

Past-paper walk-through

The Production Function ■ 3.1

eXercise

Questions in this set

- 2024 Set 2 Q2
- 2023 Set 2 Q2
- 2017 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 2

2024 Set 2 ■ Q2

2. The table provided shows the short-run production function for Lowen Feline, a profit-maximizing firm that produces cat food.

Number of Workers	Total Quantity of Cat Food (bags)
0	0
1	5
2	12
3	18
4	23
5	27
6	30
7	32

Question 2

8	33
9	32

Lowen Feline sells as many bags of cat food as it wants at a market price of \$10 per bag and hires as many workers as it wants at a market wage of \$18.

- (a) Lowen Feline's fixed cost is \$90. Calculate the average fixed cost if Lowen Feline hires 6 workers. Show your work.
- (b) Assume labor is the only variable input to Lowen Feline. Calculate the marginal cost if Lowen Feline increases output from 27 to 30 units. Show your work.
- (c) With the hiring of which worker do diminishing marginal returns begin? Explain using numbers.
- (d) Determine the profit-maximizing number of workers Lowen Feline will hire. Explain using marginal analysis.

Question 2

(e) In the long run, a rival company, Gato Food, increases its production from 40 to 50 units, and its total cost increases from \$600 to \$900. Over the output range of 40 to 50 units, is Gato Food experiencing economies of scale, diseconomies of scale, or constant returns to scale? Explain using numbers.

Begin your response to this question at the top of a new page in the separate Free Response booklet and fill in the appropriate circle at the top of each page to indicate the question number.

Solution 2

(a)

Mark scheme

- Average Fixed Cost = Total Fixed Cost / Quantity of Output = $90/30 = 3$. (6 workers correspond to 30 units of output on Lowen Feline's production schedule.)

Solution 2

(b)

Mark scheme

- Marginal Cost = Change in Total Variable Cost / Change in Output = $(108 - 90) / (30 - 27) = 18/3 = 6$. (Equivalently, Marginal Cost = Change in Total Cost / Change in Output = $(198 - 180) / (30 - 27) = 18/3 = 6$.)

Solution 2

(c)

Mark scheme

- Diminishing marginal returns to labor begin with the hiring of the 3rd worker. Explanation using numbers: the marginal product of the 1st worker is 5 bags, the marginal product of the 2nd worker increases to 7 bags, and the marginal product of the 3rd worker decreases to 6 bags — since marginal product starts falling with the 3rd worker, that is where diminishing marginal returns begin.

Solution 2

(d)

Mark scheme

- The profit-maximizing number of workers is 7. Explanation using marginal analysis: a firm hires labor up to the point where the marginal revenue product (MRP) of the last worker is at least equal to the marginal factor cost (MFC) of that worker (the wage). The MRP of the 7th worker (20) is greater than the MFC of the 7th worker (wage = 18), so hiring the 7th worker adds to profit; hiring an 8th worker would decrease profit because the MRP of the 8th worker (10) is less than the MFC of the 8th worker (wage = 18). So Lowen Feline stops hiring at 7 workers.

Solution 2

(e)

Mark scheme

- Gato Food is experiencing diseconomies of scale. Explanation using numbers: as output rises from 40 to 50 units, long-run average total cost rises from 15 *per unit* ($600 / 40 = 15$) to 18 per unit ($900 / 50 = 18$). Because long-run average total cost increases as output increases over this range, Gato Food is experiencing diseconomies of scale.

Question 2

2023 Set 2 ■ Q2

2. Keepdry produces and sells rain jackets in a perfectly competitive product market at the price of \$5 per jacket and hires all the workers it needs in a perfectly competitive labor market at the wage rate of \$15. Labor is the only variable input, and the firm's production schedule is provided in the table.

Number of Workers	Quantity of Output
0	0
1	9
2	20
3	27
4	32
5	34

Question 2

6	35
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- (a) Calculate the marginal revenue product of the second worker. Show your work.
 - (b) Diminishing marginal returns will begin with the hiring of which worker?
 - (c) Determine the profit-maximizing number of workers the firm should hire. Explain using marginal analysis.
 - (d) Assuming Keepdry's fixed cost is \$40, calculate Keepdry's economic profit when hiring the profit-maximizing number of workers. Show your work.
 - (e) Suppose Keepdry's fixed cost increases to \$80. Will the profit-maximizing number of workers hired in the short run increase, decrease, or stay the same? Explain.
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Question 2

Begin your response to this question at the top of a new page in the separate Free Response booklet and fill in the appropriate circle at the top of each page to indicate the question number.

Solution 2

(a)

Mark scheme

- MRP of the 2nd worker =

$$55. \text{MRP}(2\text{nd worker}) = \text{Marginal Product} \times \text{Marginal Revenue} = (20 - 9) \times 5 = 11 \times 5 = 55.$$

Solution 2

(b)

Mark scheme

- Diminishing marginal returns begin with the hiring of the 3rd worker (that is where the marginal product of labor starts to decline).

Solution 2

(c)

Mark scheme

- The profit-maximizing number of workers is 4. Using marginal analysis, a firm hires an additional worker as long as $MRP \geq MFC$ (the wage). The MRP of the 4th worker is 25, *which is greater than the marginal factor cost of 15*, so hiring the 4th worker adds to profit. The MRP of the 5th worker is only 10, *which is less than the 15 wage/MFC*, so hiring the 5th worker would decrease profit. Therefore the firm hires 4 workers.

Solution 2

(d)

Mark scheme

- Economic profit = 60. $Economic Profit = Total Revenue - Total Cost = Total Revenue - Total Fixed Cost - Total Variable Cost = (5 \times 32) - 40 - (15 \times 4) = 160 - 40 - 60 = 160 - 100 = 60.$

Solution 2

(e)

Mark scheme

- The number of workers hired will stay the same (4 workers). An increase in fixed cost does not affect the marginal factor cost (wage) of hiring labor or the marginal revenue product of labor, and fixed costs do not enter the marginal hire/no-hire decision, so the profit-maximizing quantity of labor is unchanged in the short run.

Question 2

2017 ■ Q2

The table below shows the output a firm produces using different amounts of capital (K) and labor (L). The markets for capital and labor are perfectly competitive. The rental rate of capital is \$75 per unit, and the wage rate is \$200 per unit. In the short run, capital is fixed and labor is variable.

Labor	Output with K=1	Output with K=2
0	0	0
1	10	20
2	25	50
3	38	75

Question 2

- (a)** If the firm uses one unit of capital and one unit of labor, will it be operating with constant, increasing, or decreasing returns to scale? Explain using numbers from the table.
- (b)** Assume now that the firm currently has two units of capital and is using three units of labor.
- (b)(i)** Calculate the marginal product for the third unit of labor. Show your work.
- (b)(ii)** Did the firm experience diminishing marginal returns with the addition of the third unit of labor? Explain using numbers from the table.
- (b)(iii)** Calculate the firm's average total cost for its current level of production. Show your work.
- (b)(iv)** If the firm's output is sold in a competitive market, what is the lowest output price at which the third unit of labor would be hired?

Question 2

Labor	Output with $K=1$	Output with $K=2$
0	0	0
1	10	20
2	25	50
3	38	75

Solution 2

(a)

Mark scheme

- With one unit of capital and one unit of labor, the firm is operating with increasing returns to scale. Doubling both inputs (to two units of capital and two units of labor) more than doubles output – output rises from 10 units to 50 units, a five-fold increase versus only a doubling of inputs. Equivalently, average total cost falls from 27.50 to 11 when labor and capital both double, which also indicates increasing returns to scale.

Solution 2

(b)

Mark scheme

- Setup/context sentence only ('the firm currently has two units of capital and is using three units of labor') – establishes the scenario for parts (b)(i)-(iv) and carries no points of its own.

(b)(i)

Mark scheme

- Marginal product of the third unit of labor = $(\text{change in output})/(\text{change in labor}) = (75-50)/(3-2) = 25$ units. Show this work.

Solution 2

(b)(ii)

Mark scheme

- Yes, the firm experienced diminishing marginal returns with the addition of the third unit of labor, because the marginal product of the third worker (25 units) is less than the marginal product of the second worker (30 units).

Solution 2

(b)(iii)

Mark scheme

- Average total cost at 2 units of capital and 3 units of labor: $(75 \times 2)/75 + (200 \times 3)/75 = 2 + 8 = 10$ (*equivalently*, $((75 \times 2) + (200 \times 3))/75 = 750/75 = 10$). Show the capital-cost-per-unit-output plus labor-cost-per-unit-output work.

Solution 2

(b)(iv)

Mark scheme

- The lowest output price at which the firm would hire the third unit of labor is \$8. This is the price at which the value of the marginal product of the third worker ($\text{price} \times \text{MP} = \text{price} \times 25$) just covers the cost of hiring that worker, consistent with the \$200 labor cost used in the ATC calculation: $\text{price} \times 25 \geq 200$ implies $\text{price} \geq \$8$.