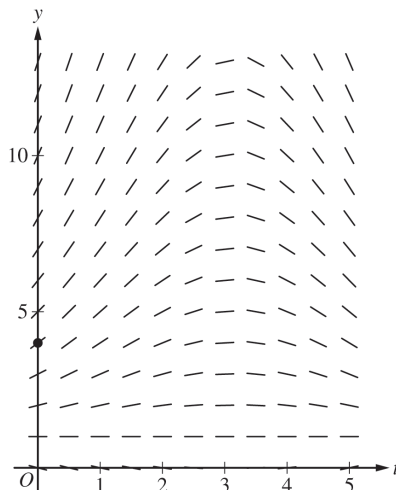


3. The depth of seawater at a location can be modeled by the function  $H$  that satisfies the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right), \text{ where } H(t) \text{ is measured in feet and } t \text{ is measured in hours after noon } (t = 0). \text{ It is}$$

known that  $H(0) = 4$ .

- (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve,  $y = H(t)$ , through the point  $(0, 4)$ .



- (b) For  $0 < t < 5$ , it can be shown that  $H(t) > 1$ . Find the value of  $t$ , for  $0 < t < 5$ , at which  $H$  has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.
- (c) Use separation of variables to find  $y = H(t)$ , the particular solution to the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right) \text{ with initial condition } H(0) = 4.$$

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing

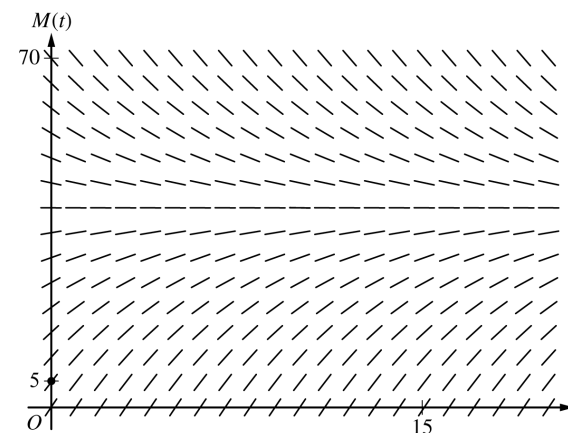
function  $M$  models the temperature of the milk at time  $t$ , where  $M(t)$  is measured in degrees Celsius ( $^{\circ}\text{C}$ )

and  $t$  is the number of minutes since the bottle was placed in the pan.  $M$  satisfies the differential equation

$$\frac{dM}{dt} = \frac{1}{4}(40 - M). \text{ At time } t = 0, \text{ the temperature of the milk is } 5^{\circ}\text{C}. \text{ It can be shown that } M(t) < 40 \text{ for all}$$

values of  $t$ .

- (a) A slope field for the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$  is shown. Sketch the solution curve through the point  $(0, 5)$ .

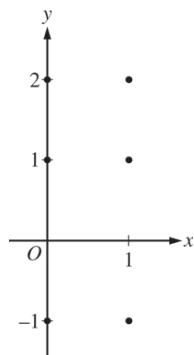


- (b) Use the line tangent to the graph of  $M$  at  $t = 0$  to approximate  $M(2)$ , the temperature of the milk at time  $t = 2$  minutes.
- (c) Write an expression for  $\frac{d^2M}{dt^2}$  in terms of  $M$ . Use  $\frac{d^2M}{dt^2}$  to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of  $M(2)$ . Give a reason for your answer.
- (d) Use separation of variables to find an expression for  $M(t)$ , the particular solution to the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$  with initial condition  $M(0) = 5$ .

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.

(d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.