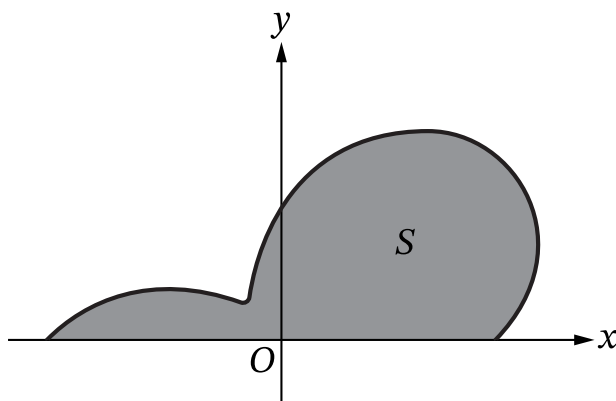


2. Let S be the shaded region bounded by the graph of the polar curve $r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $0 \leq \theta \leq \pi$, as shown in the figure.



It can be shown that $r'(\theta) = 4 \cos(2\theta) - 2 \sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

Part A

Find the area of region S . Show the setup for your calculations.

Part B

There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

Part C

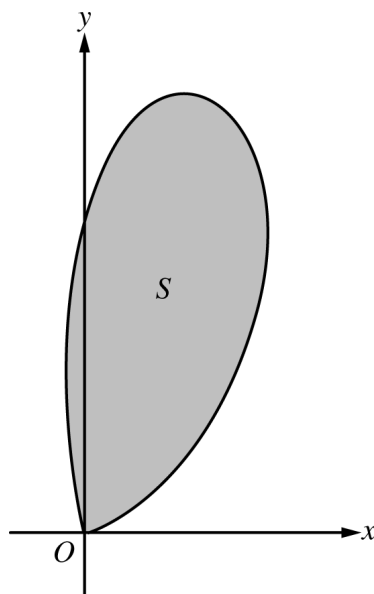
- Find the value of θ in the interval $0 < \theta < \frac{\pi}{2}$ at which r has a critical point.
- Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for r .

Part D

Find the average distance from the origin to a point on the polar curve

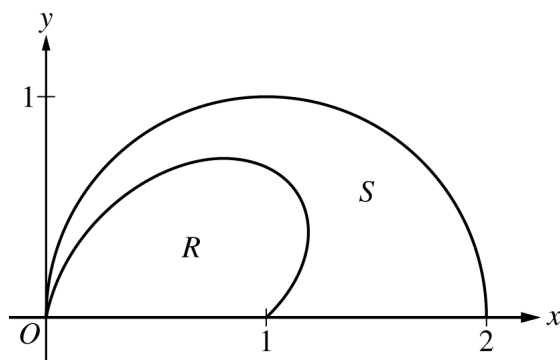
$r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $\frac{\pi}{2} \leq \theta \leq \pi$. Show the setup for your calculations.

END OF PART A



2. Let S be the region bounded by the graph of the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$, as shown in the figure above.
- (a) Find the area of S .
- (b) What is the average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$?
- (c) There is a line through the origin with positive slope m that divides the region S into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of m .
- (d) For $k > 0$, let $A(k)$ be the area of the portion of region S that is also inside the circle $r = k \cos \theta$. Find $\lim_{k \rightarrow \infty} A(k)$.
-

END OF PART A OF SECTION II



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin \theta \cos(2\theta)$ and $r = g(\theta) = 2 \cos \theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x -axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x -axis.
- (a) Find the area of R .
- (b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .
- (c) For each θ , $0 \leq \theta \leq \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A , the average value of $w(\theta)$ over the interval $0 \leq \theta \leq \frac{\pi}{2}$.
- (d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

END OF PART A OF SECTION II