Part B (BC): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The Taylor series for a function f about $x = 4$ is given by

$$\sum_{n=1}^{\infty}\frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2\cdot 3} + \frac{(x-4)^3}{3\cdot 3^2} + \frac{(x-4)^4}{4\cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$$

and converges to $f(x)$ on its interval of convergence.

	Model Solution	Scoring	
A	Using the ratio test, find the interval of convergence of the Taylor series for f about $x = 4$. Justify your answer.		
	$\left\lvert \frac{(x-4)^{n+2}}{(n+2)3^{n+1}} \cdot \frac{(n+1)3^n}{(x-4)^{n+1}} \right\rvert = \left\lvert \frac{(x-4)(n+1)}{3(n+2)} \right\rvert$	Sets up ratio	**Point 1 (P1)**
	$\lim_{n\to\infty}\left\lvert \frac{(x-4)(n+1)}{3(n+2)} \right\rvert = \left\lvert \frac{x-4}{3} \right\rvert \lim_{n\to\infty}\frac{n+1}{n+2} = \left\lvert \frac{x-4}{3} \right\rvert$	Limit of ratio	**Point 2 (P2)**
	$\left\lvert \frac{x-4}{3} \right\rvert < 1$ when $-3 < x - 4 < 3$. Thus, the series converges when $1 < x < 7$.	Interior of interval of convergence	**Point 3 (P3)**
	When $x = 1$, the series is $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 3}{n+1}$, which converges by the alternating series test.	Considers both endpoints	**Point 4 (P4)**
	When $x = 7$, the series is $\sum_{n=1}^{\infty}\frac{3}{n+1}$, which diverges by limit comparison to the harmonic series. Therefore, the series converges for $1 \le x < 7$.	Analysis and interval of convergence	**Point 5 (P5)**

Scoring Notes for Part A

- **P1** is earned by presenting a correct ratio with or without absolute values. Once earned, **P1** is banked (i.e., subsequent errors in simplification or evaluation do not impact scoring for **P1**). **P2** is not earned if there are any errors in simplification or evaluation of the limit.
- **P1** is earned for ratios mathematically equivalent to any of the following:

$$\frac{(x-4)^{n+2}}{(n+2)3^{n+1}}\cdot\frac{(n+1)3^n}{(x-4)^{n+1}},\ \frac{\frac{(x-4)^{n+2}}{(n+2)3^{n+1}}}{\frac{(x-4)^{n+1}}{(n+1)3^n}},\ \frac{(x-4)^{n+1}}{(n+1)3^n}\cdot\frac{n3^{n-1}}{(x-4)^n},\text{ or }\frac{\frac{(x-4)^{n+1}}{(n+1)3^n}}{\frac{(x-4)^n}{n3^{n-1}}}.$$

- **P1** is also earned for ratios mathematically equivalent to the following reciprocal ratios:

$$\frac{(x-4)^{n+1}}{(n+1)3^n}\cdot\frac{(n+2)3^{n+1}}{(x-4)^{n+2}},\ \frac{\frac{(x-4)^{n+1}}{(n+1)3^n}}{\frac{(x-4)^{n+2}}{(n+2)3^{n+1}}},\ \frac{(x-4)^n}{n3^{n-1}}\cdot\frac{(n+1)3^n}{(x-4)^{n+1}},\text{ or }\frac{\frac{(x-4)^n}{n3^{n-1}}}{\frac{(x-4)^{n+1}}{(n+1)3^n}}.$$

  A response that presents any of these reciprocal ratios earns **P1** and is eligible for **P2**, **P3**, and **P4**, but not **P5**.
- A response that does not present a ratio is not eligible for **P2**.
- A response that does not use the absolute value of a ratio can earn both **P1** and **P2**.
- A response that does not use absolute value in computing the limit is eligible for **P3** if the expression $-1 < \frac{x-4}{3} < 1$ or an equivalent inequality is presented.
- A response that presents an incorrect limit of form $\frac{|x-4|}{b}$, where $b > 0$, is eligible for **P3**.

  P3 is then earned for correctly finding an interval with interior $(1, 7)$ or $(4-b, 4+b)$.

  Note: $|x-4| < b$ is not sufficient to earn **P3**. $x-4 < b$ can earn **P3** if this is resolved to $-b+4 < x < b+4$.
- **P4** is earned for considering both endpoints of the correct interval or both endpoints of an incorrect interval that has earned **P3**.
- To earn **P5**, a response must correctly analyze the series at $x = 1$ and $x = 7$, and present the correct interval of convergence. Naming of an appropriate test is sufficient for the analysis at each endpoint. In addition to the tests listed in the model solution, the direct comparison test to an appropriate series or the integral test may also be used.

B Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 4$.

$f'(x) = \dfrac{(x-4)}{3} + \dfrac{(x-4)^2}{9} + \dfrac{(x-4)^3}{27} + \cdots + \dfrac{(x-4)^n}{3^n} + \cdots$	First three terms	**Point 6 (P6)**
	General term	**Point 7 (P7)**

Scoring Notes for Part B

- **P6** is earned by presenting the first three nonzero terms in a list or as part of a polynomial or series.
- **P7** is earned by identifying the correct general term (either individually or as part of a polynomial or series).

C The Taylor series for f' described in part B is a geometric series. For all x in the interval of convergence of the Taylor series for f', show that $f'(x) = \dfrac{x-4}{7-x}$.

The Taylor series for f' is a geometric series with first term $\dfrac{x-4}{3}$ and common ratio $\dfrac{x-4}{3}$.

$$f'(x) = \frac{\frac{x-4}{3}}{1 - \frac{x-4}{3}} = \frac{x-4}{3-(x-4)} = \frac{x-4}{7-x}$$

Verification **Point 8 (P8)**

Scoring Notes for Part C

- A response of $f'(x) = \dfrac{\frac{x-4}{3}}{1 - \frac{x-4}{3}}$ is sufficient to earn **P8**.

D	It is known that the radius of convergence of the Taylor series for f about $x = 4$ is the same as the radius of convergence of the Taylor series for f' about $x = 4$. Does the Taylor series for f' described in part B converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.		
	It follows from the work in part A that the interior of the interval of convergence of the Taylor series for f' is $1 < x < 7$. Therefore, $x = 8$ would be outside the interval of convergence of f', and the Taylor series for f' would not converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$.	Answer with reason	**Point 9 (P9)**

Scoring Notes for Part D

- A response of "no, $x = 8$ is outside the interval of convergence" is sufficient to earn **P9**.
- **P9** can be earned with a response consistent with an incorrect interval of convergence imported from part A.
- Alternate solutions:
  - Because the series for $f'(x)$ is geometric, this converges to $f(x)$ for all values of x such that the common ratio $\frac{x-4}{3}$ is between -1 and 1.

    $-1 < \frac{x-4}{3} < 1 \Rightarrow -3 < x - 4 < 3 \Rightarrow 1 < x < 7$
  - Because $x = 8$ is outside the interval $1 < x < 7$, the series for f' does not converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$.
  - For $x = 8$, the series is given by $\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$, which is a geometric series with $r = \frac{4}{3} > 1$. Therefore, the series diverges for $x = 8$.

Part B (BC): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

The function g has derivatives of all orders for all real numbers. The Maclaurin series for g is given by $g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3}$ on its interval of convergence.

(a) State the conditions necessary to use the integral test to determine convergence of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$. Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.

Model Solution	Scoring	
e^{-x} is positive, decreasing, and continuous on the interval $[0, \infty)$.	Conditions	**1 point**
To use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, show that $\int_0^{\infty} e^{-x}\,dx$ is finite (converges).	Improper integral	**1 point**
$\int_0^{\infty} e^{-x}\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\,dx = \lim_{b\to\infty} \left(-e^{-x}\Big\|_0^b\right) = \lim_{b\to\infty} \left(-e^{-b} + e^0\right) = 1$ Because the integral $\int_0^{\infty} e^{-x}\,dx$ converges, the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.	Evaluation	**1 point**

Scoring notes:

- To earn the first point a response must list all three conditions: e^{-x} is positive, decreasing, and continuous.
- The second point is earned for correctly writing the improper integral or for presenting a correct limit equivalent to the improper integral (for example, $\lim_{b\to\infty} \int_0^b e^{-x}\, dx$).
- To earn the third point a response must correctly use limit notation to evaluate the improper integral, find an evaluation of e^0 (or 1), and conclude that the integral converges or that the series converges.
- If an incorrect lower limit of 1 is used in the improper integral, then the second point is not earned. In this case, if the correct limit $(1/e)$ is presented, then the response is eligible for the third point.
- If the response only relies on using a geometric series approach, then no points are earned $[0\text{-}0\text{-}0]$.
- A response that presents an evaluation with ∞, such as $e^{-\infty} = 0$, does not earn the third point.

Total for part (a) 3 points

(b) Use the limit comparison test with the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ to show that the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

$$\lim_{n\to\infty} \frac{\frac{1}{e^n}}{\left\lvert \frac{(-1)^n}{2e^n + 3} \right\rvert} = \lim_{n\to\infty} \frac{2e^n + 3}{e^n} = 2$$	Sets up limit comparison	**1 point**
The limit exists and is positive. Therefore, because the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, the series $\sum_{n=0}^{\infty} \left\lvert \frac{(-1)^n}{2e^n + 3} \right\rvert$ converges by the limit comparison test. Thus, the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.	Explanation	**1 point**

Scoring notes:

- The first point is earned for setting up the limit comparison, with or without absolute values. Limit notation is required to earn this point.
- The reciprocal of the given ratio is an acceptable alternative; the limit in this case is $1/2$.
- The second point cannot be earned without the use of absolute value symbols, which can occur explicitly or implicitly (e.g., a response might set up the limit comparison initially as $\lim_{n\to\infty} \frac{\frac{1}{e^n}}{\frac{1}{2e^n + 3}}$).
- Earning the second point requires correctly evaluating the limit and noting that the limit is a positive number. For example, $L = 2 > 0$ or $L = 1/2 > 0$. Therefore, comparing the limit L to 1 does not earn the explanation point.
- A response does not have to repeat that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.
- A response that draws a conclusion based only on the sequence (such as $\frac{1}{e^n}$) without referencing a series does not earn the second point.
- If the response does not explicitly use the limit comparison test, then no points are earned in this part.
- A response cannot earn the second point for just concluding that "the series" converges absolutely because there are multiple series in this part of the problem. The response must specify that the series $g(1)$ or $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

Total for part (b) 2 points

(c) Determine the radius of convergence of the Maclaurin series for g.

$\left\lvert \dfrac{(-1)^{n+1} x^{n+1}}{2e^{n+1}+3} \cdot \dfrac{2e^n+3}{(-1)^n x^n} \right\rvert = \left\lvert \dfrac{\left(2e^n+3\right) x^{n+1}}{\left(2e^{n+1}+3\right) x^n} \right\rvert = \dfrac{2e^n+3}{2e^{n+1}+3}\lvert x \rvert$	Sets up ratio	**1 point**
$\lim\limits_{n\to\infty} \dfrac{2e^n+3}{2e^{n+1}+3}\lvert x \rvert = \dfrac{1}{e}\lvert x \rvert$	Computes limit of ratio	**1 point**
$\dfrac{1}{e}\lvert x \rvert < 1 \Rightarrow \lvert x \rvert < e$ The radius of convergence is $R = e.$	Answer	**1 point**

Scoring notes:

- The first point is earned for $\dfrac{(-1)^{n+1} x^{n+1}}{2e^{n+1}+3} \cdot \dfrac{2e^n+3}{(-1)^n x^n}$ or the equivalent. Once earned, this point cannot be lost.
- The second point cannot be earned without the first point.
- To be eligible for the third point the response must have found a limit for a presented ratio such that the limiting value of the coefficient on $\lvert x \rvert$ is finite and not $0.$ The third point is earned for setting up an inequality such that the limit is less than $1,$ solving for $\lvert x \rvert,$ and interpreting the result to find the radius of convergence.
- The radius of convergence must be explicitly presented, for example, $R = e.$ The third point cannot be earned by presenting an interval, for example $-e < x < e,$ with no identification of the radius of convergence.

Total for part (c) 3 points

(d) The first two terms of the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ are used to approximate $g(1)$. Use the alternating series error bound to determine an upper bound on the error of the approximation.

The terms of the alternating series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ decrease in magnitude to 0. The alternating series error bound for the error of the approximation is the absolute value of the third term of the series. $\text{Error} \le \left\lvert \frac{(-1)^2}{2e^2 + 3} \right\rvert = \frac{1}{2e^2 + 3}$	Answer	**1 point**

Scoring notes:

- A response of $\frac{1}{2e^2 + 3}$ earns this point.

Total for part (d) **1 point**

Total for question 6 **9 points**

2017

AP® CollegeBoard

AP Calculus BC

Scoring Guidelines

2017 SCORING GUIDELINES

Question 1

1 : units in parts (a), (c), and (d)

(a) Volume $= \int_0^{10} A(h)\, dh$

$$\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$$
$$= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$$
$$= 176.3 \text{ cubic feet}$$

$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.

1 : overestimate with reason

(c) $\int_0^{10} f(h)\, dh = 101.325338$

The volume is 101.325 cubic feet.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Using the model, $V(h) = \int_0^h f(x)\, dx$.

$$\left.\frac{dV}{dt}\right|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= f(5)\cdot 0.26 = 1.694419$$

When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.

$3 : \begin{cases} 2 : \dfrac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2017 SCORING GUIDELINES

Question 2

(a) $\frac{1}{2}\int_0^{\pi/2}(f(\theta))^2\,d\theta = 0.648414$

The area of R is 0.648.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\int_0^k\left((g(\theta))^2 - (f(\theta))^2\right)d\theta = \frac{1}{2}\int_0^{\pi/2}\left((g(\theta))^2 - (f(\theta))^2\right)d\theta$

— OR —

$\int_0^k\left((g(\theta))^2 - (f(\theta))^2\right)d\theta = \int_k^{\pi/2}\left((g(\theta))^2 - (f(\theta))^2\right)d\theta$

$$2 : \begin{cases} 1 : \text{integral expression} \\ \quad\;\; \text{for one region} \\ 1 : \text{equation} \end{cases}$$

(c) $w(\theta) = g(\theta) - f(\theta)$

$$w_A = \frac{\int_0^{\pi/2} w(\theta)\,d\theta}{\frac{\pi}{2} - 0} = 0.485446$$

The average value of $w(\theta)$ on the interval $\left[0, \frac{\pi}{2}\right]$ is 0.485.

$$3 : \begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$$

(d) $w(\theta) = w_A$ for $0 \le \theta \le \frac{\pi}{2} \Rightarrow \theta = 0.517688$

$w(\theta) = w_A$ at $\theta = 0.518$ (or 0.517).

$w'(0.518) < 0 \Rightarrow w(\theta)$ is decreasing at $\theta = 0.518$.

$$2 : \begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$$

2017 SCORING GUIDELINES

Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

$$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$$

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

$$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$$

(d) $f''(-5) = \dfrac{2 - 0}{-6 - (-2)} = -\dfrac{1}{2}$

$$\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because
$$\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

$$2 : \begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \quad\ \text{with explanation} \end{cases}$$

2017 SCORING GUIDELINES

Question 4

(a) $H'(0) = -\frac{1}{4}(91 - 27) = -16$

$H(0) = 91$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

$$3 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$$

(b) $\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$H > 27$ for $t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0$ for $t > 0$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

1 : underestimate with reason

(c) $\frac{dG}{(G - 27)^{2/3}} = -dt$

$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1)\, dt$

$3(G - 27)^{1/3} = -t + C$

$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$

$3(G - 27)^{1/3} = 12 - t$

$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3$ for $0 \le t < 10$

The internal temperature of the potato at time $t = 3$ minutes is $27 + \left(\frac{12 - 3}{3}\right)^3 = 54$ degrees Celsius.

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad\ \text{uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{cases}$$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

2017 SCORING GUIDELINES

Question 5

(a) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2}$

$f'(3) = \dfrac{(-3)(5)}{(18-21+5)^2} = -\dfrac{15}{4}$

2 : $f'(3)$

(b) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2} = 0 \Rightarrow x = \dfrac{7}{4}$

The only critical point in the interval $1 < x < 2.5$ has x-coordinate $\dfrac{7}{4}$.

f' changes sign from positive to negative at $x = \dfrac{7}{4}$.

Therefore, f has a relative maximum at $x = \dfrac{7}{4}$.

2 : $\begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \quad\ \text{with justification} \end{cases}$

(c) $\displaystyle\int_5^\infty f(x)\,dx = \lim_{b\to\infty}\int_5^b \frac{3}{2x^2-7x+5}\,dx = \lim_{b\to\infty}\int_5^b \left(\frac{2}{2x-5} - \frac{1}{x-1}\right) dx$

$\displaystyle = \lim_{b\to\infty}\Big[\ln(2x-5) - \ln(x-1)\Big]_5^b = \lim_{b\to\infty}\left[\ln\left(\frac{2x-5}{x-1}\right)\right]_5^b$

$\displaystyle = \lim_{b\to\infty}\left[\ln\left(\frac{2b-5}{b-1}\right) - \ln\left(\frac{5}{4}\right)\right] = \ln 2 - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\displaystyle\int_5^\infty \frac{3}{2x^2-7x+5}\,dx$ converges.

— OR —

$\dfrac{3}{2n^2-7n+5} > 0$ and $\dfrac{1}{n^2} > 0$ for $n \ge 5$.

Since $\displaystyle\lim_{n\to\infty}\frac{\frac{3}{2n^2-7n+5}}{\frac{1}{n^2}} = \frac{3}{2}$ and the series $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n^2}$ converges,

the series $\displaystyle\sum_{n=5}^{\infty}\frac{3}{2n^2-7n+5}$ converges by the limit comparison test.

2 : answer with conditions

2017 SCORING GUIDELINES

Question 6

(a) $f(0) = 0$

$f'(0) = 1$

$f''(0) = -1(1) = -1$

$f'''(0) = -2(-1) = 2$

$f^{(4)}(0) = -3(2) = -6$

The first four nonzero terms are

$$0 + 1x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}.$$

The general term is $\dfrac{(-1)^{n+1}x^n}{n}$.

$$3 : \begin{cases} 1 : f''(0),\ f'''(0), \text{ and } f^{(4)}(0) \\ 1 : \text{verify terms} \\ 1 : \text{general term} \end{cases}$$

(b) For $x = 1$, the Maclaurin series becomes $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

The series does not converge absolutely because the harmonic series diverges.

The series alternates with terms that decrease in magnitude to 0, and therefore the series converges conditionally.

2 : converges conditionally with reason

(c)
$$\int_0^x f(t)\,dt = \int_0^x \left(t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots + \frac{(-1)^{n+1}t^n}{n} + \cdots\right) dt$$
$$= \left[\frac{t^2}{2} - \frac{t^3}{3\cdot 2} + \frac{t^4}{4\cdot 3} - \frac{t^5}{5\cdot 4} + \cdots + \frac{(-1)^{n+1}t^{n+1}}{(n+1)n} + \cdots\right]_{t=0}^{t=x}$$
$$= \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \cdots + \frac{(-1)^{n+1}x^{n+1}}{(n+1)n} + \cdots$$

$$3 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$$

(d) The terms alternate in sign and decrease in magnitude to 0. By the alternating series error bound, the error $\left|P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right)\right|$ is bounded by the magnitude of the first unused term, $\left|-\dfrac{(1/2)^5}{20}\right|$.

Thus, $\left|P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right)\right| \le \left|-\dfrac{(1/2)^5}{20}\right| = \dfrac{1}{32\cdot 20} < \dfrac{1}{500}$.

1 : error bound