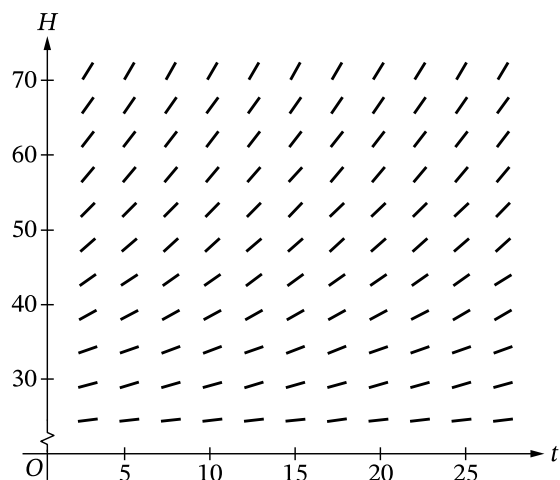


3. A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

Part A

Explain why the following could not be a slope field for the differential equation

$$\frac{dH}{dt} = -\frac{1}{15}(H - 20).$$

**Part B**

Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

Part C

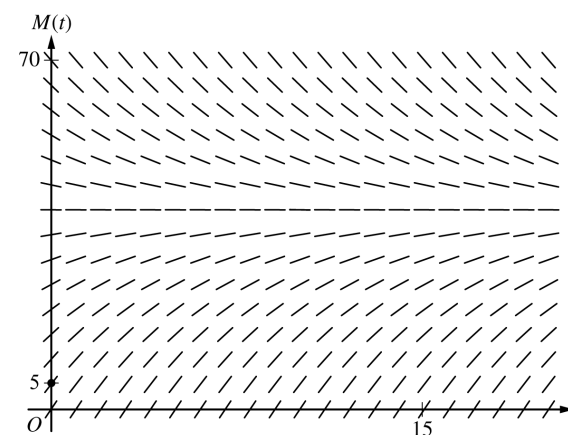
It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

Part D

Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t , where $M(t)$ is measured in degrees Celsius ($^{\circ}\text{C}$) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C . It can be shown that $M(t) < 40$ for all values of t .

- (a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



- (b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.
- (c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M . Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.
- (d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ($^{\circ}\text{C}$) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

- (a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.
- (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.
- (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

5. Consider the function $f(x) = \frac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}.$$

- (a) Let $k = 3$, so that $f(x) = \frac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x -coordinate is 4.
- (b) Let $k = 4$, so that $f(x) = \frac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.
- (c) Find the value of k for which f has a critical point at $x = -5$.
- (d) Let $k = 6$, so that $f(x) = \frac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f . Find $\int f(x) dx$.

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.
 - (a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
 - (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
 - (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
 - (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.