

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.		
	$\frac{1}{4 - 0} \int_0^4 C(t) dt$	Average value formula	Point 1 (P1)
	$= \frac{1}{4}(11.112896) = 2.778224$	Answer	Point 2 (P2)
	From time $t = 0$ to $t = 4$ weeks, the average number of acres affected by the invasive species was 2.778 acres.		

Scoring Notes for Part A

- **P1** is earned for the correct integral, with or without the differential, along with evidence of division by 4. In the presence of the correct integral, the correct answer will suffice as evidence of division by 4. These may appear all in one step, as in the model solution, or in multiple steps.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:

○ $\int_0^4 C(t) dt = 11.112896$ so the average velocity is 2.778224.

Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer.

○ $\int_0^4 C(t) dt = \frac{11.112896}{4} = 2.778224$

Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)

○ $\int_0^4 C(t) dt = 2.778224$

Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)

- Note that the values $\frac{1}{4}(11.112)$ and $\frac{1}{4}(11.113)$ are accurate to three digits after the decimal and therefore earn **P2**.

B Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.	
$\frac{C(4) - C(0)}{4 - 0} = 1.282008$	Uses average rate of change Point 3 (P3)
$C'(t) = \frac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298$ The instantaneous rate of change of C equals the average rate of change of C over the interval $0 \leq t \leq 4$ at time $t = 2.154$.	Answer with supporting work Point 4 (P4)

Scoring Notes for Part B

- P3** may be earned by presenting the expression or value for the average rate of change. Note that because $C(0) = 0$ and the interval is $0 \leq t \leq 4$, any of the following will earn **P3**: $\frac{\int_0^4 C'(t) dt}{4}$, $\frac{C(4) - C(0)}{4 - 0}$, $\frac{C(4)}{4}$, $\frac{5.128 - 0}{4 - 0}$, $\frac{5.128}{4}$, or 1.282. However, neither **P3** nor **P4** is earned by just presenting $t = 1.282$.
 - P4** is earned for the correct answer supported by the appropriate equation. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- The following response, for example, earns both **P3** and **P4**: $C'(t) = \frac{C(4) - C(0)}{4}$ when $t = 2.154$.

C Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.	
$\lim_{t \rightarrow \infty} C'(t) = \lim_{t \rightarrow \infty} \frac{38}{25 + t^2}$	Limit expression Point 5 (P5)
$= 0$	Value Point 6 (P6)

Scoring Notes for Part C

- P5** can be earned for either $\lim_{t \rightarrow \infty} C'(t)$ or $\lim_{t \rightarrow \infty} C(t)$.
- A response that includes $\lim_{t \rightarrow \infty} C(t)$ is not eligible to earn **P6**.
- For **P6**, arithmetic with infinity, e.g., $\frac{38}{25 + \infty^2} = 0$, will be considered as scratch work and will not be considered in scoring.

- D** At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A , defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x) \, dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t , for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.

$$A'(t) = C'(t) - 0.1 \cdot \ln t$$

Considers $A'(t) = 0$ **Point 7 (P7)**

For $4 \leq t \leq 36$, the maximum value of $A(t)$ occurs when $A'(t) = 0$ or at an endpoint.

$$A'(t) = C'(t) - 0.1 \cdot \ln t = 0 \Rightarrow C'(t) = 0.1 \cdot \ln t$$

$$\Rightarrow t = 11.441700$$

Justification **Point 8 (P8)**

t	$A(t)$
4	5.128031
11.441700	7.316978
36	1.743056

Therefore, the number of acres affected by the species is a maximum at time $t = 11.442$ (or 11.441) weeks.

Answer with supporting work **Point 9 (P9)**

Scoring Notes for Part D

- P7** is earned for considering $A'(t) = 0$, $C'(t) - 0.1 \cdot \ln t = 0$, or $C'(t) = 0.1 \cdot \ln t$. **P7** is not earned by just presenting $t = 11.441700$.
A response that discusses the sign of $A'(t)$ changing or uses the phrase “critical points of A ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by correctly evaluating $A(t)$ at $t = 4$, $t = 11.441700$, and $t = 36$. The evaluations must be correct to the first digit after the decimal, rounded or truncated.
- Alternate justifications:
 - $A'(t) > 0$ for $4 < t < 11.442$, and $A'(t) < 0$ for $11.442 < t < 36$. Therefore, $t = 11.442$ is the location of the absolute maximum for A on the interval $4 \leq t \leq 36$.
 - Because $A'(t)$ changes sign from positive to negative at $t = 11.442$ (this might be presented as “ $A'(t) > 0$ for $t < 11.442$, and $A'(t) < 0$ for $t > 11.442$ ”), it is the location of a relative maximum for A . And because $t = 11.442$ is the only critical point of A in the interval $4 \leq t \leq 36$, it is the location of the absolute maximum for A on the interval.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

	Model Solution	Scoring	
A	Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.		
	$R'(1) \approx \frac{R(2) - R(0)}{2 - 0}$ $= \frac{100 - 90}{2} = \frac{10}{2} = 5 \text{ words per minute per minute}$	Answer with setup	Point 1 (P1)
		Units	Point 2 (P2)

Scoring Notes for Part A

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
 - $\frac{100 - 90}{2 - 0}$, $\frac{10}{2 - 0}$, $\frac{100 - 90}{2}$, or $\frac{R(2) - R(0)}{2 - 0} = 5$ is sufficient to earn **P1**.
 - $\frac{R(2) - R(0)}{2 - 0}$ by itself is not sufficient to earn **P1**.
- P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- P2** is also earned for the units “words / minute² .”

B Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

R is differentiable implies R is continuous.

Differentiable
implies continuous

Point 3 (P3)

$$R(0) = 90 < 155 < R(10) = 162$$

Answer with
justification

Point 4 (P4)

Therefore, by the Intermediate Value Theorem, there must be a value c , with $0 < c < 10$, such that $R(c) = 155$.

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states “ R is continuous” without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that “ R is continuous,” and answer “yes” in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

- C Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) dt$. Show the work that leads to your answer.

$\int_0^{10} R(t) dt$ $\approx \frac{R(0) + R(2)}{2}(2 - 0) + \frac{R(2) + R(8)}{2}(8 - 2)$ $+ \frac{R(8) + R(10)}{2}(10 - 8)$	Form of trapezoidal sum	Point 5 (P5)
$= \frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ $= \frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2) = 190 + 750 + 312 = 1252$	Answer with supporting work	Point 6 (P6)

Scoring Notes for Part C

- Read “=” as “ $\approx$ ” for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
 - $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{1}{2}((R(0) + R(2))(2) + (R(2) + R(8))(6) + (R(8) + R(10))(2))$ earns **P5** and is eligible for **P6**.
 - $\frac{90 + 100}{2}(2) + \frac{100 + 150}{2}(2) + \frac{150 + 162}{2}(2)$ earns **P5** but is not eligible for **P6**.
(Note that the factor of 2 in the second term of this expression is incorrect.)
- Special case:** A response of $(90 + 100) + (100 + 150)3 + (150 + 162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.
Special case: A response of $95 \cdot 2 + 125 \cdot 6 + 156 \cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ or equivalent earns **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
- A response of $\frac{(90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2) + (100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2 = 1080$) or a completely correct right Riemann sum (e.g., $100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2 = 1424$) earns **P5** but does not earn **P6**.

- D** A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

$$\int_0^{10} W(t) \, dt = \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100 \right) dt$$

Integrand **Point 7 (P7)**

$$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t \right) \Big|_0^{10}$$

Antiderivative **Point 8 (P8)**

$$= \left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right) \\ = 1300$$

Answer **Point 9 (P9)**

Based on the model, the teacher has read 1300 words by the end of the 10 minutes.

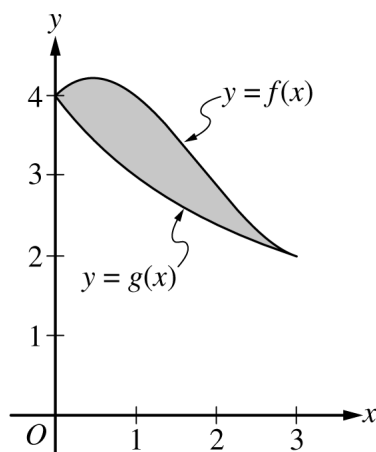
Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt .
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Part B (BC): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The graphs of the functions f and g are shown in the figure for $0 \leq x \leq 3$. It is known that $g(x) = \frac{12}{3+x}$ for $x \geq 0$. The twice-differentiable function f , which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x) \, dx = 10$.

Model Solution**Scoring**

(a) Find the area of the shaded region enclosed by the graphs of f and g .

$\text{Area} = \int_0^3 (f(x) - g(x)) \, dx = \int_0^3 f(x) \, dx - \int_0^3 g(x) \, dx$	Integrand	1 point
$= 10 - \int_0^3 \frac{12}{3+x} \, dx = 10 - 12[\ln 3+x]_0^3$	Antiderivative of $g(x)$	1 point
$= 10 - 12(\ln 6 - \ln 3) = 10 - 12(\ln 2)$	Answer	1 point

Scoring notes:

- The first point is earned for any of the integrands $f(x) - g(x)$, $g(x) - f(x)$, $|f(x) - g(x)|$, or $|g(x) - f(x)|$ in any definite integral. If the limits are incorrect, the response does not earn the third point.

- The first point is earned with an implied integrand for f and explicit integrand for g , such as $10 - \int_0^3 g(x) dx$.
- The second point is earned for finding $a \int \frac{dx}{3+x} = a \cdot \ln|3+x|$ or $a \cdot \ln(3+x)$.
- A response is eligible for the third point only if it has earned the first 2 points. The third point is earned only for the correct answer. The answer does not need to be simplified; however, if simplification is attempted, it must be correct.
- A response is not eligible for the third point with incorrect limits of integration for u -substitution, for example, $\int_0^3 \frac{12}{3+x} dx = \int_0^3 \frac{12}{u} du = 12[\ln(x+3)]_0^3$.
- A response with incorrect communication, such as “Area = $\int_0^3 (g(x) - f(x)) dx = 10 - 12(\ln 2)$,” does not earn the third point. However, a response of “ $\int_0^3 (g(x) - f(x)) dx = 12(\ln 2) - 10$, so the area is $10 - 12(\ln 2)$ ” earns all 3 points.

Total for part (a) 3 points

- (b) Evaluate the improper integral $\int_0^\infty (g(x))^2 dx$, or show that the integral diverges.

$\int_0^\infty (g(x))^2 dx = \lim_{b \rightarrow \infty} \int_0^b \frac{144}{(3+x)^2} dx$	Limit notation	1 point
$= \lim_{b \rightarrow \infty} \left(-\frac{144}{(3+x)} \Big _0^b \right)$	Antiderivative	1 point
$= \lim_{b \rightarrow \infty} \left(-\frac{144}{3+b} + \frac{144}{3} \right) = 48$	Answer	1 point

Scoring notes:

- To earn the first point a response must correctly use limit notation throughout the problem and not include arithmetic with infinity, for example, $\left[-\frac{144}{3+x} \right]_0^\infty$ or $-\frac{144}{3+\infty} + 48$.
- The second point can be earned by finding an antiderivative of the form $-\frac{a}{(3+x)}$ for $a > 0$, from an indefinite or improper integral, with or without correct limit notation. If $a \neq 144$, the response does not earn the third point.
- The third point is earned only for an answer of 48 (or equivalent).
- A response is not eligible for the third point with incorrect limits of integration for u -substitution, for example, $\lim_{b \rightarrow \infty} \int_0^b \frac{144}{u^2} du = \lim_{b \rightarrow \infty} \left[-\frac{144}{3+x} \right]_0^b$.

Total for part (b) 3 points

- (c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x) \, dx$.

Using integration by parts, $u = x \quad dv = f'(x) \, dx$ $du = dx \quad v = f(x)$	u and dv	1 point
$\int h(x) \, dx = \int x \cdot f'(x) \, dx = x \cdot f(x) - \int f(x) \, dx$	$\int h(x) \, dx$ $= x \cdot f(x) - \int f(x) \, dx$	1 point
$\int_0^3 h(x) \, dx = \int_0^3 x \cdot f'(x) \, dx = x \cdot f(x) \Big _0^3 - \int_0^3 f(x) \, dx$ $= (3 \cdot f(3) - 0 \cdot f(0)) - 10 = 3 \cdot 2 - 0 - 10 = -4$	Answer	1 point

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $x \cdot f(x) - \int f(x) \, dx$ or $x \cdot f(x) \Big|_0^3 - 10$.
- Limits of integration may be present, omitted, or partially present in the work for the first and second points.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by having columns (labeled or unlabeled) that begin with x and $f'(x)$. The second point is earned for $x \cdot f(x) - \int f(x) \, dx$.
- The third point is earned only for the correct answer and can only be earned if the first 2 points were earned.

Total for part (c) 3 points

Total for question 5 9 points