

5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (3 - x)y^2$ with initial condition $f(1) = -1$.

- A. Find $f''(1)$, the value of $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. Show the work that leads to your answer.
- B. Write the second-degree Taylor polynomial for f about $x = 1$.
- C. The second-degree Taylor polynomial for f about $x = 1$ is used to approximate $f(1.1)$. Given that $|f'''(x)| \leq 60$ for all x in the interval $1 \leq x \leq 1.1$, use the Lagrange error bound to show that this approximation differs from $f(1.1)$ by at most 0.01.
- D. Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the work that leads to your answer.

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6. The function f has derivatives of all orders for all real numbers. It is known that $f(0) = 2$, $f'(0) = 3$,

$$f''(x) = -f(x^2), \text{ and } f'''(x) = -2x \cdot f'(x^2).$$

(a) Find $f^{(4)}(x)$, the fourth derivative of f with respect to x . Write the fourth-degree Taylor polynomial for f about $x = 0$. Show the work that leads to your answer.

(b) The fourth-degree Taylor polynomial for f about $x = 0$ is used to approximate $f(0.1)$. Given that

$$|f^{(5)}(x)| \leq 15 \text{ for } 0 \leq x \leq 0.5, \text{ use the Lagrange error bound to show that this approximation is within } \frac{1}{10^5} \text{ of the exact value of } f(0.1).$$

(c) Let g be the function such that $g(0) = 4$ and $g'(x) = e^x f(x)$. Write the second-degree Taylor polynomial for g about $x = 0$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

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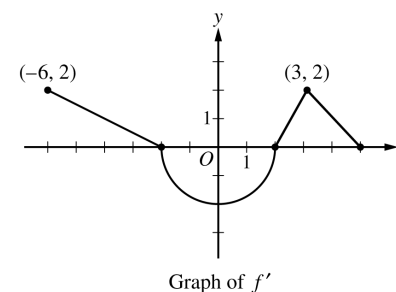
CALCULUS BC

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.

- (a) Find the values of $f(-6)$ and $f(5)$.
- (b) On what intervals is f increasing? Justify your answer.
- (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
- (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

- (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .
- (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.
- (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x + 1)^2} \right)$. Show the work that leads to your answer.
- (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.