

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

Model Solution	Scoring
(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).	
The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t) dt$.	Answer 1 point
Scoring notes: - The response must be a definite integral with correct lower and upper limits to earn this point. - Because $A(t) = A(t)$ for $1 \leq t \leq 5$, a response of $\int_1^5 450\sqrt{\sin(0.62t)} dt$ or $\int_1^5 A(t) dt$ earns the point. - A response missing dt or using dx is eligible to earn the point. - A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t) dt$.	
Total for part (a) 1 point	

- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

Average = $\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966$	Uses average value formula: $\frac{1}{b-a} \int_a^b A(t) dt$ 1 point
The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.	Answer 1 point

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points:
 $\int_1^5 A(t) dt = 1502.147865$, so the average value is 375.536966 .
- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t) dt = 1502.147865 = 375.536966$.
- The answer must be correct to three decimal places. For example,
 $\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966 \approx 376$ earns only the first point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4} \int_1^5 A(t) dt = 79.416068$.
- Special case: $\frac{1}{5} \int_1^5 A(t) dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$A'(1) = 148.947272$	Considers $A'(1)$	1 point
Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.	Answer with reason	1 point

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
 - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
 - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$N'(t) = A(t) - 400 = 0$ $\Rightarrow A(t) = 400 \Rightarrow t = 1.469372, t = 3.597713$ $a = 1.469372$ $b = 3.597713$	Considers $N'(t) = 0$	1 point								
	$t = a$ and $t = b$	1 point								
<table><tr><td>t</td><td>$N(t) = \int_a^t (A(x) - 400) dx$</td></tr><tr><td>$a$</td><td>0</td></tr><tr><td>b</td><td>71.254129</td></tr><tr><td>4</td><td>62.338346</td></tr></table>	t	$N(t) = \int_a^t (A(x) - 400) dx$	a	0	b	71.254129	4	62.338346	Answer	1 point
t	$N(t) = \int_a^t (A(x) - 400) dx$									
a	0									
b	71.254129									
4	62.338346									
	Justification	1 point								

The greatest number of vehicles in line is 71.

Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of “ $A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$ ” will earn the first 2 points. A response of “ $A(t) \geq 400$ ” along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of “ $A(t) \geq 400$ ” by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254*** only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:
For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.
Thus, $N(t) = \int_a^t (A(x) - 400) dx$ is greatest at $t = b$.
 $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Total for part (d) 4 points**Total for question 1 9 points**

2018

AP Calculus BC

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) \, dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) \, dt = 20 + \int_0^{300} r(t) \, dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by $20 + \int_0^t r(x) \, dx - 0.7t$.

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

2018 SCORING GUIDELINES

Question 2

(a) $p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$$

(b) $\int_0^{30} 3p(h) \, dh = 1675.414936$

There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(c) $\int_{30}^K 3f(h) \, dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters.

The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h) \, dh + \int_{30}^K 3f(h) \, dh$.

Because $0 \leq f(h) \leq u(h)$ for all $h \geq 30$,

$$3 \int_{30}^K f(h) \, dh \leq 3 \int_{30}^K u(h) \, dh \leq 3 \int_{30}^{\infty} u(h) \, dh = 3 \cdot 105 = 315.$$

The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \leq 2000$ million.

$$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$$

(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2} \, dt = 757.455862$

The total distance traveled by the boat over the time interval $0 \leq t \leq 1$ is 757.456 (or 757.455) meters.

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$$

2018 SCORING GUIDELINES

Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx$
 $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\int_1^6 g(x) \, dx = \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx$
 $= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx$
 $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$

$$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f''(x) = g(x)$ is increasing on those intervals.

$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$$

(d) The graph of f has a point of inflection at $x = 4$ because $f''(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

2018 SCORING GUIDELINES

Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

(c) The average height of the tree over the time interval $2 \leq t \leq 10$ is given by $\frac{1}{10 - 2} \int_2^{10} H(t) \, dt$.

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

2 : $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

3 : $\begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

$$(a) \quad \text{Area} = \frac{1}{2} \int_{\pi/3}^{5\pi/3} (4^2 - (3 + 2\cos \theta)^2) \, d\theta$$

$$(b) \quad \frac{dr}{d\theta} = -2\sin \theta \Rightarrow \left. \frac{dr}{d\theta} \right|_{\theta=\pi/2} = -2$$

$$r\left(\frac{\pi}{2}\right) = 3 + 2\cos\left(\frac{\pi}{2}\right) = 3$$

$$y = r\sin \theta \Rightarrow \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin \theta + r\cos \theta$$

$$x = r\cos \theta \Rightarrow \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos \theta - r\sin \theta$$

$$\left. \frac{dy}{dx} \right|_{\theta=\pi/2} = \frac{dy/d\theta}{dx/d\theta} \Big|_{\theta=\pi/2} = \frac{-2\sin\left(\frac{\pi}{2}\right) + 3\cos\left(\frac{\pi}{2}\right)}{-2\cos\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos \theta$

at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

— OR —

$$y = r\sin \theta = (3 + 2\cos \theta)\sin \theta \Rightarrow \frac{dy}{d\theta} = 3\cos \theta + 2\cos^2 \theta - 2\sin^2 \theta$$

$$x = r\cos \theta = (3 + 2\cos \theta)\cos \theta \Rightarrow \frac{dx}{d\theta} = -3\sin \theta - 4\sin \theta \cos \theta$$

$$\left. \frac{dy}{dx} \right|_{\theta=\pi/2} = \frac{dy/d\theta}{dx/d\theta} \Big|_{\theta=\pi/2} = \frac{3\cos\left(\frac{\pi}{2}\right) + 2\cos^2\left(\frac{\pi}{2}\right) - 2\sin^2\left(\frac{\pi}{2}\right)}{-3\sin\left(\frac{\pi}{2}\right) - 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos \theta$

at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

$$(c) \quad \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = -2\sin \theta \cdot \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{dr}{dt} \cdot \frac{1}{-2\sin \theta}$$

$$\left. \frac{d\theta}{dt} \right|_{\theta=\pi/3} = 3 \cdot \frac{1}{-2\sin\left(\frac{\pi}{3}\right)} = \frac{3}{-\sqrt{3}} = -\sqrt{3} \text{ radians per second}$$

3 : $\begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \end{cases}$

3 : $\begin{cases} 1 : \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin \theta + r\cos \theta \\ \text{or } \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos \theta - r\sin \theta \\ 1 : \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin \theta + r\cos \theta}{\frac{dr}{d\theta}\cos \theta - r\sin \theta} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} \\ 1 : \frac{d\theta}{dt} = \frac{dr}{dt} \cdot \frac{1}{-2\sin \theta} \\ 1 : \text{answer with units} \end{cases}$

2018 SCORING GUIDELINES

Question 6

(a) The first four nonzero terms are $\frac{x^2}{3} - \frac{x^3}{2 \cdot 3^2} + \frac{x^4}{3 \cdot 3^3} - \frac{x^5}{4 \cdot 3^4}$.

The general term is $(-1)^{n+1} \frac{x^{n+1}}{n \cdot 3^n}$.

(b)
$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+2} x^{n+2}}{(n+1)(3^{n+1})}}{\frac{(-1)^{n+1} x^{n+1}}{n \cdot 3^n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{-x}{3} \cdot \frac{n}{(n+1)} \right| = \left| \frac{x}{3} \right|$$

$\left| \frac{x}{3} \right| < 1$ for $|x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

— OR —

The radius of convergence of the Maclaurin series for $\ln(1+x)$ is 1, so the series for $f(x) = x \ln\left(1 + \frac{x}{3}\right)$ converges absolutely for $\left|\frac{x}{3}\right| < 1$.

$\left|\frac{x}{3}\right| < 1 \Rightarrow |x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

When $x = -3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-3)^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{3}{n}$, which

diverges by comparison to the harmonic series.

When $x = 3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n}$, which

converges by the alternating series test.

The interval of convergence of the Maclaurin series for f is $-3 < x \leq 3$.

(c) By the alternating series error bound, an upper bound for $|P_4(2) - f(2)|$ is the magnitude of the next term of the alternating series.

$|P_4(2) - f(2)| < \left| -\frac{2^5}{4 \cdot 3^4} \right| = \frac{8}{81}$

2 : $\begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

5 : $\begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

— OR —

5 : $\begin{cases} 1 : \text{radius for } \ln(1+x) \text{ series} \\ 1 : \text{substitutes } \frac{x}{3} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

2 : $\begin{cases} 1 : \text{uses fifth-degree term as error bound} \\ 1 : \text{answer} \end{cases}$