

3.18 Undecidable Problems

Name: _____ Class: _____ Date: _____

Total: 8 marks

KEY WORDS decidable 可判定 • undecidable 不可判定 • inefficient 低效 • algorithm 算法
• efficiency 效率

Objective

Build the skills to answer exam questions on **undecidable problems**.

You must be able to:

- explain how a **decidable** 可判定 problem has an algorithm that always gives a correct yes-or-no answer
- explain how an **undecidable** 不可判定 problem has no algorithm that solves every case
- describe the difference between undecidable and merely **inefficient** 低效
- recognize undecidability as a **limit** on computation

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Decidable vs undecidable

- A **decidable** problem has an algorithm that **always** gives a correct yes-or-no answer.
- An **undecidable** problem has **no** algorithm that solves **every** case correctly.

■ Undecidable is not just slow

Undecidability is stronger than inefficiency: an inefficient problem can still be solved (given enough time), but an undecidable one **cannot** be solved for all cases at all.

■ A limit on computation

Undecidability shows there are things computers simply **cannot** do.

2 Practice

2.1 Define a decidable problem. [1]

2.2 Define an undecidable problem. [1]

2.3 State the difference between an undecidable problem and a merely inefficient one. [1]

3 Exam-style questions

3.1 A decidable problem has an algorithm that [1]

- | | |
|---|---|
| A | B |
| C | D |
- A always gives a correct yes-or-no answer
B never finishes
C gives a random answer
D is simply slow

3.2 An undecidable problem [1]

- | | |
|---|---|
| A | B |
| C | D |
- A can always be solved slowly
B has no algorithm that solves every case correctly
C is a syntax error
D is a data type

3.3 A problem has no algorithm that can correctly answer it in **all** cases.

(a) Name this kind of problem. [1]

(b) State whether this is the same as being slow. [1]

(c) State what it shows about computation. [1]