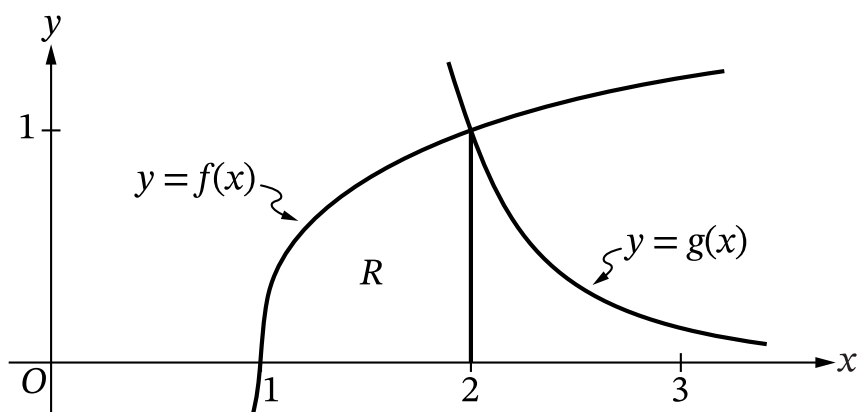


5. Let f be the function defined by $f(x) = \sqrt[3]{x-1}$, and let g be the function defined by $g(x) = e^{(-2x+4)}$. The graphs of f and g intersect at the point $(2, 1)$. Let R be the region bounded by the graph of f , the x -axis, and the vertical line $x = 2$, as shown in the figure.

**Part A**

Evaluate $\int_1^2 f(x) dx$. Show the work that leads to your answer.

Part B

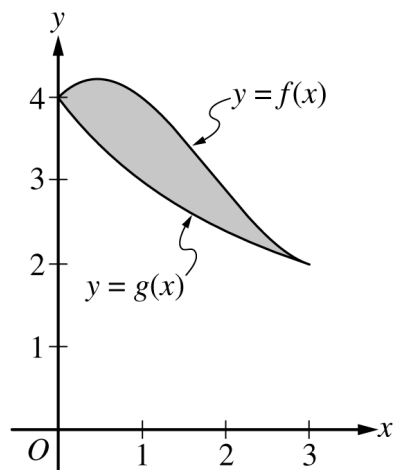
Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when region R is rotated about the x -axis.

Part C

Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R .

Part D

Evaluate $\int_2^\infty g(x) dx$. Show the work that leads to your answer.



5. The graphs of the functions f and g are shown in the figure for $0 \leq x \leq 3$. It is known that $g(x) = \frac{12}{3+x}$ for $x \geq 0$. The twice-differentiable function f , which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x) \, dx = 10$.
- (a) Find the area of the shaded region enclosed by the graphs of f and g .
- (b) Evaluate the improper integral $\int_0^\infty (g(x))^2 \, dx$, or show that the integral diverges.
- (c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x) \, dx$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

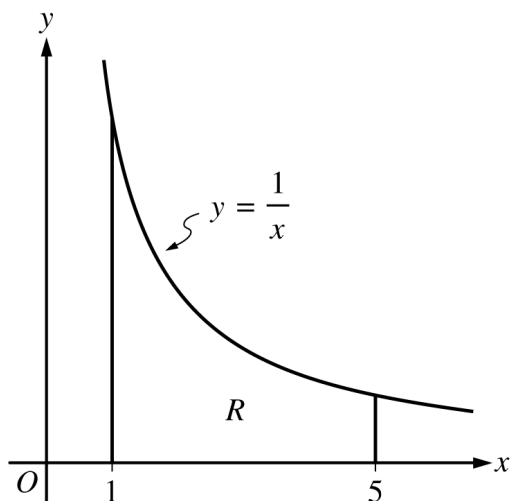


Figure 1

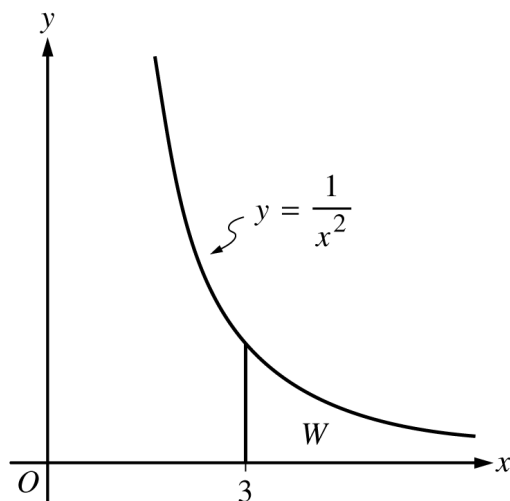


Figure 2

5. Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x -axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x -axis that lies to the right of the vertical line $x = 3$.

- (a) Find the area of region R .
- (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.
- (c) Find the volume of the solid generated when the unbounded region W is revolved about the x -axis.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Consider the family of functions $f(x) = \frac{1}{x^2 - 2x + k}$, where k is a constant.

(a) Find the value of k , for $k > 0$, such that the slope of the line tangent to the graph of f at $x = 0$ equals 6.

(b) For $k = -8$, find the value of $\int_0^1 f(x) \, dx$.

(c) For $k = 1$, find the value of $\int_0^2 f(x) \, dx$ or show that it diverges.

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \leq h \leq 30$ and is modeled by $f(h)$ for $h \geq 30$. The continuous function f is not explicitly given.
- (a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.
- (b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?
- (c) There is a function u such that $0 \leq f(h) \leq u(h)$ for all $h \geq 30$ and $\int_{30}^{\infty} u(h) \, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.
- (d) The boat is moving on the surface of the sea. At time $t \geq 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662 \sin(5t)$ and $y'(t) = 880 \cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \leq t \leq 1$.
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END OF PART A OF SECTION II

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.

- (a) Find the slope of the line tangent to the graph of f at $x = 3$.
- (b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) \, dx$ or show that the integral diverges.
- (d) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.
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