# AP Calculus BC

**Revision Sheet · 考前速记**

*Learn these by heart before the exam.* 考试前把这些内容背熟。

## BC-only integration techniques

$\int u\,dv = uv - \int v\,du$ choose $u$ by LIATE: log, inverse trig, algebraic, trig, exponential 分部积分

$\dfrac{P(x)}{(x-a)(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{x-b}$ only for NON-repeated linear factors on the BC exam 部分分式

$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$ write the limit explicitly; the setup earns the mark 反常积分

$\int_1^\infty \dfrac{1}{x^p} dx$ converges $\iff p > 1$ p-积分判别

## Differential equations (BC)

$\dfrac{dy}{dt} = ky\left(1 - \dfrac{y}{L}\right), \quad y = \dfrac{L}{1 + Ae^{-kt}}$ fastest growth at $y = L/2$; $y \to L$ as $t \to \infty$ 逻辑斯蒂增长

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$ under-estimates when the solution is concave up 欧拉法

## Parametric, polar, and vectors

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}, \quad \dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \Big/ \dfrac{dx}{dt}$ the second derivative divides by $dx/dt$ again, NOT by $d^2x/dt^2$ 参数方程求导

$L = \int_a^b \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\, dt$ 参数曲线弧长

$\vec{v}(t) = \langle x'(t), y'(t) \rangle, \quad \text{speed} = |\vec{v}| = \sqrt{x'^2 + y'^2}$ 向量值函数的速度与速率

$\text{distance} = \int_a^b |\vec{v}(t)|\, dt$ 路程

$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2$ 极坐标与直角坐标

$A = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$ for the area BETWEEN two curves use $\frac{1}{2}\int (r_{out}^2 - r_{in}^2)\, d\theta$ 极坐标面积

## Sequences and series - convergence tests

$\sum_{n=0}^\infty ar^n = \dfrac{a}{1-r}, \quad |r| < 1$ 几何级数

$\sum \dfrac{1}{n^p}$ converges $\iff p > 1$ $p = 1$ is the harmonic series: it DIVERGES p-级数

*nth-term test: if* $a_n \not\to 0$ *the series diverges. If* $a_n \to 0$ *it proves NOTHING.*

*Ratio test:* $L = \lim |a_{n+1}/a_n|$. $L < 1$ *converges,* $L > 1$ *diverges,* $L = 1$ *inconclusive.*

*Alternating series: converges if terms decrease in size AND tend to 0. Error is smaller than the first omitted term.*

*Absolute convergence implies convergence. Conditional means the series converges but the absolute one does not.*

*Comparison and limit comparison: compare with a p-series or geometric series you already know.*

## Power series and Taylor

$f(x) = \sum_{n=0}^\infty \dfrac{f^{(n)}(a)}{n!}(x-a)^n$ Maclaurin is the case $a = 0$ 泰勒级数

$e^x = \sum \dfrac{x^n}{n!}, \quad \sin x = \sum \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum \dfrac{(-1)^n x^{2n}}{(2n)!}$ 必背的麦克劳林级数

$\dfrac{1}{1-x} = \sum_{n=0}^\infty x^n, \quad |x| < 1$ differentiate, integrate or substitute into this to build new series fast 几何级数形式

$|R_n(x)| \le \dfrac{\max|f^{(n+1)}(z)|}{(n+1)!}|x-a|^{n+1}$ 拉格朗日余项

*Radius of convergence: apply the ratio test and solve* $L < 1$. *Then TEST BOTH ENDPOINTS separately.*

## Exam technique

*Name the test before using it: 'by the ratio test' or 'by the alternating series error bound'. Unnamed work loses marks.*

*For series, show the general term and the limit you took. A bare 'converges' earns nothing.*

*BC still examines all of AB. Revise limits, FTC, and volumes alongside this sheet.*