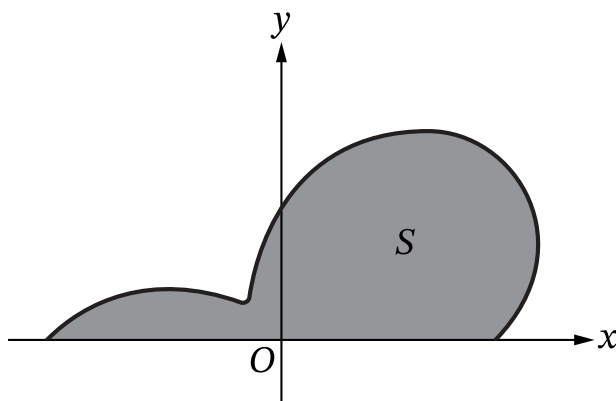


2. Let S be the shaded region bounded by the graph of the polar curve $r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $0 \leq \theta \leq \pi$, as shown in the figure.



It can be shown that $r'(\theta) = 4 \cos(2\theta) - 2 \sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

Part A

Find the area of region S . Show the setup for your calculations.

Part B

There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

Part C

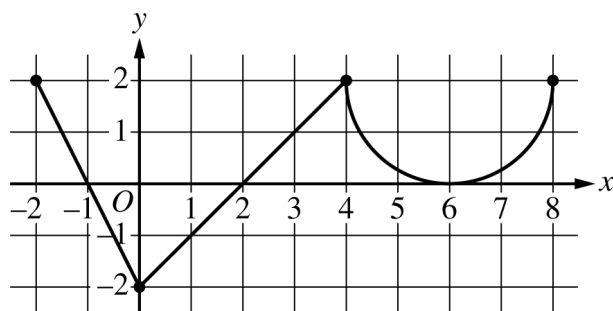
- Find the value of θ in the interval $0 < \theta < \frac{\pi}{2}$ at which r has a critical point.
- Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for r .

Part D

Find the average distance from the origin to a point on the polar curve

$r(\theta) = 3 + 2 \sin(2\theta) + \cos(2\theta)$ for $\frac{\pi}{2} \leq \theta \leq \pi$. Show the setup for your calculations.

END OF PART A

Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.
- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.
- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.
- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.
- (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .
- (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.
- (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right)$. Show the work that leads to your answer.
- (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.