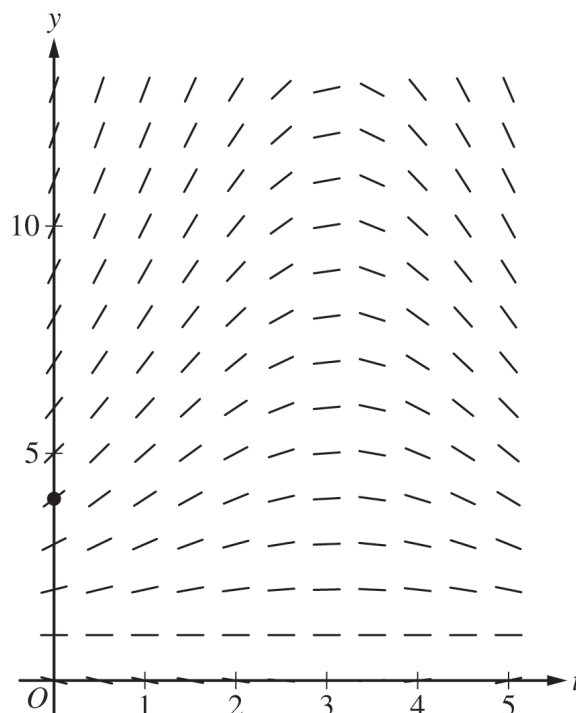


3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right), \text{ where } H(t) \text{ is measured in feet and } t \text{ is measured in hours after noon } (t = 0). \text{ It is}$$

known that $H(0) = 4$.

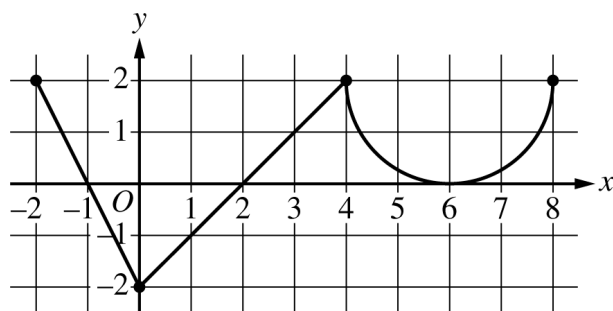
- (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



- (b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t , for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.
- (c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right) \text{ with initial condition } H(0) = 4.$$

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f'

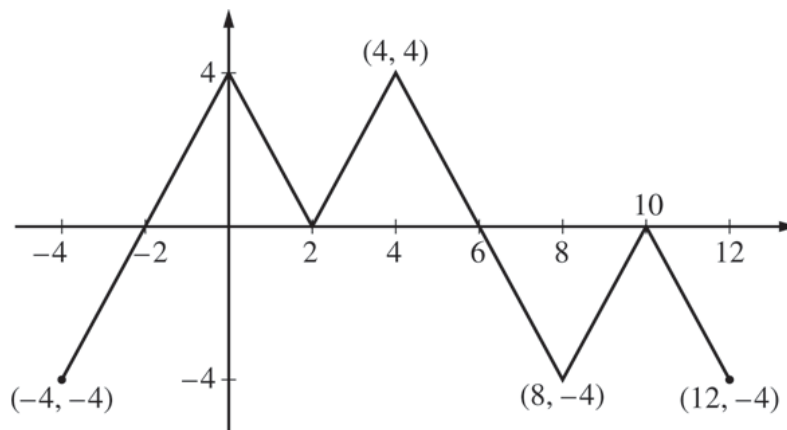
4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.
- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.
- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.
- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.
- (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.
- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.
- (a) Find the slope of the line tangent to the graph of f at $x = 3$.
- (b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^\infty f(x) dx$ or show that the integral diverges.
- (d) Determine whether the series $\sum_{n=5}^\infty \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

CALCULUS BC
SECTION II, Part B**Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**Graph of f

3. The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t) dt$.

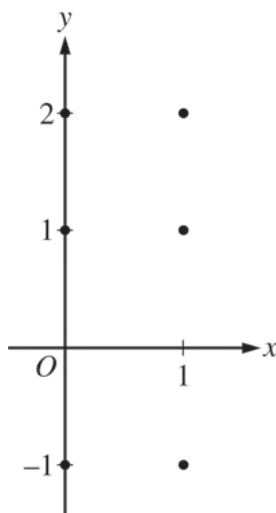
- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
 - (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
 - (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
 - (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.
-

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

- (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .
 - (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.
 - (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right)$. Show the work that leads to your answer.
 - (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.
-

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.
-

5. Consider the function $f(x) = \frac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}.$$

- (a) Let $k = 3$, so that $f(x) = \frac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x -coordinate is 4.
- (b) Let $k = 4$, so that $f(x) = \frac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.
- (c) Find the value of k for which f has a critical point at $x = -5$.
- (d) Let $k = 6$, so that $f(x) = \frac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f .
Find $\int f(x) dx$.
-