

1. Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M , where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

Part A

Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \leq t \leq 10$. Show the work that leads to your answer, and indicate units of measure.

Part B

- Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t) dt$. Show the work that leads to your answer.
- Interpret the meaning of $\int_0^{30} M(t) dt$ in the context of the problem.

Part C

The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

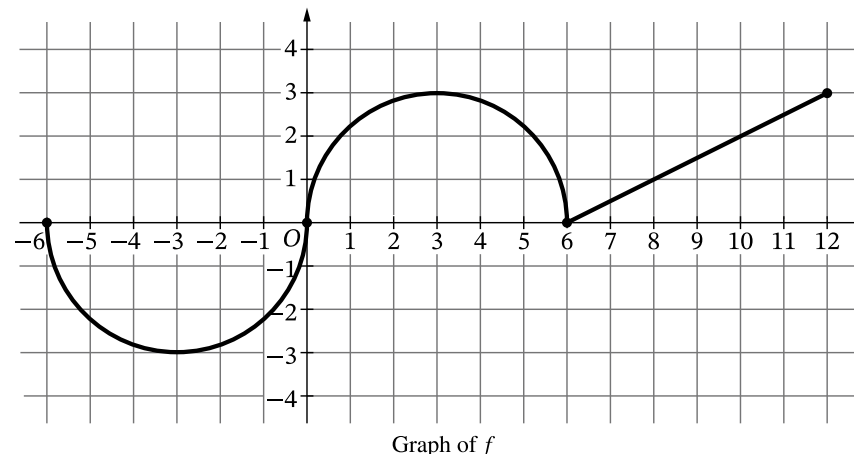
$$F(t) = \begin{cases} 0 & \text{for } 0 \leq t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \leq t \leq 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

Part D

On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.

4. The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.



Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

- Find $g'(8)$. Give a reason for your answer.
- Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
- Find $g(12)$ and $g(0)$. Label your answers.
- Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the

value of $\int_0^{12} C(t) \, dt$. Interpret the meaning of $\frac{1}{12} \int_0^{12} C(t) \, dt$ in the context of the problem.

(c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$,

where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$.

Show the setup for your calculations.

(d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$,

determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate.

Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. A particle moving along a curve in the xy -plane has position $(x(t), y(t))$ at time t seconds, where $x(t)$ and $y(t)$ are measured in centimeters. It is known that $x'(t) = 8t - t^2$ and $y'(t) = -t + \sqrt{t^{1.2} + 20}$. At time $t = 2$ seconds, the particle is at the point $(3, 6)$.

(a) Find the speed of the particle at time $t = 2$ seconds. Show the setup for your calculations.

(b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 2$. Show the setup for your calculations.

(c) Find the y -coordinate of the position of the particle at the time $t = 0$. Show the setup for your calculations.

(d) For $2 \leq t \leq 8$, the particle remains in the first quadrant. Find all times t in the interval $2 \leq t \leq 8$ when the particle is moving toward the x -axis. Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

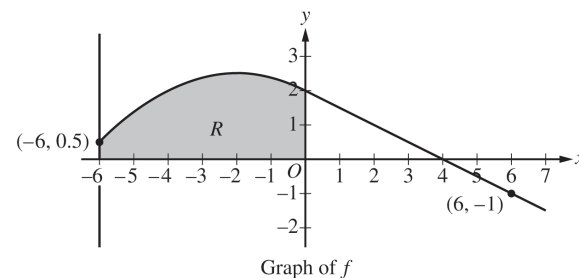
CALCULUS BC

SECTION II, Part B

Time—1 hour

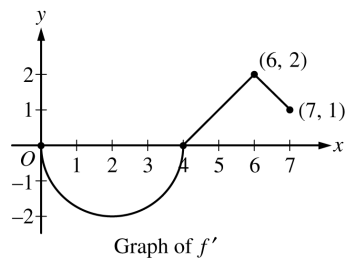
4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



4. The graph of the differentiable function f , shown for $-6 \leq x \leq 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \leq x \leq 7$. Let R be the region in the second quadrant bounded by the graph of f , the vertical line $x = -6$, and the x - and y -axes. Region R has area 12.
- (a) The function g is defined by $g(x) = \int_0^x f(t) \, dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.
- (b) For the function g defined in part (a), find all values of x in the interval $0 \leq x \leq 6$ at which the graph of g has a critical point. Give a reason for your answer.
- (c) The function h is defined by $h(x) = \int_{-6}^x f'(t) \, dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.
- Find $f(0)$ and $f(5)$.
 - Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.
 - Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.
 - For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

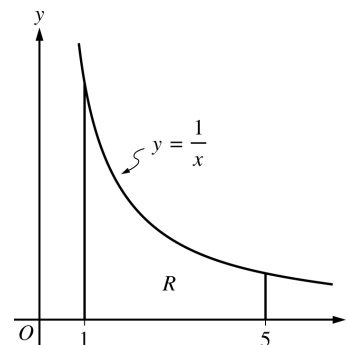


Figure 1

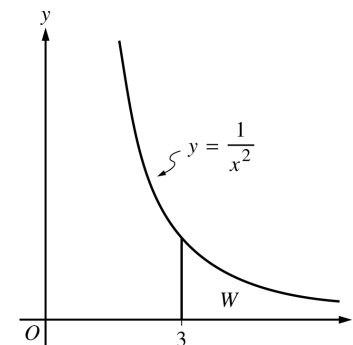


Figure 2

5. Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x -axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x -axis that lies to the right of the vertical line $x = 3$.
- Find the area of region R .
 - Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.
 - Find the volume of the solid generated when the unbounded region W is revolved about the x -axis.

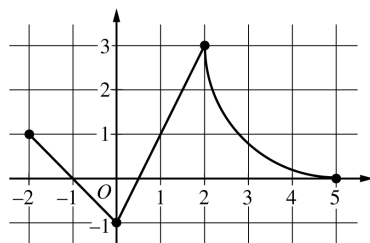
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS BC
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f , consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f .

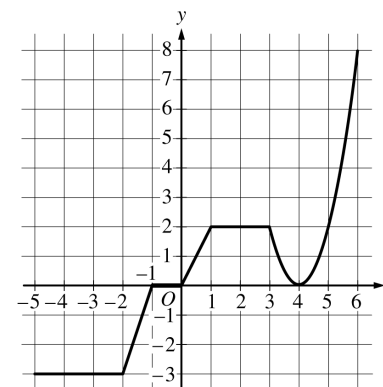
- (a) If $\int_{-6}^5 f(x) \, dx = 7$, find the value of $\int_{-6}^{-2} f(x) \, dx$. Show the work that leads to your answer.
- (b) Evaluate $\int_3^5 (2f'(x) + 4) \, dx$.
- (c) The function g is given by $g(x) = \int_{-2}^x f(t) \, dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.
- (d) Find $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.

CALCULUS BC
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g , the derivative of the function f , is shown above. The function g is piecewise linear for $-5 \leq x < 3$, and $g(x) = 2(x - 4)^2$ for $3 \leq x \leq 6$.
- (a) If $f(1) = 3$, what is the value of $f(-5)$?
- (b) Evaluate $\int_1^6 g(x) \, dx$.
- (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.
- (d) Find the x -coordinate of each point of inflection of the graph of f . Give a reason for your answer.

CALCULUS BC
SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

- A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A , where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

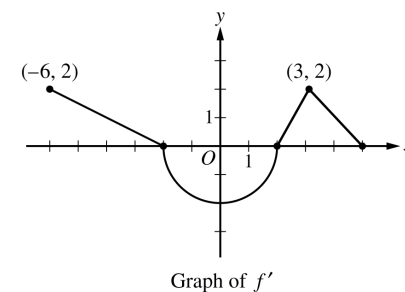
 - Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.
 - Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.
 - The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.
 - Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

CALCULUS BC
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

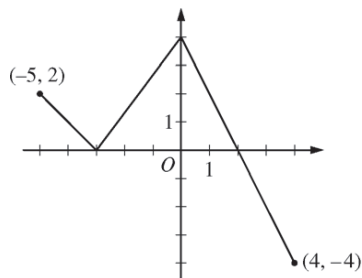


- The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.

 - Find the values of $f(-6)$ and $f(5)$.
 - On what intervals is f increasing? Justify your answer.
 - Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
 - For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.
- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.