

Part B (BC): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

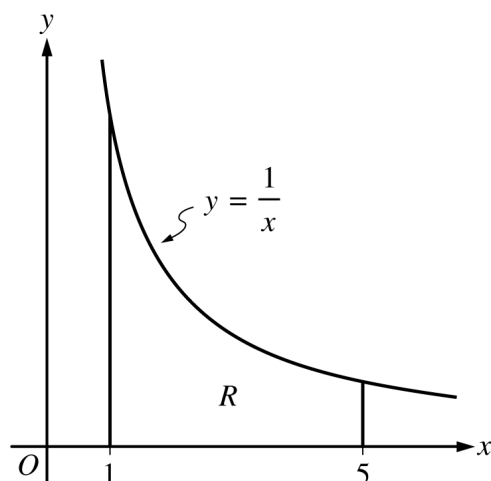


Figure 1

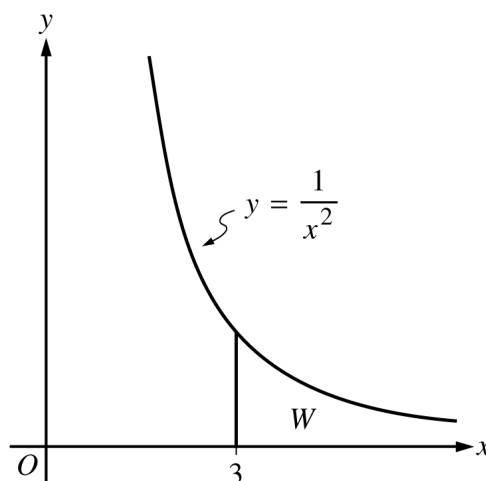


Figure 2

Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x -axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x -axis that lies to the right of the vertical line $x = 3$.

Model Solution**Scoring**

- (a) Find the area of region R .

Area = $\int_1^5 \frac{1}{x} dx$	Integral	1 point
$= \ln x \Big _1^5$	Answer	1 point
$= \ln 5 - \ln 1 = \ln 5$		

Scoring notes:

- A definite integral with incorrect bounds does not earn either point.
- An unevaluated indefinite integral does not earn either point.
- An indefinite integral that is evaluated in a later step may earn one or both points. For example,
 $\int \frac{1}{x} dx = \ln 5 - \ln 1$ (or $\ln 5$) does not earn the first point but does earn the second. However,
 $\int \frac{1}{x} dx = \ln x + C \Rightarrow \text{Area} = \ln 5 - \ln 1$ earns both points.

Total for part (a) 2 points

- (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.

Volume = $\int_1^5 xe^{x/5} dx$	Definite integral	1 point
Using integration by parts, $u = x \quad dv = e^{x/5} dx$ $du = dx \quad v = 5e^{x/5}$	u and dv	1 point
$\int xe^{x/5} dx = 5xe^{x/5} - \int 5e^{x/5} dx$ $= 5xe^{x/5} - 25e^{x/5} + C$ $= 5e^{x/5}(x - 5) + C$	$\int xe^{x/5} dx$ $= 5xe^{x/5} - \int 5e^{x/5} dx$	1 point
Volume = $5e^{x/5}(x - 5) \Big _1^5$ $= 5e(0) - 5e^{1/5}(-4) = 20e^{1/5}$	Answer	1 point

Scoring notes:

- The first point is earned for $c \int_1^5 xe^{x/5} dx$, where $c \neq 0$. Errors of $c \neq 1$, for example $c = \pi$, will not earn the fourth point.
- Incorrect integrals that require integration by parts are still eligible for the second and third points. Both of these points will be earned with at least one correct application of integration by parts.
- The second point will be earned with an implied u and dv in the presence of $5xe^{x/5} - \int 5e^{x/5} dx$.
- The tabular method may be used to show integration by parts. In this case, the second point is earned by having columns (labeled or unlabeled) that begin with x and $e^{x/5}$. The third point is earned for either $5xe^{x/5} - \int 5e^{x/5} dx$ or $5xe^{x/5} - 25e^{x/5}$.
- Limits of integration may be present, omitted, or partially present in the work for the second and third points.
- The fourth point is earned only for the correct answer.

Total for part (b) 4 points

- (c) Find the volume of the solid generated when the unbounded region W is revolved about the x -axis.

Volume = $\pi \int_3^{\infty} \left(\frac{1}{x^2}\right)^2 dx = \pi \lim_{b \rightarrow \infty} \int_3^b \frac{1}{x^4} dx$	Improper integral	1 point
$= \pi \lim_{b \rightarrow \infty} \left(\frac{1}{-3x^3} \Big _3^b \right)$	Antiderivative	1 point
$= \pi \lim_{b \rightarrow \infty} \left(\frac{1}{-3} \right) \left[\frac{1}{b^3} - \frac{1}{3^3} \right]$ $= \pi \left(\frac{1}{-3} \right) \left(0 - \frac{1}{3^3} \right) = \frac{\pi}{81}$	Answer	1 point

Scoring notes:

- The first point is earned for either $c \int_3^{\infty} \left(\frac{1}{x^2}\right)^2 dx$ or $\lim_{b \rightarrow \infty} c \int_3^b \frac{1}{x^4} dx$, where $c \neq 0$. Errors of $c \neq \pi$ will not earn the third point.
- The second point is earned for a correct antiderivative of any integrand of the form $\frac{1}{x^n}$, for any integer $n \geq 2$.
- To earn the answer point, a response must use correct limit notation and cannot include arithmetic with infinity, such as $\frac{1}{\infty^3}$.

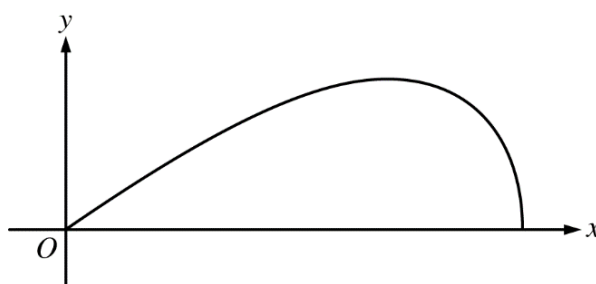
Total for part (c) 3 points

Total for question 5 9 points

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.



A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant.

The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.

Model Solution	Scoring
(a) Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.	
$6x\sqrt{4 - x^2} = 0 \Rightarrow x = 0, x = 2$ $\text{Area} = \int_0^2 6x\sqrt{4 - x^2} \, dx$	Integrand 1 point
Let $u = 4 - x^2$. $du = -2x \, dx \Rightarrow -\frac{1}{2} du = x \, dx$ $x = 0 \Rightarrow u = 4 - 0^2 = 4$ $x = 2 \Rightarrow u = 4 - 2^2 = 0$ $\int_0^2 6x\sqrt{4 - x^2} \, dx = \int_4^0 6\left(-\frac{1}{2}\right)\sqrt{u} \, du = -3\int_4^0 u^{1/2} \, du = 3\int_0^4 u^{1/2} \, du$ $= 2u^{3/2} \Big _{u=0}^{u=4} = 2 \cdot 8 = 16$	Antiderivative 1 point
The area of the region is 16 square inches.	Answer 1 point

Scoring notes:

- Units are not required for any points in this question and are not read if presented (correctly or incorrectly) in any part of the response.
- The first point is earned for presenting $cx\sqrt{4 - x^2}$ or $6x\sqrt{4 - x^2}$ as the integrand in a definite integral. Limits of integration (numeric or alphanumeric) must be presented (as part of the definite integral) but do not need to be correct in order to earn the first point.
- If an indefinite integral is presented with an integrand of the correct form, the first point can be earned if the antiderivative (correct or incorrect) is eventually evaluated using the correct limits of integration.
- The second point can be earned without the first point. The second point is earned for the presentation of a correct antiderivative of a function of the form $Ax\sqrt{4 - x^2}$, for any nonzero constant A . If the response has subsequent errors in simplification of the antiderivative or sign errors, the response will earn the second point but will not earn the third point.
- Responses that use u -substitution and have incorrect limits of integration or do not change the limits of integration from x - to u -values are eligible for the second point.
- The response is eligible for the third point only if it has earned the second point.
- The third point is earned only for the answer 16 or equivalent. In the case where a response only presents an indefinite integral, the use of the correct limits of integration to evaluate the antiderivative must be shown to earn the third point.
- The response cannot correct -16 to $+16$ in order to earn the third point; there is no possible reversal here.

Total for part (a) 3 points

(b)

It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

The cross-sectional circular slice with the largest radius occurs where $cx\sqrt{4 - x^2}$ has its maximum on the interval $0 < x < 2$.

Sets $\frac{dy}{dx} = 0$ **1 point**

$$\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0 \Rightarrow x = \sqrt{2}$$

$$x = \sqrt{2} \Rightarrow y = c\sqrt{2}\sqrt{4 - (\sqrt{2})^2} = 2c$$

$$2c = 1.2 \Rightarrow c = 0.6$$

Answer **1 point**

Scoring notes:

- The first point is earned for setting $\frac{dy}{dx} = 0$, $\frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0$, or $c(4 - 2x^2) = 0$.
- An unsupported $x = \sqrt{2}$ does not earn the first point.
- The second point can be earned without the first point but is earned only for the answer $c = 0.6$ with supporting work.

Total for part (b) 2 points

- (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

$\text{Volume} = \int_0^2 \pi \left(cx\sqrt{4-x^2} \right)^2 dx = \pi c^2 \int_0^2 x^2 (4-x^2) dx$	Form of the integrand	1 point
	Limits and constant	1 point
$= \pi c^2 \int_0^2 (4x^2 - x^4) dx = \pi c^2 \left(\frac{4}{3}x^3 - \frac{1}{5}x^5 \right) \Big _0^2$	Antiderivative	1 point
$= \pi c^2 \left(\frac{32}{3} - \frac{32}{5} \right) = \frac{64\pi c^2}{15}$ $\frac{64\pi c^2}{15} = 2\pi \Rightarrow c^2 = \frac{15}{32} \Rightarrow c = \sqrt{\frac{15}{32}}$	Answer	1 point

Scoring notes:

- The first point is earned for presenting an integrand of the form $A(x\sqrt{4-x^2})^2$ in a definite integral with any limits of integration (numeric or alphanumeric) and any nonzero constant A . Mishandling the c will result in the response being ineligible for the fourth point.
- The second point can be earned without the first point. The second point is earned for the limits of integration, $x = 0$ and $x = 2$, and the constant π (but not for 2π) as part of an integral with a correct or incorrect integrand.
- If an indefinite integral is presented with the correct constant π , the second point can be earned if the antiderivative (correct or incorrect) is evaluated using the correct limits of integration.
- A response that presents $2 = \int_0^2 (cx\sqrt{4-x^2})^2 dx$ earns the first and second points.
- The third point is earned for presenting a correct antiderivative of the presented integrand of the form $A(x\sqrt{4-x^2})^2$ for any nonzero A . If there are subsequent errors in simplification of the antiderivative, linkage errors, or sign errors, the response will not earn the fourth point.
- The fourth point cannot be earned without the third point. The fourth point is earned only for the correct answer. The expression does not need to be simplified to earn the fourth point.

Total for part (c) 4 points

Total for question 3 9 points

AP[®] Calculus BC

2016 Scoring Guidelines

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2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{3 - 1} = -120 \text{ liters/hr}^2$

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t) dt$.

$$\begin{aligned} \int_0^8 R(t) dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $\text{Total} \approx 50000 + \int_0^8 W(t) dt - 8050$
 $= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

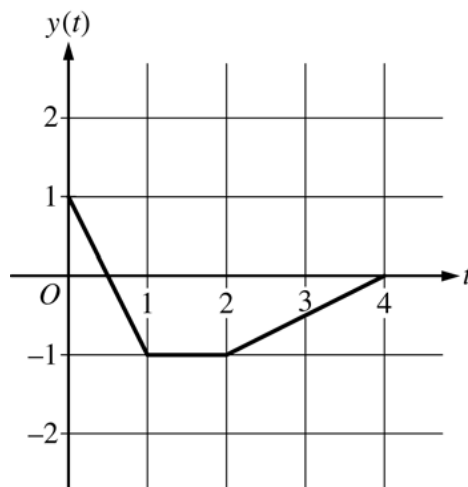
2 : $\begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

Therefore, the Intermediate Value Theorem guarantees at least one time t , $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t , the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

2016 SCORING GUIDELINES

Question 2



At time t , the position of a particle moving in the xy -plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y , consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

- Find the position of the particle at $t = 3$.
- Find the slope of the line tangent to the path of the particle at $t = 3$.
- Find the speed of the particle at $t = 3$.
- Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$(a) \quad x(3) = x(0) + \int_0^3 x'(t) dt = 5 + 9.377035 = 14.377$$

$$y(3) = -\frac{1}{2}$$

The position of the particle at $t = 3$ is $(14.377, -0.5)$.

$$(b) \quad \text{Slope} = \frac{y'(3)}{x'(3)} = \frac{0.5}{9.956376} = 0.05$$

$$(c) \quad \text{Speed} = \sqrt{(x'(3))^2 + (y'(3))^2} = 9.969 \text{ (or } 9.968)$$

$$\begin{aligned} (d) \quad \text{Distance} &= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2} dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2} dt \\ &= 2.237871 + 2.112003 = 4.350 \text{ (or } 4.349) \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

1 : slope

2 : $\begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{expression for distance} \\ 1 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

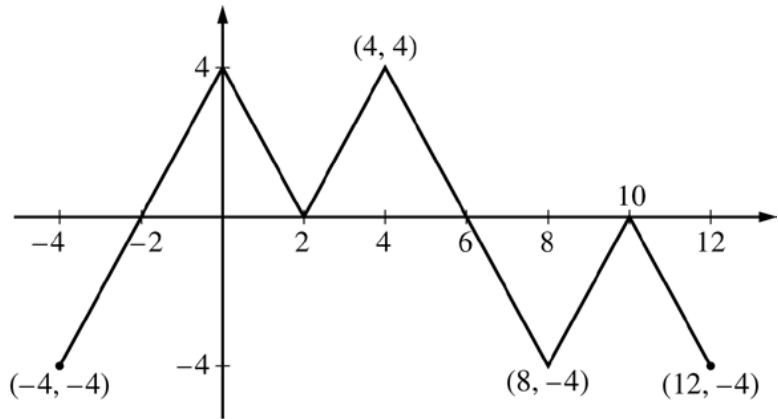
2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

$$g(x) = \int_2^x f(t) \, dt.$$

- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
- (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Graph of f

- (a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.
- (b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$.
- (c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

- (d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

4 : $\left\{ \begin{array}{l} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{array} \right.$

2 : intervals

2016 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find

$$\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right).$$
 Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

$$(a) \quad \frac{d^2y}{dx^2} = 2x - \frac{1}{2} \frac{dy}{dx} = 2x - \frac{1}{2} \left(x^2 - \frac{1}{2}y \right)$$

$$2 : \frac{d^2y}{dx^2} \text{ in terms of } x \text{ and } y$$

$$(b) \quad \left. \frac{dy}{dx} \right|_{(x,y)=(-2,8)} = (-2)^2 - \frac{1}{2} \cdot 8 = 0$$

2 : conclusion with justification

$$\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-2,8)} = 2(-2) - \frac{1}{2} \left((-2)^2 - \frac{1}{2} \cdot 8 \right) = -4 < 0$$

Thus, the graph of f has a relative maximum at the point $(-2, 8)$.

$$(c) \quad \lim_{x \rightarrow -1} (g(x) - 2) = 0 \text{ and } \lim_{x \rightarrow -1} 3(x+1)^2 = 0$$

$$3 : \begin{cases} 2 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right) = \lim_{x \rightarrow -1} \left(\frac{g'(x)}{6(x+1)} \right)$$

$$\lim_{x \rightarrow -1} g'(x) = 0 \text{ and } \lim_{x \rightarrow -1} 6(x+1) = 0$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow -1} \left(\frac{g'(x)}{6(x+1)} \right) = \lim_{x \rightarrow -1} \left(\frac{g''(x)}{6} \right) = \frac{-2}{6} = -\frac{1}{3}$$

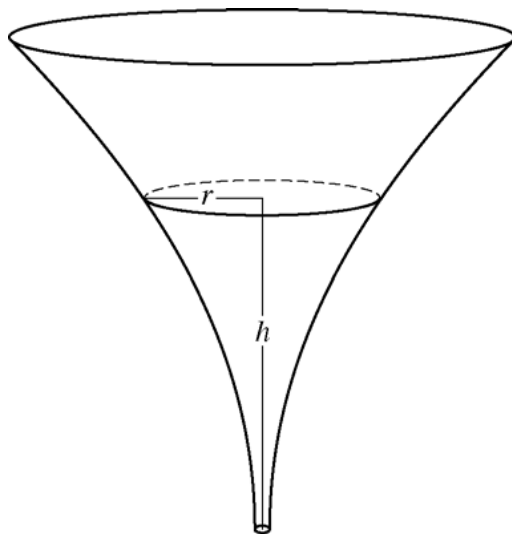
$$(d) \quad h\left(\frac{1}{2}\right) \approx h(0) + h'(0) \cdot \frac{1}{2} = 2 + (-1) \cdot \frac{1}{2} = \frac{3}{2}$$

$$h(1) \approx h\left(\frac{1}{2}\right) + h'\left(\frac{1}{2}\right) \cdot \frac{1}{2} \approx \frac{3}{2} + \left(-\frac{1}{2}\right) \cdot \frac{1}{2} = \frac{5}{4}$$

$$2 : \begin{cases} 1 : \text{Euler's method} \\ 1 : \text{approximation} \end{cases}$$

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\begin{aligned} \text{(a) Average radius} &= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10} \\ &= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^{10} \left(\left(\frac{1}{20} \right) (3 + h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh \\ &= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10} \\ &= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \frac{dr}{dt} &= \frac{1}{20}(2h) \frac{dh}{dt} \\ -\frac{1}{5} &= \frac{3}{10} \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec} \end{aligned}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence.

It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the n th derivative of f at $x = 1$ is given by

$$f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n} \text{ for } n \geq 2.$$

- (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
- (b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.
- (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.
- (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

$$\begin{aligned} \text{(a)} \quad f(1) = 1, \quad f'(1) = -\frac{1}{2}, \quad f''(1) = \frac{1}{2^2}, \quad f'''(1) = -\frac{2}{2^3} \\ f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2}(x-1)^2 - \frac{1}{2^3 \cdot 3}(x-1)^3 + \cdots \\ + \frac{(-1)^n}{2^n \cdot n}(x-1)^n + \cdots \end{aligned}$$

$$4 : \begin{cases} 1 : \text{first two terms} \\ 1 : \text{third term} \\ 1 : \text{fourth term} \\ 1 : \text{general term} \end{cases}$$

- (b) $R = 2$. The series converges on the interval $(-1, 3)$.

When $x = -1$, the series is $1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$.

Since the harmonic series diverges, this series diverges.

When $x = 3$, the series is $1 - 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \cdots$.

Since the alternating harmonic series converges, this series converges.

Therefore, the interval of convergence is $-1 < x \leq 3$.

$$2 : \begin{cases} 1 : \text{identifies both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$$

$$\text{(c)} \quad f(1.2) \approx 1 - \frac{1}{2}(0.2) + \frac{1}{8}(0.2)^2 = 1 - 0.1 + 0.005 = 0.905$$

1 : approximation

- (d) The series for $f(1.2)$ alternates with terms that decrease in magnitude to 0.

$$|f(1.2) - T_2(1.2)| \leq \left| \frac{-1}{2^3 \cdot 3}(0.2)^3 \right| = \frac{1}{3000} \leq 0.001$$

$$2 : \begin{cases} 1 : \text{error form} \\ 1 : \text{analysis} \end{cases}$$