

Part B (BC): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	π	2π
$f'(x)$	5	6	0

The function f is twice differentiable for all x with $f(0) = 0$. Values of f' , the derivative of f , are given in the table for selected values of x .

	Model Solution	Scoring
(a)	For $x \geq 0$, the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2} dt$. Find the value of $h'(\pi)$. Show the work that leads to your answer.	
	$h'(x) = \sqrt{1 + (f'(x))^2}$	Fundamental Theorem of Calculus 1 point
	$h'(\pi) = \sqrt{1 + (f'(\pi))^2} = \sqrt{1 + 6^2} = \sqrt{37}$	Answer 1 point
Scoring notes:		
- A response of $\sqrt{1 + (f'(\pi))^2}$ earns the first point. - A response of $\sqrt{1 + 6^2}$ alone earns both points. - A response such as $h'(x) = \sqrt{1 + (f'(x))^2} = \sqrt{37}$, that equates a variable expression to a numeric value, earns at most 1 of the 2 points. - A response that equates $h'(x)$ or $h'(\pi)$ to a derivative of a constant, such as $h'(x) = \frac{d}{dx} \int_0^\pi \sqrt{1 + (f'(t))^2} dt$, earns at most 1 of the 2 points.		
Total for part (a)		2 points

- (b) What information does $\int_0^\pi \sqrt{1 + (f'(x))^2} \, dx$ provide about the graph of f ?

$\int_0^\pi \sqrt{1 + (f'(x))^2} \, dx$ is the arc length of the graph of f on $[0, \pi]$.	Arc length of f	1 point
	Interval $[0, \pi]$	1 point

Scoring notes:

- A response of “arc length” or “length” earns the first point. Such a response does not need to reference f . However, if the response references a different function, the response does not earn the first point and is eligible to earn the second point.
- A response referring to distance explicitly connected to the graph or f (or equivalent) earns the first point. For example, a response of “distance along the curve” or “distance traveled by a particle moving along f ” earns the first point and is eligible to earn the second point.
- A response referring to distance that is not explicitly connected to the graph of f does not earn the first point but is eligible to earn the second point. For example, a response of “distance” or “distance traveled” does not earn the first point but is eligible to earn the second point.
- To earn the second point a response must connect the interval $[0, \pi]$ to arc length, length, or distance.

Total for part (b) 2 points

- (c) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(2\pi)$. Show the computations that lead to your answer.

$f(\pi) \approx f(0) + \pi f'(0) = 0 + 5\pi = 5\pi$	Euler's method	1 point
$f(2\pi) \approx f(\pi) + \pi f'(\pi)$		
$\approx 5\pi + 6\pi = 11\pi$	Answer	1 point

Scoring notes:

- To earn the first point a response must demonstrate two Euler's steps, with use of the correct expression for $\frac{dy}{dx}$, and at most one error. If there is an error, the second point is not earned.
- In order to earn the first point, a response that presents a single error in computing the approximation of $f(\pi)$ must import the incorrect value in computing the approximation of $f(2\pi)$.
- The two Euler's steps may be explicit expressions or may be presented in a table. For example:

x	y	$\frac{dy}{dx} \cdot \Delta x$ (or $\frac{dy}{dx} \cdot \pi$)
0	0	5π
π	5π	6π
2π	11π	

- In the presence of a correct answer, a table does not need to be labeled in order to earn both points. In the presence of no answer or an incorrect answer, such a table must be correctly labeled in order to earn the first point.
- Both points are earned for $5\pi + 6\pi$.
- The response may report the final answer as $(2\pi, 11\pi)$.

Total for part (c) 2 points

- (d) Find $\int (t + 5)\cos\left(\frac{t}{4}\right) dt$. Show the work that leads to your answer.

$u = t + 5 \quad dv = \cos\left(\frac{t}{4}\right) dt$ $du = dt \quad v = 4\sin\left(\frac{t}{4}\right)$	u and dv	1 point
$\int (t + 5)\cos\left(\frac{t}{4}\right) dt = 4(t + 5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$	$uv - \int v du$	1 point
$= 4(t + 5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$	Answer	1 point

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $4(t + 5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by columns (labeled or unlabeled) that begin with $t + 5$ and $\cos\left(\frac{t}{4}\right)$. The second point is earned for $4(t + 5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The third point is earned only for an expression mathematically equivalent to $4(t + 5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$ (such as $4t\sin\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$) in the presence of correct supporting work.
- To earn the third point a response must have a final answer that includes a constant of integration.
- Alternate solution:

$$\int (t + 5)\cos\left(\frac{t}{4}\right) dt = \int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$u = t \quad dv = \cos\left(\frac{t}{4}\right) dt$$

$$du = dt \quad v = 4\sin\left(\frac{t}{4}\right)$$

$$\int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt = 4t\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$= 4t\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + C$$

- A response can earn the first and second points for correctly applying integration by parts to $\int t\cos\left(\frac{t}{4}\right) dt$. The tabular method may be used to show integration by parts. The third point is earned for the correct answer.

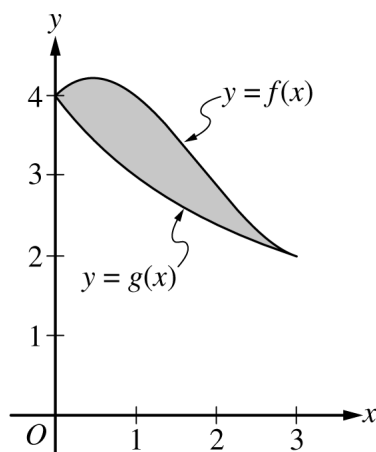
Total for part (d) 3 points

Total for question 5 9 points

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The graphs of the functions f and g are shown in the figure for $0 \leq x \leq 3$. It is known that $g(x) = \frac{12}{3+x}$ for $x \geq 0$. The twice-differentiable function f , which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x) \, dx = 10$.

Model Solution**Scoring**

- (a) Find the area of the shaded region enclosed by the graphs of f and g .

$\text{Area} = \int_0^3 (f(x) - g(x)) \, dx = \int_0^3 f(x) \, dx - \int_0^3 g(x) \, dx$	Integrand	1 point
$= 10 - \int_0^3 \frac{12}{3+x} \, dx = 10 - 12[\ln 3+x]_0^3$	Antiderivative of $g(x)$	1 point
$= 10 - 12(\ln 6 - \ln 3) = 10 - 12(\ln 2)$	Answer	1 point

Scoring notes:

- The first point is earned for any of the integrands $f(x) - g(x)$, $g(x) - f(x)$, $|f(x) - g(x)|$, or $|g(x) - f(x)|$ in any definite integral. If the limits are incorrect, the response does not earn the third point.

- The first point is earned with an implied integrand for f and explicit integrand for g , such as $10 - \int_0^3 g(x) dx$.
- The second point is earned for finding $a \int \frac{dx}{3+x} = a \cdot \ln|3+x|$ or $a \cdot \ln(3+x)$.
- A response is eligible for the third point only if it has earned the first 2 points. The third point is earned only for the correct answer. The answer does not need to be simplified; however, if simplification is attempted, it must be correct.
- A response is not eligible for the third point with incorrect limits of integration for u -substitution, for example, $\int_0^3 \frac{12}{3+x} dx = \int_0^3 \frac{12}{u} du = 12[\ln(x+3)]_0^3$.
- A response with incorrect communication, such as “Area = $\int_0^3 (g(x) - f(x)) dx = 10 - 12(\ln 2)$,” does not earn the third point. However, a response of “ $\int_0^3 (g(x) - f(x)) dx = 12(\ln 2) - 10$, so the area is $10 - 12(\ln 2)$ ” earns all 3 points.

Total for part (a) 3 points

- (b) Evaluate the improper integral $\int_0^\infty (g(x))^2 dx$, or show that the integral diverges.

$\int_0^\infty (g(x))^2 dx = \lim_{b \rightarrow \infty} \int_0^b \frac{144}{(3+x)^2} dx$	Limit notation	1 point
$= \lim_{b \rightarrow \infty} \left(-\frac{144}{(3+x)} \Big _0^b \right)$	Antiderivative	1 point
$= \lim_{b \rightarrow \infty} \left(-\frac{144}{3+b} + \frac{144}{3} \right) = 48$	Answer	1 point

Scoring notes:

- To earn the first point a response must correctly use limit notation throughout the problem and not include arithmetic with infinity, for example, $\left[-\frac{144}{3+x} \right]_0^\infty$ or $-\frac{144}{3+\infty} + 48$.
- The second point can be earned by finding an antiderivative of the form $-\frac{a}{(3+x)}$ for $a > 0$, from an indefinite or improper integral, with or without correct limit notation. If $a \neq 144$, the response does not earn the third point.
- The third point is earned only for an answer of 48 (or equivalent).
- A response is not eligible for the third point with incorrect limits of integration for u -substitution, for example, $\lim_{b \rightarrow \infty} \int_0^b \frac{144}{u^2} du = \lim_{b \rightarrow \infty} \left[-\frac{144}{3+x} \right]_0^b$.

Total for part (b) 3 points

- (c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x) \, dx$.

Using integration by parts, $u = x \quad dv = f'(x) \, dx$ $du = dx \quad v = f(x)$	u and dv	1 point
$\int h(x) \, dx = \int x \cdot f'(x) \, dx = x \cdot f(x) - \int f(x) \, dx$	$\int h(x) \, dx$ $= x \cdot f(x) - \int f(x) \, dx$	1 point
$\int_0^3 h(x) \, dx = \int_0^3 x \cdot f'(x) \, dx = x \cdot f(x) \Big _0^3 - \int_0^3 f(x) \, dx$ $= (3 \cdot f(3) - 0 \cdot f(0)) - 10 = 3 \cdot 2 - 0 - 10 = -4$	Answer	1 point

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $x \cdot f(x) - \int f(x) \, dx$ or $x \cdot f(x) \Big|_0^3 - 10$.
- Limits of integration may be present, omitted, or partially present in the work for the first and second points.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by having columns (labeled or unlabeled) that begin with x and $f'(x)$. The second point is earned for $x \cdot f(x) - \int f(x) \, dx$.
- The third point is earned only for the correct answer and can only be earned if the first 2 points were earned.

Total for part (c) 3 points

Total for question 5 9 points