

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r , where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \leq t \leq 12$.

- (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \leq t \leq 10$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $0 \leq t \leq 3$, for which $r'(t) = -6$? Justify your answer.
- (c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of

$$\int_0^{12} r'(t) dt.$$

- (d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H , where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.
- (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.
- (b) Explain why there must be at least one time t , for $2 < t < 10$, such that $H'(t) = 2$.
- (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \leq t \leq 10$.
- (d) The height of the tree, in meters, can also be modeled by the function G , given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?
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