AP Calculus BC

Formula Sheet · 公式表

No formula sheet is supplied. BC assumes the whole of AB from memory and adds four areas that carry their own recall load: series with their convergence tests, parametric and polar calculus, the extra integration techniques, and the error bounds. This booklet covers the BC-only material and the conditions each test needs. 无公式表; 本册专列 BC 独有内容及各判别法的适用条件。

Integration techniques beyond AB

Integration by parts 分部积分 — $\int u\,dv = uv - \int v\,du$. Choose u so that differentiating it simplifies. For $\int x^2 e^x dx$ you apply it twice; for $\int e^x \sin x\,dx$ you apply it twice and solve for the original integral.

Partial fractions 部分分式 — Only for a proper rational function with distinct linear factors on this exam. Split, then each piece integrates to a logarithm.

Improper integrals 反常积分 — Rewrite the infinite limit as a limit of a definite integral and evaluate it. Writing the limit notation is itself a scored step; converging or diverging must then be stated.

Arc length 弧长 — $L = \int_a^b \sqrt{1 + (dy/dx)^2}\,dx$, and for a parametric curve $L = \int \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$.

Logistic growth 逻辑斯蒂增长 — $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ with carrying capacity L. Growth is fastest at $P = L/2$, and $P \to L$ as $t \to \infty$ – both are asked in words far more often than the solution is asked for.

Euler's method 欧拉方法 — Step forward with $y_{n+1} = y_n + h\,f(x_n, y_n)$. Show the table of steps; the arithmetic alone earns little without it.

Parametric, vector and polar

Parametric derivatives 参数方程导数 — $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, and the second derivative is $\frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$ – not the ratio of the second derivatives, which is the standard trap.

Vector-valued motion 向量运动 — Position $\langle x(t), y(t)\rangle$; velocity is the derivative componentwise; SPEED is $\sqrt{(dx/dt)^2 + (dy/dt)^2}$, a scalar. Total distance is the integral of the speed.

Polar area 极坐标面积 — $A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$. Finding the correct limits – where the curve traces the region exactly once – carries more of the credit than the integral does.

Series – the tests and their conditions

Geometric 几何级数 — Converges exactly when $|r| < 1$, to $\frac{a}{1-r}$. The only common series whose sum you can state.

p-series p 级数 — $\sum 1/n^p$ converges when $p > 1$. The harmonic series $p = 1$ diverges, and knowing that decides many comparison questions immediately.

nth-term test 通项判别法 — If the terms do not tend to zero, the series diverges. It can NEVER prove convergence, and claiming it does is a lost point.

Ratio test 比值判别法 — Compute $\lim |a_{n+1}/a_n|$: below 1 converges, above 1 diverges, exactly 1 is inconclusive and you must switch tests. This is the standard tool for a radius of convergence.

Comparison and limit comparison 比较判别法 — Both require positive terms – say so. Name the series you are comparing with and why it converges or diverges.

Alternating series 交错级数 — Converges when the terms decrease in absolute value and tend to zero; both conditions must be stated. The error is then smaller than the first omitted term.

Absolute versus conditional 绝对与条件收敛 — If $\sum |a_n|$ converges the series converges absolutely; if the series converges but $\sum |a_n|$ does not, it converges conditionally. The exam asks which, so answer with the word.

Taylor and Maclaurin

The general form 泰勒级数 — $\sum \frac{f^{(n)}(a)}{n!}(x-a)^n$. Building this from a table of derivative values is a recurring free-response part.

The four to memorise 必背四个展开 — $e^x = \sum \frac{x^n}{n!}$; $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$; $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$; $\frac{1}{1-x} = \sum x^n$ for $|x| < 1$. Every other series in the exam is built from these by substitution, differentiation or integration.

Interval of convergence 收敛区间 — Use the ratio test for the radius, then TEST BOTH ENDPOINTS separately. The endpoints are where the marks are, and they are the step most often skipped.

Lagrange error bound 拉格朗日余项 — $|R_n(x)| \le \frac{\max|f^{(n+1)}|}{(n+1)!}|x-a|^{n+1}$. State what bound you are using for the maximum derivative and why it is valid on the interval.

How BC free-response is scored

Name the test before applying it, and state its conditions. "By the alternating series test, since the terms decrease and tend to zero" is two scored statements; "it converges" is none.

For series questions, show the general term. A pattern written out as four terms with dots is accepted only when a general term accompanies it.

Set up every integral in full even when the calculator evaluates it, with correct limits and the variable of integration, then give the value to three decimal places.