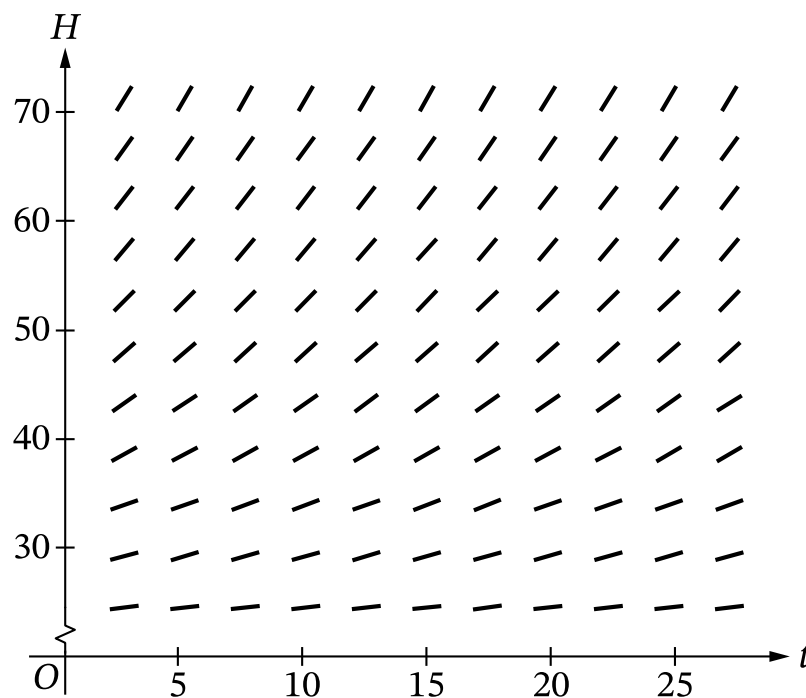


3. A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 75$ . For  $t > 0$ , it is known that  $20 < H(t) < 75$ .

**Part A**

Explain why the following could not be a slope field for the differential equation

$$\frac{dH}{dt} = -\frac{1}{15}(H - 20).$$

**Part B**

Find the slope of the line tangent to the graph of  $H$  at time  $t = 0$ . Show the work that leads to your answer.

**Part C**

It can be shown that  $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$ . The line tangent to the graph of  $H$  at time  $t = 0$  is used to approximate  $H(5)$ , the internal temperature of the pie at time  $t = 5$ . Is this approximation an overestimate or an underestimate for the actual value of  $H(5)$ ? Give a reason for your answer.

**Part D**

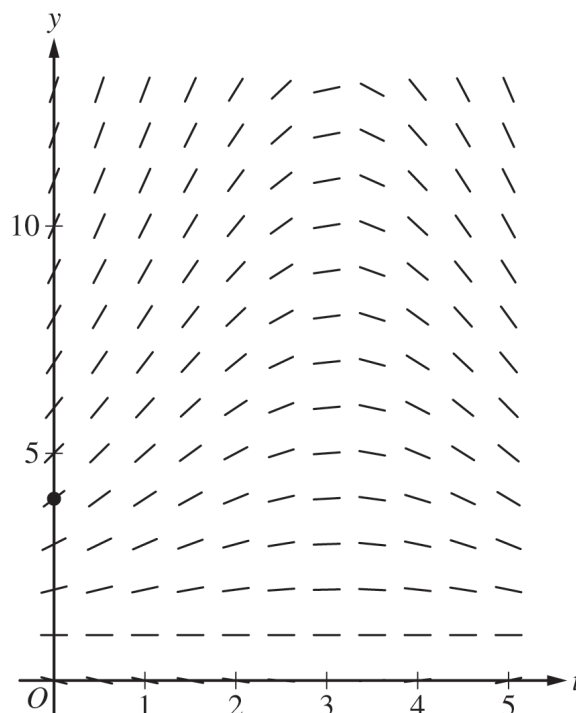
Use separation of variables to find an expression for  $H(t)$ , the particular solution to the given differential equation with initial condition  $H(0) = 75$ .

3. The depth of seawater at a location can be modeled by the function  $H$  that satisfies the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right), \text{ where } H(t) \text{ is measured in feet and } t \text{ is measured in hours after noon } (t = 0). \text{ It is}$$

known that  $H(0) = 4$ .

- (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve,  $y = H(t)$ , through the point  $(0, 4)$ .



- (b) For  $0 < t < 5$ , it can be shown that  $H(t) > 1$ . Find the value of  $t$ , for  $0 < t < 5$ , at which  $H$  has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.
- (c) Use separation of variables to find  $y = H(t)$ , the particular solution to the differential equation

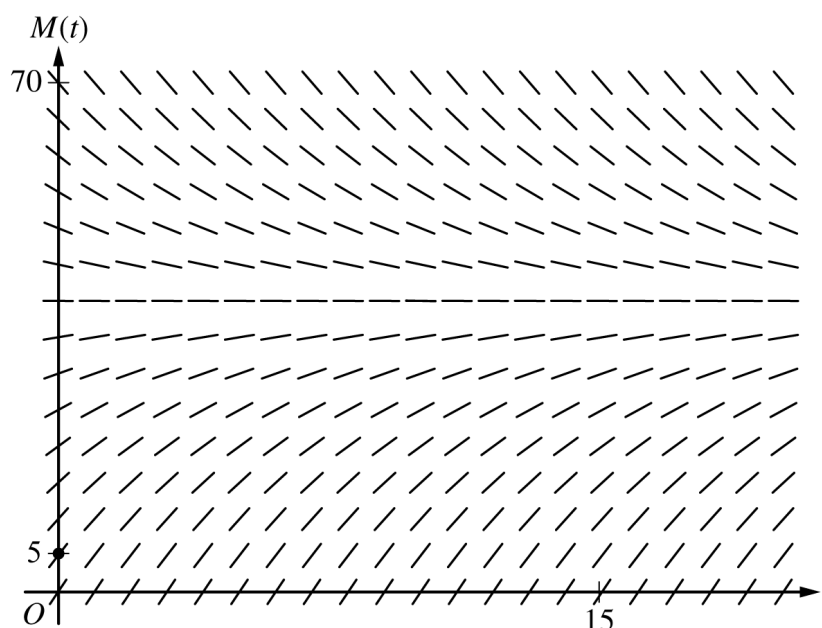
$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right) \text{ with initial condition } H(0) = 4.$$

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function  $M$  models the temperature of the milk at time  $t$ , where  $M(t)$  is measured in degrees Celsius ( $^{\circ}\text{C}$ ) and  $t$  is the number of minutes since the bottle was placed in the pan.  $M$  satisfies the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ . At time  $t = 0$ , the temperature of the milk is  $5^{\circ}\text{C}$ . It can be shown that  $M(t) < 40$  for all values of  $t$ .

- (a) A slope field for the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$  is shown. Sketch the solution curve through the point  $(0, 5)$ .



- (b) Use the line tangent to the graph of  $M$  at  $t = 0$  to approximate  $M(2)$ , the temperature of the milk at time  $t = 2$  minutes.
- (c) Write an expression for  $\frac{d^2M}{dt^2}$  in terms of  $M$ . Use  $\frac{d^2M}{dt^2}$  to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of  $M(2)$ . Give a reason for your answer.
- (d) Use separation of variables to find an expression for  $M(t)$ , the particular solution to the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$  with initial condition  $M(0) = 5$ .

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

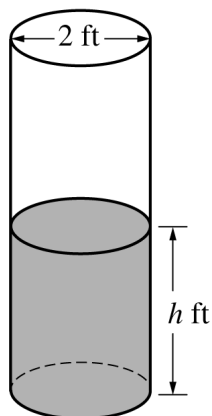
5. Let  $y = f(x)$  be the particular solution to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition

$f(1) = 4$ . It can be shown that  $f''(1) = 4$ .

- (a) Write the second-degree Taylor polynomial for  $f$  about  $x = 1$ . Use the Taylor polynomial to approximate  $f(2)$ .
- (b) Use Euler's method, starting at  $x = 1$  with two steps of equal size, to approximate  $f(2)$ . Show the work that leads to your answer.
- (c) Find the particular solution  $y = f(x)$  to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition  $f(1) = 4$ .

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height  $h$  of the water in the barrel with respect to time  $t$  is modeled by  $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$ , where  $h$  is measured in feet and  $t$  is measured in seconds. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)
- (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.
- (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.
- (c) At time  $t = 0$  seconds, the height of the water is 5 feet. Use separation of variables to find an expression for  $h$  in terms of  $t$ .
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4. At time  $t = 0$ , a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ( $^{\circ}\text{C}$ ) at time  $t = 0$ , and the internal temperature of the potato is greater than  $27^{\circ}\text{C}$  for all times  $t > 0$ . The internal temperature of the potato at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 91$ .
- (a) Write an equation for the line tangent to the graph of  $H$  at  $t = 0$ . Use this equation to approximate the internal temperature of the potato at time  $t = 3$ .
- (b) Use  $\frac{d^2H}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time  $t = 3$ .
- (c) For  $t < 10$ , an alternate model for the internal temperature of the potato at time  $t$  minutes is the function  $G$  that satisfies the differential equation  $\frac{dG}{dt} = -(G - 27)^{2/3}$ , where  $G(t)$  is measured in degrees Celsius and  $G(0) = 91$ . Find an expression for  $G(t)$ . Based on this model, what is the internal temperature of the potato at time  $t = 3$  ?
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