AP Calculus BC

Revision Sheet · 考前速记

Learn these by heart before the exam. 考试前把这些内容背熟。

BC-only integration techniques

$\int u\,dv = uv - \int v\,du$ choose u by LIATE: log, inverse trig, algebraic, trig, exponential 分部积分

$\dfrac{P(x)}{(x-a)(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{x-b}$ only for NON-repeated linear factors on the BC exam 部分分式

$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$ write the limit explicitly; the setup earns the mark 反常积分

$\int_1^\infty \dfrac{1}{x^p}dx$ converges $\iff p > 1$ p-积分判别

Differential equations (BC)

$\dfrac{dy}{dt} = ky\left(1 - \dfrac{y}{L}\right), \quad y = \dfrac{L}{1 + Ae^{-kt}}$ fastest growth at $y = L/2$; $y \to L$ as $t \to \infty$ 逻辑斯蒂增长

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$ under-estimates when the solution is concave up 欧拉法

Parametric, polar, and vectors

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}, \quad \dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)\Big/\dfrac{dx}{dt}$ the second derivative divides by dx/dt again, NOT by d^2x/dt^2 参数方程求导

$L = \int_a^b \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$ 参数曲线弧长

$\vec{v}(t) = \langle x'(t), y'(t)\rangle, \quad \text{speed} = |\vec{v}| = \sqrt{x'^2 + y'^2}$ 向量值函数的速度与速率

$\text{distance} = \int_a^b |\vec{v}(t)|\,dt$ 路程

$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2$ 极坐标与直角坐标

$A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$ for the area BETWEEN two curves use $\frac{1}{2}\int(r_{out}^2 - r_{in}^2)\,d\theta$ 极坐标面积

Sequences and series - convergence tests

$\sum_{n=0}^\infty ar^n = \dfrac{a}{1-r}, \quad |r| < 1$ 几何级数

$\sum \dfrac{1}{n^p}$ converges $\iff p > 1$ $p = 1$ is the harmonic series: it DIVERGES p-级数

nth-term test: if $a_n \not\to 0$ the series diverges. If $a_n \to 0$ it proves NOTHING.

Ratio test: $L = \lim|a_{n+1}/a_n|$. $L < 1$ converges, $L > 1$ diverges, $L = 1$ inconclusive.

Alternating series: converges if terms decrease in size AND tend to 0. Error is smaller than the first omitted term.

Absolute convergence implies convergence. Conditional means the series converges but the absolute one does not.

Comparison and limit comparison: compare with a p-series or geometric series you already know.

Power series and Taylor

$f(x) = \sum_{n=0}^\infty \dfrac{f^{(n)}(a)}{n!}(x-a)^n$ Maclaurin is the case $a = 0$ 泰勒级数

$e^x = \sum \dfrac{x^n}{n!}, \quad \sin x = \sum \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum \dfrac{(-1)^n x^{2n}}{(2n)!}$ 必背的麦克劳林级数

$\dfrac{1}{1-x} = \sum_{n=0}^\infty x^n, \quad |x| < 1$ differentiate, integrate or substitute into this to build new series fast 几何级数形式

$|R_n(x)| \le \dfrac{\max|f^{(n+1)}(z)|}{(n+1)!}|x-a|^{n+1}$ 拉格朗日余项

Radius of convergence: apply the ratio test and solve $L < 1$. Then TEST BOTH ENDPOINTS separately.

Exam technique

Name the test before using it: 'by the ratio test' or 'by the alternating series error bound'. Unnamed work loses marks.

For series, show the general term and the limit you took. A bare 'converges' earns nothing.

BC still examines all of AB. Revise limits, FTC, and volumes alongside this sheet.