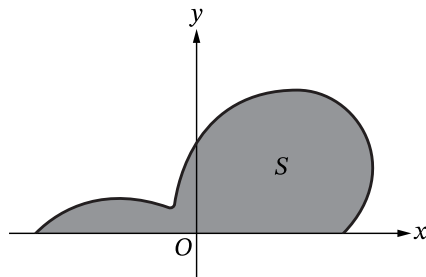


2. Let S be the shaded region bounded by the graph of the polar curve $r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $0 \leq \theta \leq \pi$, as shown in the figure.



It can be shown that $r'(\theta) = 4 \cos(2\theta) - 2 \sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

Part A

Find the area of region S . Show the setup for your calculations.

Part B

There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

Part C

- Find the value of θ in the interval $0 < \theta < \frac{\pi}{2}$ at which r has a critical point.
- Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for r .

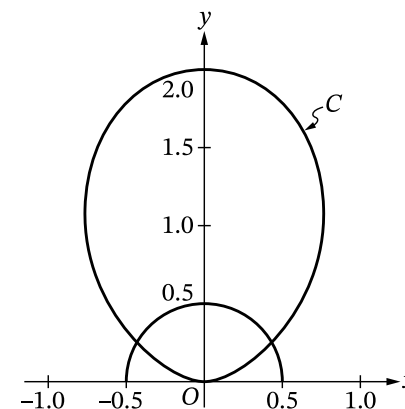
Part D

Find the average distance from the origin to a point on the polar curve

$r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $\frac{\pi}{2} \leq \theta \leq \pi$. Show the setup for your calculations.

END OF PART A

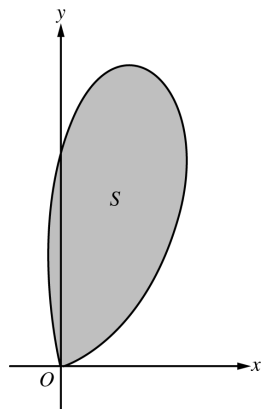
2. Curve C is defined by the polar equation $r(\theta) = 2 \sin^2 \theta$ for $0 \leq \theta \leq \pi$. Curve C and the semicircle $r = \frac{1}{2}$ for $0 \leq \theta \leq \pi$ are shown in the xy -plane.



(Note: Your calculator should be in radian mode.)

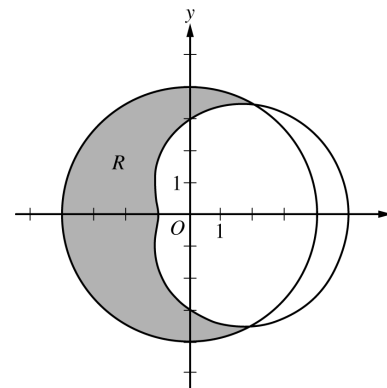
- Find the rate of change of r with respect to θ at the point on curve C where $\theta = 1.3$. Show the setup for your calculations.
- Find the area of the region that lies inside curve C but outside the graph of the polar equation $r = \frac{1}{2}$. Show the setup for your calculations.
- It can be shown that $\frac{dx}{d\theta} = 4 \sin \theta \cos^2 \theta - 2 \sin^3 \theta$ for curve C . For $0 \leq \theta \leq \frac{\pi}{2}$, find the value of θ that corresponds to the point on curve C that is farthest from the y -axis. Justify your answer.
- A particle travels along curve C so that $\frac{d\theta}{dt} = 15$ for all times t . Find the rate at which the particle's distance from the origin changes with respect to time when the particle is at the point where $\theta = 1.3$. Show the setup for your calculations.

END OF PART A

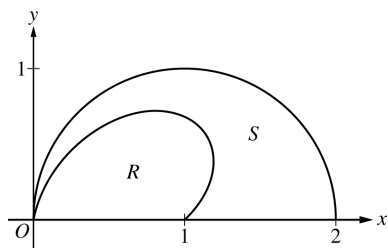


2. Let S be the region bounded by the graph of the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$, as shown in the figure above.
- (a) Find the area of S .
- (b) What is the average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$?
- (c) There is a line through the origin with positive slope m that divides the region S into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of m .
- (d) For $k > 0$, let $A(k)$ be the area of the portion of region S that is also inside the circle $r = k \cos \theta$. Find $\lim_{k \rightarrow \infty} A(k)$.

END OF PART A OF SECTION II

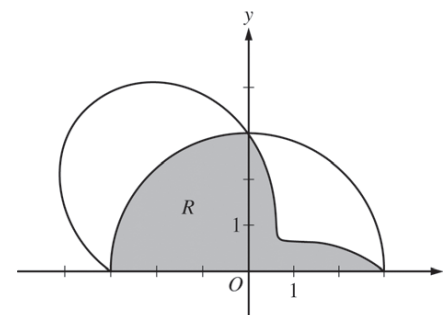


5. The graphs of the polar curves $r = 4$ and $r = 3 + 2 \cos \theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.
- (a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2 \cos \theta$, as shown in the figure above. Write an expression involving an integral for the area of R .
- (b) Find the slope of the line tangent to the graph of $r = 3 + 2 \cos \theta$ at $\theta = \frac{\pi}{2}$.
- (c) A particle moves along the portion of the curve $r = 3 + 2 \cos \theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin \theta \cos(2\theta)$ and $r = g(\theta) = 2 \cos \theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x -axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x -axis.
- Find the area of R .
 - The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .
 - For each θ , $0 \leq \theta \leq \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A , the average value of $w(\theta)$ over the interval $0 \leq \theta \leq \frac{\pi}{2}$.
 - Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

END OF PART A OF SECTION II



2. The graphs of the polar curves $r = 3$ and $r = 3 - 2 \sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.
- Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2 \sin(2\theta)$. Find the area of R .
 - For the curve $r = 3 - 2 \sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.
 - The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.
 - A particle is moving along the curve $r = 3 - 2 \sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

END OF PART A OF SECTION II