Part A (BC): Graphing calculator required
Question 2 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t seconds, where $x(t)$ and $y(t)$ are measured in centimeters. It is known that $x'(t) = 8t - t^2$ and $y'(t) = -t + \sqrt{t^{1.2} + 20}$. At time $t = 2$ seconds, the particle is at the point $(3, 6)$.

Model Solution	Scoring	
(a) Find the speed of the particle at time $t = 2$ seconds. Show the setup for your calculations.		
$\sqrt{(x'(2))^2 + (y'(2))^2}$	Setup for speed	**1 point**
$= 12.3048506$ The speed of the particle at time $t = 2$ seconds is 12.305 (or 12.304) centimeters per second.	Answer	**1 point**

Scoring notes:

- The first point is earned for the expression $\sqrt{(x'(2))^2 + (y'(2))^2}$, $\sqrt{(x'(t))^2 + (y'(t))^2}$, or equivalent.
- A response that presents just the exact answer, $\sqrt{144 + \left(-2 + \sqrt{2^{1.2} + 20}\right)^2}$, earns both points.
- The second point is earned only for the answer 12.305 (or 12.304) regardless of whether the first point is earned.
- A response that includes a linkage error, such as
$\sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{144 + \left(-2 + \sqrt{2^{1.2} + 20}\right)^2}$ or $\sqrt{(x'(t))^2 + (y'(t))^2} = 12.305$ (or 12.304),
earns at most 1 of the 2 points.
- Missing or incorrect units will not affect scoring in this part.

Total for part (a) — 2 points

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 2$. Show the setup for your calculations.

$\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$	Integral	**1 point**
$= 15.901715$ The total distance traveled by the particle over the time interval $0 \le t \le 2$ is 15.902 (or 15.901) centimeters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for an integral of $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ (or the mathematical equivalent), with or without the differential.
  - Note: $\int_0^2 \sqrt{x'(t)^2 + y'(t)^2}\, dt$ is not read as a parenthesis error.
- The second point is earned only for an answer of 15.902 (or 15.901), regardless of whether the first point is earned.
- Missing or incorrect units will not affect scoring in this part.

Total for part (b) 2 points

(c) Find the y-coordinate of the position of the particle at the time $t = 0$. Show the setup for your calculations.

$y(0) = 6 + \int_2^0 y'(t)\, dt = 6 - 7.173613 = -1.173613$	Definite integral	**1 point**
	Uses initial condition	**1 point**
The y-coordinate of the position of the particle at time $t = 0$ is -1.174 (or -1.173).	Answer	**1 point**

Scoring notes:

- An answer of -1.174 (or -1.173) with no supporting work does not earn any points.
- The first point is earned for either of the definite integrals $\int_2^0 y'(t)\, dt$ or $\int_0^2 y'(t)\, dt$.
- The second point is earned for any of:
  - $y(2) \pm \int_2^0 y'(t)\, dt,\ 6 \pm \int_2^0 y'(t)\, dt,$
  - $y(2) \pm \int_0^2 y'(t)\, dt,\ 6 \pm \int_0^2 y'(t)\, dt,$
  - $y(2) \pm 7.173613,$ or $6 \pm 7.173613.$
- A response that attempts to evaluate $\int y'(t)\, dt$ does not earn the first or the third point.
  - Such a response can earn the second point by attempting to solve for the constant of integration by presenting an expression as an antiderivative for $y'(t)$, evaluating this expression at $t = 2$, and setting this expression equal to 6.

- A response that reverses the limits of integration, e.g., $y(0) = 6 + \int_0^2 y'(t)\,dt$ or $6 + 7.173613,$ earns the second point but does not earn the third point.
- In order to earn the third point, a response must have earned at least 1 of the first 2 points.
- A response containing any linkage error can earn at most 2 of the 3 points. For example:
  - Equating two unequal quantities: $\int_2^0 y'(t)\,dt = -1.174,$ $\int_2^0 y'(t)\,dt = 6 - 7.173613,$ $6 + \int_0^2 y'(t)\,dt = 6 - 7.173613,$ or $6 + \int_0^2 y'(t)\,dt = 6 + 7.173613 = -1.173613$
  - Equating an expression to a numerical value: $y(t) = 6 + \int_2^0 y'(t)\,dt = -1.174$
- Missing differentials (dt):
  - Unambiguous responses of $y(2) + \int_2^0 y'(t),$ $y(2) - \int_0^2 y'(t),$ $6 + \int_2^0 y'(t),$ or $6 - \int_0^2 y'(t)$ earn the first 2 points and would earn the third point for the correct numerical answer.
  - Unambiguous responses of $y(2) + \int_0^2 y'(t)$ or $6 + \int_0^2 y'(t)$ with reversed limits of integration and missing differential earn the first 2 points but cannot earn the third point.
  - Ambiguous responses of $\int_2^0 y'(t) + y(2),$ $-\int_0^2 y'(t) + y(2),$ $\int_2^0 y'(t) + 6,$ or $-\int_0^2 y'(t) + 6$ earn the first point, do not earn the second point, but do earn the third point if a correct numeric answer is provided. If no numeric answer is given, none of these responses earn the third point.
  - Ambiguous responses of $\int_0^2 y'(t) + y(2)$ or $\int_0^2 y'(t) + 6$ with reversed limits of integration and no differential earn 1 out of 3 points.
- If a response provides work for both the x- and y-coordinates, the work for the x-coordinate will not affect scoring.
- However, a response that reports only a completely correct x-coordinate of the particle's position at time $t = 0$ with all supporting work, e.g., $x(0) = 3 + \int_2^0 x'(t)\,dt = -\frac{31}{3} = -10.333,$ earns 2 out of 3 points.

Total for part (c) 3 points

(d) For $2 \le t \le 8$, the particle remains in the first quadrant. Find all times t in the interval $2 \le t \le 8$ when the particle is moving toward the x-axis. Give a reason for your answer.

Because $y(t) > 0$ when $2 \le t \le 8$, the particle will be moving toward the x-axis when $y'(t) < 0$. This occurs when 5.222 (or 5.221) $< t < 8$.	Considers sign of $y'(t)$	**1 point**
	Answer with reason	**1 point**

Scoring notes:

- The first point can be earned by stating $y'(t) = 0$, $y'(t) < 0$, $y'(t) > 0$, or $t = 5.222$ (or 5.221).

  Note: $y'(t)$ may be written as $\frac{dy}{dt}$.
- The second point cannot be earned without the first.
- To earn the second point, a response must identify the correct interval (and no additional intervals in $[2, 8]$) and explicitly state the need for $y'(t) < 0$. The interval can be open, closed, or half-open.

Total for part (d) 2 points

Total for question 2 9 points

Part A (BC): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

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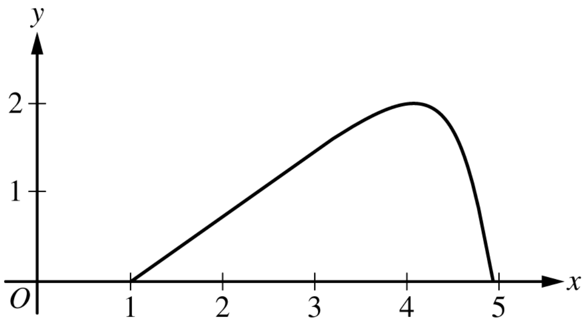
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



For $0 \le t \le \pi$, a particle is moving along the curve shown so that its position at time t is $(x(t), y(t))$, where $x(t)$ is not explicitly given and $y(t) = 2\sin t$. It is known that $\frac{dx}{dt} = e^{\cos t}$. At time $t = 0$, the particle is at position $(1, 0)$.

Model Solution	Scoring	
(a) Find the acceleration vector of the particle at time $t = 1$. Show the setup for your calculations.		
$x''(1) = \frac{d}{dt}\left(e^{\cos t}\right)\Big\vert_{t=1} = -1.444407$	$x''(1)$ with setup	**1 point**
$y(t) = 2\sin t \Rightarrow y'(t) = 2\cos t$ $y''(1) = \frac{d}{dt}(2\cos t)\Big\vert_{t=1} = -1.682942$ The acceleration vector at time $t = 1$ is $a(1) = \langle -1.444, -1.683 \text{ (or } -1.682)\rangle$.	$y''(1)$ with setup	**1 point**

Scoring notes:

- The exact answer is $\langle x''(1), y''(1)\rangle = \left\langle -e^{\cos 1}\sin 1, -2\sin 1\right\rangle$.
- $\left\langle -e^{\cos t}\sin t, -2\sin t\right\rangle$ together with an incorrect or missing evaluation at $t = 1$ earns 1 of the 2 points.

- A response of $\langle -e^{\cos t}\sin t, -2\sin t\rangle = \langle -e^{\cos 1}\sin 1, -2\sin 1\rangle$ or equivalent earns only 1 of the 2 points because it equates an expression to a numerical value.
- An unsupported correct acceleration vector earns 1 of the 2 points.
- The acceleration vector may be presented with other symbols, for example (,) or [,].
- The components may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x''(1) = -0.000828$ or -0.047433 and $y''(1) = -0.000609$ or -0.034905. A response that presents one of these values with correct setups earns 1 of the 2 points.

Total for part (a) 2 points

(b) For $0 \le t \le \pi$, find the first time t at which the speed of the particle is 1.5. Show the work that leads to your answer.

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}$ $0 \le t \le \pi$ and $\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} = 1.5$	$\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} = 1.5$	**1 point**
$\Rightarrow t = 1.254472,\ t = 2.358077$ The first time at which the speed of the particle is 1.5 is $t = 1.254$.	Answer	**1 point**

Scoring notes:

- A response with an implied equation is eligible for both points. For example, a response of "Speed $= \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}$ and is first equal to 1.5 at $t = 1.254$" earns both points.
- $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.5$ earns the first point. Speed $= 1.5$ by itself does not earn the first point. Both of these responses are eligible to earn the second point.
- A response need not consider the value $t = 2.358077$.
- A response of $t = 1.254$ alone does not earn either point.
- A response with a parenthesis error(s) in either $\left(e^{\cos t}\right)^2$ or $(2\cos t)^2$ does not earn the first point but does earn the second point for the correct answer. Note: $\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$ is not considered a parenthesis error.

- Degree mode: In degree mode, $\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} = 1.5$ has no solution for $0 \le t \le \pi$.

  A response that finds no time t at which the speed of the particle is 1.5 cannot be assumed to be working in degree mode.

Total for part (b) 2 points

(c) Find the slope of the line tangent to the path of the particle at time $t = 1$. Find the x-coordinate of the position of the particle at time $t = 1$. Show the work that leads to your answers.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{e^{\cos t}}$ $\left.\frac{dy}{dx}\right\|_{t=1} = \frac{2\cos 1}{e^{\cos 1}} = 0.629530$ The slope of the line tangent to the curve at $t = 1$ is 0.630 (or 0.629).	Slope with supporting work	**1 point**
$x(1) = x(0) + \int_0^1 \frac{dx}{dt}\,dt = 1 + \int_0^1 e^{\cos t}\,dt = 3.341575$ The x-coordinate of the position at $t = 1$ is 3.342 (or 3.341).	$\int_0^1 e^{\cos t}\,dt$	**1 point**
	$x(1)$	**1 point**

Scoring notes:

- To earn the first point, the response must communicate $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; for example:
  - $\frac{dy}{dx} = \frac{2\cos 1}{e^{\cos 1}}$
  - $\frac{dy/dt}{dx/dt} = 0.63$
  - $x'(1) = 1.716526,\ \ y'(1) = 1.080605,\ \ \text{slope} = 0.63$
  - $\frac{dy}{dt} = 2\cos t,\ \ \text{slope} = 0.63$
- A response may import an incorrect expression for $y'(t)$ or value of $y'(1)$ from part (a), provided it was declared in part (a).
- The second point is earned for a response that presents the definite integral $\int_0^1 e^{\cos t}\,dt$ or $\int_0^1 \frac{dx}{dt}\,dt$ with or without the initial condition.

- For the second point, if the differential is missing:
  - $\int_0^1 e^{\cos t}$ earns the second point and is eligible for the third point.
  - $x(1) = \int_0^1 e^{\cos t}$ earns the second point but is not eligible for the third point.
  - $x(1) = 1 + \int_0^1 e^{\cos t}$ earns the second point and is eligible for the third point.
  - $x(1) = \int_0^1 e^{\cos t} + 1$ does not earn the second point but earns the third point for the correct answer.
- The third point is not earned for a response that presents an incorrect statement, such as $x(1) = \int_0^1 e^{\cos t}\, dt = 1 + 2.342.$
- Degree mode: In degree mode, $\frac{dy}{dx} = 0.735759$ or 0.012841 and $1 + \int_0^1 e^{\cos t}\, dt = 3.718144.$

Total for part (c) 3 points

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le \pi$. Show the setup for your calculations.

$\int_0^{\pi} \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}\, dt$	Integral	**1 point**
$= 6.034611$ The total distance traveled by the particle over $0 \le t \le \pi$ is 6.035 (or 6.034).	Answer	**1 point**

Scoring notes:

- The first point is earned for presenting the correct integrand in a definite integral.
- Parentheses errors were assessed in part (b) and, therefore, will not affect the scoring in part (d).
- If the integrand is an incorrect speed function imported from part (b), the response earns the first point and does not earn the second point.
- An unsupported answer of 6.035 (or 6.034) does not earn either point.
- Degree mode: In degree mode, the total distance is 10.596835 or 8.536161.

Total for part (d) 2 points

Total for question 2 9 points

Part A (BC): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time $t > 0$. The particle moves in such a way that $\frac{dx}{dt} = \sqrt{1 + t^2}$ and $\frac{dy}{dt} = \ln\left(2 + t^2\right)$. At time $t = 4$, the particle is at the point $(1, 5)$.

	Model Solution	Scoring	
(a)	Find the slope of the line tangent to the path of the particle at time $t = 4$.		
	$\left.\frac{dy}{dx}\right\|_{t=4} = \frac{y'(4)}{x'(4)} = \frac{\ln 18}{\sqrt{17}} = 0.701018$ The slope of the line tangent to the path of the particle at time $t = 4$ is 0.701.	Answer	**1 point**

Scoring notes:

- To earn the point, the setup used to perform the calculation must be evident in the response. The following examples earn the point: $\frac{y'(4)}{x'(4)} = 0.701$, $\frac{\ln\left(2 + 4^2\right)}{\sqrt{1 + 4^2}}$, or $\frac{\ln 18}{\sqrt{17}}$.
- Note: A response with an incorrect equation of the form "function = constant", such as $\frac{y'(t)}{x'(t)} = \frac{\ln(18)}{\sqrt{17}}$, will not earn the point. However, such a response will be eligible for any points for similar errors in subsequent parts.

Total for part (a) — 1 point

(b) Find the speed of the particle at time $t = 4$, and find the acceleration vector of the particle at time $t = 4$.

$\sqrt{(x'(4))^2 + (y'(4))^2} = \sqrt{17 + (\ln 18)^2} = 5.035300$ The speed of the particle at time $t = 4$ is 5.035.	Speed	**1 point**
$a(4) = \langle x''(4), y''(4)\rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{4}{9}\right\rangle = \langle 0.970143, 0.444444\rangle$ The acceleration vector of the particle at time $t = 4$ is $\langle 0.970, 0.444\rangle$.	First component of acceleration	**1 point**
	Second component of acceleration	**1 point**

Scoring notes:

- To earn any of these points, the setup used to perform the calculation must be evident in the response. For example, $\sqrt{(x'(4))^2 + (y'(4))^2} = 5.035$ or $\sqrt{17 + (\ln 18)^2}$ earns the first point, and $\langle x''(4), y''(4)\rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{4}{9}\right\rangle$ earns both the second and third points.
- The second and third points can be earned independently.
- If the acceleration vector is not presented as an ordered pair, the x- and y-components must be labeled.
- If the components of the acceleration vector are reversed, the response does not earn either of the last 2 points.
- A response which correctly calculates expressions for both $x''(t) = \frac{t}{\sqrt{1 + t^2}}$ and $y''(t) = \frac{2t}{2 + t^2}$, but which fails to evaluate both of these expressions at $t = 4$, earns only 1 of the last 2 points.
- An unsupported acceleration vector earns only 1 of the last 2 points.

Total for part (b) 3 points

(c) Find the y-coordinate of the particle's position at time $t = 6$.

$y(6) = y(4) + \int_4^6 \ln\left(2 + t^2\right) dt$	Integrand	**1 point**
	Uses $y(4)$	**1 point**
$= 5 + 6.570517 = 11.570517$ The y-coordinate of the particle's position at time $t = 6$ is 11.571 (or 11.570).	Answer	**1 point**

Scoring notes:

- For the first point, an integrand of $\ln\left(2 + t^2\right)$ can appear in either an indefinite integral or an incorrect definite integral.
- A definite integral with incorrect limits is not eligible for the answer point.
- Similarly, an indefinite integral is not eligible for the answer point.
- For the second point, the value for $y(4)$ must be added to a definite integral.
- A response that reports the correct x-coordinate of the particle's position at time $t = 6$ as $x(6) = x(4) + \int_4^6 \sqrt{1 + t^2}\, dt = 11.200$ (or 11.201) instead of the y-coordinate, earns 2 out of the 3 points.
- A response that earns the first point but not the second can earn the third point with an answer of 6.571 (or 6.570).
- If the differential is missing:
  - $y(6) = \int_4^6 \ln\left(2 + t^2\right)$ earns the first point and is eligible for the third.
  - $y(6) = \int_4^6 \ln\left(2 + t^2\right) + y(4)$ does not earn the first point but is eligible for the second and third points in the presence of the correct answer.
  - $y(6) = y(4) + \int_4^6 \ln\left(2 + t^2\right)$ earns the first two points and is eligible for the third.

Total for part (c) **3 points**

(d) Find the total distance the particle travels along the curve from time $t = 4$ to time $t = 6$.

$\int_4^6 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	Integrand	**1 point**
$= 12.136228$ The total distance the particle travels along the curve from time $t = 4$ to time $t = 6$ is 12.136.	Answer	**1 point**

Scoring notes:

- The first point is earned for presenting the correct integrand in a definite integral.
- To earn the second point, a response must have earned the first point and must present the value 12.136.
- An unsupported answer of 12.136 does not earn either point.

Total for part (d) **2 points**

Total for question 2 **9 points**

Part A (BC): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

For time $t \ge 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right) \right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

Model Solution	Scoring	
(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.		
$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271488$ At time $t = 1.2$, the speed of the particle is 1.271.	Speed	**1 point**
$\langle x''(1.2), y''(1.2) \rangle = \langle 6.246630, 0.405125 \rangle$ At time $t = 1.2$, the acceleration vector of the particle is $\langle 6.247 \text{ (or } 6.246), 0.405 \rangle$.	Acceleration vector	**1 point**

Scoring notes:

- Unsupported answers do not earn any points in this part.
- The acceleration vector may be presented with other symbols, for example $(\ ,\ )$ or $[\ ,\ ]$, or the coordinates may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, speed $= 0.844$ and $y''(1.2) = 0.023$ (or 0.022).

Total for part (a) — 2 points

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 1.2$.

$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.009817$	Integrand	**1 point**
The total distance traveled by the particle over the time interval $0 \le t \le 1.2$ is 1.010 (or 1.009).	Answer	**1 point**

Scoring notes:

- The first point is earned by presenting the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$ in a definite integral with any limits. A definite integral with incorrect limits is not eligible for the second point.
- Once earned, the first point cannot be lost. Even in the presence of subsequent copy errors, the correct answer will earn the second point.
- If the first point is not earned because of a copy error, the second point is still earned for a correct answer.
- Unsupported answers will not earn either point.
- Degree mode: distance $= 0.677$ (or 0.676). (See degree mode statement in part (a).)

Total for part (b) **2 points**

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \ge 0$. Explain why there is no point at which the particle is farthest to the right for $t \ge 0$.

$x'(t) = (t-1)e^{t^2} = 0 \Rightarrow t = 1$	Sets $x'(t) = 0$	**1 point**
Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1$, the particle is farthest to the left at time $t = 1$.	Explains leftmost position at $t = 1$	**1 point**
$x(1) = -2 + \int_0^1 x'(t)\,dt = -2.603511$ $y(1) = 5 + \int_0^1 y'(t)\,dt = 5.410486$	One coordinate of leftmost position	**1 point**
The particle is farthest to the left at point $(-2.604 \text{ (or } -2.603), 5.410)$.	Leftmost position	**1 point**
Because $x'(t) > 0$ for $t > 1$, the particle moves to the right for $t > 1$. Also, $x(2) = -2 + \int_0^2 x'(t)\,dt > -2 = x(0)$, so the particle's motion extends to the right of its initial position after time $t = 1$. Therefore, there is no point at which the particle is farthest to the right.	Explanation	**1 point**

Scoring notes:

- The second point is earned for presenting a valid reason why the particle is at its leftmost position at time $t = 1$. For example, a response could present the argument shown in the model solution, or it could indicate that the only critical point of $x(t)$ occurs at $t = 1$ and $x'(t)$ changes from negative to positive at this time.
- Unsupported positions $x(1)$ and/or $y(1)$ do not earn the third (or fourth) point(s).
- Writing $x(1) = \int_0^1 x'(t) - 2 = -2.603511$ does not earn the third (or fourth) point, because the missing dt makes this statement unclear or false. However, $x(1) = -2 + \int_0^1 x'(t) = -2.603511$ does earn the third point, because it is not ambiguous. Similarly, for $y(1)$.
- For the fourth point the coordinates of the leftmost point do not have to be written as an ordered pair as long as they are labeled as the x- and y-coordinates.
- To earn the last point a response must verify that the particle moves to the right of its initial position (as well as moves to the right for all $t > 1$). Note that there are several ways to demonstrate this.
- Degree mode: y-coordinate $= 5.008$ (or 5.007). (See degree mode statement in part (a).)

Total for part (c) **5 points**

Total for question 2 **9 points**

2018

AP CollegeBoard

AP Calculus BC
Scoring Guidelines

AP® CALCULUS AB/CALCULUS BC
2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t)\,dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7)\,dt = 20 + \int_0^{300} r(t)\,dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

$2 : \begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.

1 : answer

(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$.

$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

2018 SCORING GUIDELINES

Question 2

(a) $p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$

There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters.

The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$

Because $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$

The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.

$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$

(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$

The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$

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Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$

$$= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$

$$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$$

$$= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$$

$$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

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2018 SCORING GUIDELINES

Question 4

(a) $H'(6) \approx \dfrac{H(7) - H(5)}{7 - 5} = \dfrac{11 - 6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\dfrac{H(5) - H(3)}{5 - 3} = \dfrac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5,$ H is continuous on $3 \le t \le 5.$

By the Mean Value Theorem, there exists a value $c,$ $3 < c < 5,$ such that $H'(c) = 2.$

$2 : \begin{cases} 1 : \dfrac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using} \\ \quad\;\, \text{Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\dfrac{1}{10 - 2}\displaystyle\int_2^{10} H(t)\, dt.$

$$\frac{1}{8}\int_2^{10} H(t)\, dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\dfrac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1 + x)^2} \cdot \frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\dfrac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \dfrac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule

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Question 5

(a) $\text{Area} = \dfrac{1}{2}\displaystyle\int_{\pi/3}^{5\pi/3}\left(4^2 - (3 + 2\cos\theta)^2\right)d\theta$

$3 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \end{cases}$

(b) $\dfrac{dr}{d\theta} = -2\sin\theta \Rightarrow \left.\dfrac{dr}{d\theta}\right|_{\theta=\pi/2} = -2$

$r\left(\dfrac{\pi}{2}\right) = 3 + 2\cos\left(\dfrac{\pi}{2}\right) = 3$

$y = r\sin\theta \Rightarrow \dfrac{dy}{d\theta} = \dfrac{dr}{d\theta}\sin\theta + r\cos\theta$

$x = r\cos\theta \Rightarrow \dfrac{dx}{d\theta} = \dfrac{dr}{d\theta}\cos\theta - r\sin\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{-2\sin\left(\frac{\pi}{2}\right) + 3\cos\left(\frac{\pi}{2}\right)}{-2\cos\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \dfrac{\pi}{2}$ is $\dfrac{2}{3}$.

— OR —

$y = r\sin\theta = (3 + 2\cos\theta)\sin\theta \Rightarrow \dfrac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$x = r\cos\theta = (3 + 2\cos\theta)\cos\theta \Rightarrow \dfrac{dx}{d\theta} = -3\sin\theta - 4\sin\theta\cos\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{3\cos\left(\frac{\pi}{2}\right) + 2\cos^2\left(\frac{\pi}{2}\right) - 2\sin^2\left(\frac{\pi}{2}\right)}{-3\sin\left(\frac{\pi}{2}\right) - 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \dfrac{\pi}{2}$ is $\dfrac{2}{3}$.

$$3 : \begin{cases} 1 : \dfrac{dy}{d\theta} = \dfrac{dr}{d\theta}\sin\theta + r\cos\theta \\ \quad \text{or } \dfrac{dx}{d\theta} = \dfrac{dr}{d\theta}\cos\theta - r\sin\theta \\ 1 : \dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} \\ 1 : \text{answer} \end{cases}$$

(c) $\dfrac{dr}{dt} = \dfrac{dr}{d\theta}\cdot\dfrac{d\theta}{dt} = -2\sin\theta\cdot\dfrac{d\theta}{dt} \Rightarrow \dfrac{d\theta}{dt} = \dfrac{dr}{dt}\cdot\dfrac{1}{-2\sin\theta}$

$\left.\dfrac{d\theta}{dt}\right|_{\theta=\pi/3} = 3\cdot\dfrac{1}{-2\sin\left(\frac{\pi}{3}\right)} = \dfrac{3}{-\sqrt{3}} = -\sqrt{3}$ radians per second

$$3 : \begin{cases} 1 : \dfrac{dr}{dt} = \dfrac{dr}{d\theta}\cdot\dfrac{d\theta}{dt} \\ 1 : \dfrac{d\theta}{dt} = \dfrac{dr}{dt}\cdot\dfrac{1}{-2\sin\theta} \\ 1 : \text{answer with units} \end{cases}$$

2018 SCORING GUIDELINES

Question 6

(a) The first four nonzero terms are $\frac{x^2}{3} - \frac{x^3}{2 \cdot 3^2} + \frac{x^4}{3 \cdot 3^3} - \frac{x^5}{4 \cdot 3^4}$.

The general term is $(-1)^{n+1} \frac{x^{n+1}}{n \cdot 3^n}$.

$$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$$

(b) $\lim\limits_{n\to\infty} \left| \dfrac{\dfrac{(-1)^{n+2} x^{n+2}}{(n+1)\left(3^{n+1}\right)}}{\dfrac{(-1)^{n+1} x^{n+1}}{n \cdot 3^n}} \right| = \lim\limits_{n\to\infty} \left| \dfrac{-x}{3} \cdot \dfrac{n}{(n+1)} \right| = \left| \dfrac{x}{3} \right|$

$\left| \frac{x}{3} \right| < 1$ for $|x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

— OR —

The radius of convergence of the Maclaurin series for $\ln(1 + x)$ is 1, so the series for $f(x) = x \ln\left(1 + \frac{x}{3}\right)$ converges absolutely for $\left| \frac{x}{3} \right| < 1$.

$\left| \frac{x}{3} \right| < 1 \Rightarrow |x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

When $x = -3$, the series is $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{(-3)^{n+1}}{n \cdot 3^n} = \sum\limits_{n=1}^{\infty} \frac{3}{n}$, which diverges by comparison to the harmonic series.

When $x = 3$, the series is $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{3^{n+1}}{n \cdot 3^n} = \sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n}$, which converges by the alternating series test.

The interval of convergence of the Maclaurin series for f is $-3 < x \le 3$.

$$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of} \\ \quad\ \text{convergence} \end{cases}$$

— OR —

$$5 : \begin{cases} 1 : \text{radius for } \ln(1+x) \text{ series} \\ 1 : \text{substitutes } \frac{x}{3} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of} \\ \quad\ \text{convergence} \end{cases}$$

(c) By the alternating series error bound, an upper bound for $|P_4(2) - f(2)|$ is the magnitude of the next term of the alternating series.

$|P_4(2) - f(2)| < \left| -\frac{2^5}{4 \cdot 3^4} \right| = \frac{8}{81}$

$$2 : \begin{cases} 1 : \text{uses fifth-degree term} \\ \quad\ \text{as error bound} \\ 1 : \text{answer} \end{cases}$$

AP® Calculus BC
2016 Scoring Guidelines

2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3) - R(1)}{3 - 1} = \dfrac{950 - 1190}{3 - 1} = -120$ liters/hr^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\,dt$.

$$\begin{aligned}\int_0^8 R(t)\,dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters}\end{aligned}$$

This is an overestimate since R is a decreasing function.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $$\begin{aligned}\text{Total} &\approx 50000 + \int_0^8 W(t)\,dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters}\end{aligned}$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

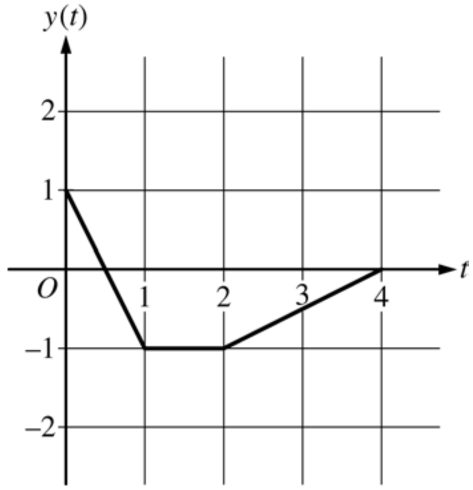
Therefore, the Intermediate Value Theorem guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

2016 SCORING GUIDELINES

Question 2



At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin\left(3t^2\right)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

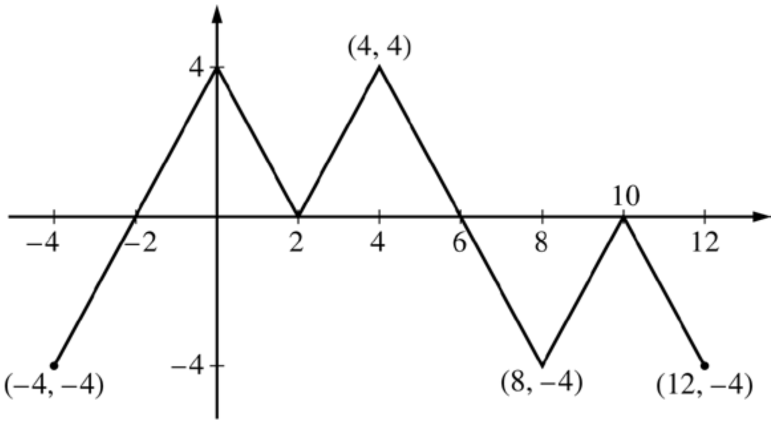
(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + 9.377035 = 14.377$ $y(3) = -\frac{1}{2}$ The position of the particle at $t = 3$ is $(14.377, -0.5)$.	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$
(b) Slope $= \frac{y'(3)}{x'(3)} = \frac{0.5}{9.956376} = 0.05$	1 : slope
(c) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 9.969$ (or 9.968)	$2 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer} \end{cases}$
(d) Distance $= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ $= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\,dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\,dt$ $= 2.237871 + 2.112003 = 4.350$ (or 4.349)	$3 : \begin{cases} 1 : \text{expression for distance} \\ 1 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

---

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

$$4 : \begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad\ \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$$

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals

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Question 4

Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right)$. Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

(a) $\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right)$

$2 : \frac{d^2y}{dx^2}$ in terms of x and y

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(-2,\,8)} = (-2)^2 - \frac{1}{2}\cdot 8 = 0$

$\left.\frac{d^2y}{dx^2}\right|_{(x,\,y)=(-2,\,8)} = 2(-2) - \frac{1}{2}\left((-2)^2 - \frac{1}{2}\cdot 8\right) = -4 < 0$

Thus, the graph of f has a relative maximum at the point $(-2, 8)$.

2 : conclusion with justification

(c) $\lim_{x \to -1}(g(x) - 2) = 0$ and $\lim_{x \to -1} 3(x+1)^2 = 0$

Using L'Hospital's Rule,

$\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right) = \lim_{x \to -1}\left(\frac{g'(x)}{6(x+1)}\right)$

$\lim_{x \to -1} g'(x) = 0$ and $\lim_{x \to -1} 6(x+1) = 0$

Using L'Hospital's Rule,

$\lim_{x \to -1}\left(\frac{g'(x)}{6(x+1)}\right) = \lim_{x \to -1}\left(\frac{g''(x)}{6}\right) = \frac{-2}{6} = -\frac{1}{3}$

$3 : \begin{cases} 2 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

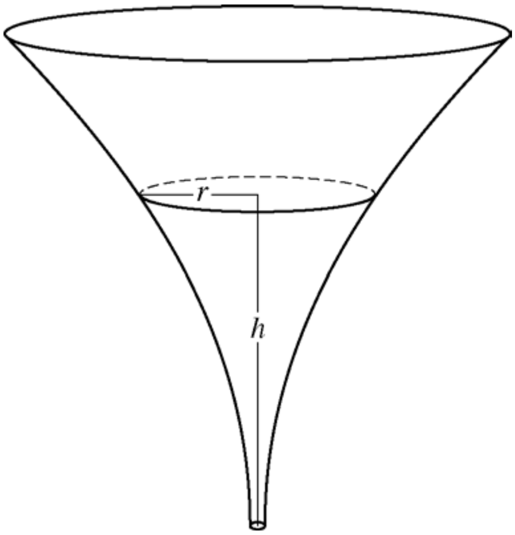
(d) $h\left(\frac{1}{2}\right) \approx h(0) + h'(0)\cdot\frac{1}{2} = 2 + (-1)\cdot\frac{1}{2} = \frac{3}{2}$

$h(1) \approx h\left(\frac{1}{2}\right) + h'\left(\frac{1}{2}\right)\cdot\frac{1}{2} \approx \frac{3}{2} + \left(-\frac{1}{2}\right)\cdot\frac{1}{2} = \frac{5}{4}$

$2 : \begin{cases} 1 : \text{Euler's method} \\ 1 : \text{approximation} \end{cases}$

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right)\, dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2 dh = \frac{\pi}{400}\int_0^{10} \left(9 + 6h^2 + h^4\right) dh$

$= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}$ in^3

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3}$ in/sec

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \ge 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

(a) $f(1) = 1,\ f'(1) = -\frac{1}{2},\ f''(1) = \frac{1}{2^2},\ f'''(1) = -\frac{2}{2^3}$

$$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2}(x-1)^2 - \frac{1}{2^3 \cdot 3}(x-1)^3 + \cdots + \frac{(-1)^n}{2^n \cdot n}(x-1)^n + \cdots$$

$4 : \begin{cases} 1 : \text{first two terms} \\ 1 : \text{third term} \\ 1 : \text{fourth term} \\ 1 : \text{general term} \end{cases}$

(b) $R = 2$. The series converges on the interval $(-1, 3)$.

When $x = -1$, the series is $1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$.
Since the harmonic series diverges, this series diverges.

When $x = 3$, the series is $1 - 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \cdots$.
Since the alternating harmonic series converges, this series converges.

Therefore, the interval of convergence is $-1 < x \le 3$.

$2 : \begin{cases} 1 : \text{identifies both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

(c) $f(1.2) \approx 1 - \frac{1}{2}(0.2) + \frac{1}{8}(0.2)^2 = 1 - 0.1 + 0.005 = 0.905$

1 : approximation

(d) The series for $f(1.2)$ alternates with terms that decrease in magnitude to 0.

$$|f(1.2) - T_2(1.2)| \le \left| \frac{-1}{2^3 \cdot 3}(0.2)^3 \right| = \frac{1}{3000} \le 0.001$$

$2 : \begin{cases} 1 : \text{error form} \\ 1 : \text{analysis} \end{cases}$

AP® Calculus BC
2015 Scoring Guidelines

2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8.$ The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8.$ There are 30 cubic feet of water in the pipe at time $t = 0.$

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time $t,$ $0 \le t \le 8,$ is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8,$ water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3),$ the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time $t,$ $0 \le t \le 8,$ is

$30 + \int_0^t [R(x) - D(x)]\,dx.$

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(a) $x(2) = 3 + \int_1^2 \cos\left(t^2\right)\,dt = 2.557$ (or 2.556)

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{e^{0.5t}}{\cos\left(t^2\right)}$

$\dfrac{e^{0.5t}}{\cos\left(t^2\right)} = 2$

$t = 0.840$

$2 : \begin{cases} 1 : \text{slope in terms of } t \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{\cos^2\left(t^2\right) + e^t}$

$\sqrt{\cos^2\left(t^2\right) + e^t} = 3$

$t = 2.196$ (or 2.195)

$2 : \begin{cases} 1 : \text{speed in terms of } t \\ 1 : \text{answer} \end{cases}$

(d) Distance $= \int_0^1 \sqrt{\cos^2\left(t^2\right) + e^t}\,dt = 1.595$ (or 1.594)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

(a) $v'(16) \approx \dfrac{240 - 200}{20 - 12} = 5$ meters/min^2

1 : approximation

(b) $\int_0^{40} |v(t)|\, dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes.

$$\begin{aligned}\int_0^{40} |v(t)|\, dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

$3 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$B'(5) = 3(25) - 12(5) = 15$ meters/min^2

$2 : \begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d)
$$\begin{aligned}\text{Avg vel} &= \frac{1}{10}\int_0^{10} \left(t^3 - 6t^2 + 300\right) dt \\ &= \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10} \\ &= \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350 \text{ meters/min}\end{aligned}$$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y.$

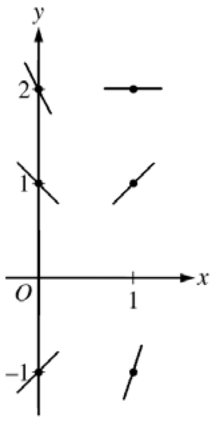
(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

(a)



$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\dfrac{d^2y}{dx^2} = 2 - \dfrac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0,$ so $2 - 2x + y > 0.$
Therefore, all solution curves are concave up in Quadrant II.

$2 : \begin{cases} 1 : \dfrac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2.$

$2 : \begin{cases} 1 : \text{considers } \left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(mx + b) = m$

$2x - y = m$

$2x - (mx + b) = m$

$(2 - m)x - (m + b) = 0$

$2 - m = 0 \Rightarrow m = 2$

$b = -m \Rightarrow b = -2$

Therefore, $m = 2$ and $b = -2$.

$3 : \begin{cases} 1 : \dfrac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 5

Consider the function $f(x) = \dfrac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$$f'(x) = \frac{k - 2x}{\left(x^2 - kx\right)^2}.$$

(a) Let $k = 3$, so that $f(x) = \dfrac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x-coordinate is 4.

(b) Let $k = 4$, so that $f(x) = \dfrac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.

(c) Find the value of k for which f has a critical point at $x = -5$.

(d) Let $k = 6$, so that $f(x) = \dfrac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f. Find $\int f(x)\,dx$.

(a) $f(4) = \dfrac{1}{4^2 - 3\cdot 4} = \dfrac{1}{4}$ $\qquad f'(4) = \dfrac{3 - 2\cdot 4}{\left(4^2 - 3\cdot 4\right)^2} = -\dfrac{5}{16}$

An equation for the line tangent to the graph of f at the point whose x-coordinate is 4 is $y = -\dfrac{5}{16}(x - 4) + \dfrac{1}{4}$.

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(b) $f'(x) = \dfrac{4 - 2x}{\left(x^2 - 4x\right)^2}$ $\qquad f'(2) = \dfrac{4 - 2\cdot 2}{\left(2^2 - 4\cdot 2\right)^2} = 0$

$f'(x)$ changes sign from positive to negative at $x = 2$.
Therefore, f has a relative maximum at $x = 2$.

$2 : \begin{cases} 1 : \text{considers } f'(2) \\ 1 : \text{answer with justification} \end{cases}$

(c) $f'(-5) = \dfrac{k - 2\cdot(-5)}{\left((-5)^2 - k\cdot(-5)\right)^2} = 0 \Rightarrow k = -10$

1 : answer

(d) $\dfrac{1}{x^2 - 6x} = \dfrac{1}{x(x-6)} = \dfrac{A}{x} + \dfrac{B}{x-6} \Rightarrow 1 = A(x-6) + Bx$

$x = 0 \Rightarrow 1 = A\cdot(-6) \Rightarrow A = -\dfrac{1}{6}$

$x = 6 \Rightarrow 1 = B\cdot(6) \Rightarrow B = \dfrac{1}{6}$

$\dfrac{1}{x(x-6)} = \dfrac{-1/6}{x} + \dfrac{1/6}{x-6}$

$\int f(x)\,dx = \int\left(\dfrac{-1/6}{x} + \dfrac{1/6}{x-6}\right)dx$

$= -\dfrac{1}{6}\ln|x| + \dfrac{1}{6}\ln|x-6| + C = \dfrac{1}{6}\ln\left|\dfrac{x-6}{x}\right| + C$

$4 : \begin{cases} 2 : \text{partial fraction decomposition} \\ 2 : \text{general antiderivative} \end{cases}$

2015 SCORING GUIDELINES

Question 6

The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n + \cdots$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

(a) Use the ratio test to find R.

(b) Write the first four nonzero terms of the Maclaurin series for f', the derivative of f. Express f' as a rational function for $|x| < R$.

(c) Write the first four nonzero terms of the Maclaurin series for e^x. Use the Maclaurin series for e^x to write the third-degree Taylor polynomial for $g(x) = e^x f(x)$ about $x = 0$.

(a) Let a_n be the nth term of the Maclaurin series.

$$\frac{a_{n+1}}{a_n} = \frac{(-3)^n x^{n+1}}{n+1} \cdot \frac{n}{(-3)^{n-1} x^n} = \frac{-3n}{n+1} \cdot x$$

$$\lim_{n\to\infty} \left| \frac{-3n}{n+1} \cdot x \right| = 3|x|$$

$$3|x| < 1 \Rightarrow |x| < \frac{1}{3}$$

The radius of convergence is $R = \frac{1}{3}$.

$$3 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{determines radius of convergence} \end{cases}$$

(b) The first four nonzero terms of the Maclaurin series for f' are $1 - 3x + 9x^2 - 27x^3$.

$$f'(x) = \frac{1}{1 - (-3x)} = \frac{1}{1 + 3x}$$

$$3 : \begin{cases} 2 : \text{first four nonzero terms} \\ 1 : \text{rational function} \end{cases}$$

(c) The first four nonzero terms of the Maclaurin series for e^x are $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$.

The product of the Maclaurin series for e^x and the Maclaurin series for f is

$$\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x - \frac{3}{2}x^2 + 3x^3 - \cdots\right)$$
$$= x - \frac{1}{2}x^2 + 2x^3 + \cdots$$

The third-degree Taylor polynomial for $g(x) = e^x f(x)$ about $x = 0$ is $T_3(x) = x - \frac{1}{2}x^2 + 2x^3$.

$$3 : \begin{cases} 1 : \text{first four nonzero terms of the Maclaurin series for } e^x \\ 2 : \text{Taylor polynomial} \end{cases}$$