

Exercise walk-through

Comparing Boolean Expressions ■ 2.6

eXercise

You must be able to

- determine whether two Boolean expressions are **equivalent** 等价
- apply **De Morgan's Laws** 德摩根定律 to rewrite $!(a \ \&\& \ b)$ and $!(a \ || \ b)$
- simplify a compound Boolean expression

Method — Equivalent expressions

Two Boolean expressions are **equivalent** if they always give the **same** result for every input.

Method — De Morgan's Laws

$$\neg(a \ \&\& \ b) = \neg a \ || \ \neg b, \quad \neg(a \ || \ b) = \neg a \ \&\& \ \neg b.$$

Method — Simplifying

$!(x > 5)$ is simply $x \leq 5$.

Practice 2.1 ■ 1 mark

State what it means for two Boolean expressions to be equivalent.

Solution 2.1

Method: Equivalent expressions

Worked steps

they give the same result for every possible input **B1**.

Practice 2.2 ■ 1 mark

Rewrite $\neg(a \wedge b)$ using De Morgan's Law.

Solution 2.2

Worked steps

`!a || !b` **B1**.

Practice 2.3 ■ 1 mark

Rewrite $\neg(x > 5)$ as a simpler comparison.

Solution 2.3

Worked steps

$x \leq 5$ **B1**.

Exam-style 3.1 ■ 1 mark

$!(a \ || \ b)$ is equivalent to

A $!a \ || \ !b$

B $!a \ \&\& \ !b$

C $a \ \&\& \ b$

D $a \ || \ b$

Solution 3.1

A !a || !b

B !a && !b



C a && b

D a || b

Worked steps

B.

Exam-style 3.2 ■ 1 mark

Two Boolean expressions are equivalent if they

- A** look the same
- B** always give the same result
- C** are both true
- D** both use `&&`

Solution 3.2

Method: Equivalent expressions

- A look the same
- B always give the same result 
- C are both true
- D both use `&&`

Worked steps

B.

Exam-style 3.3 ■ 1 mark

Simplify or rewrite the following.

- (a) Rewrite `!(p && q)`.
- (b) Rewrite `!(x >= 10)` as a comparison.
- (c) State whether `!(true)` equals `false`.

Solution 3.3

Worked steps

(a) $\neg p \vee \neg q$ **B1**. (b) $x < 10$ **B1**. (c) yes **B1**.