overestimates that speed. [2]

3.4

(a) With $I = \frac{1}{2}mR^2$ and $\omega = v/R$, $mgh = \frac{3}{4}mv^2$, so $v = \sqrt{4gh/3} = 2.80 \text{ m s}^{-1}$.

(b) Some energy becomes rotational kinetic energy; assigning all of it to translation gives a larger value, $\sqrt{2gh} = 3.43 \text{ m s}^{-1}$.