

AP[®] Calculus BC

Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

3 : $\begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$

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Question 2

(a) $\frac{1}{2} \int_0^{\sqrt{\pi}} (r(\theta))^2 d\theta = 3.534292$

The area of S is 3.534.

(b) $\frac{1}{\sqrt{\pi}-0} \int_0^{\sqrt{\pi}} r(\theta) d\theta = 1.579933$

The average distance from the origin to a point on the curve $r = r(\theta)$ for $0 \leq \theta \leq \sqrt{\pi}$ is 1.580 (or 1.579).

(c) $\tan \theta = \frac{y}{x} = m \Rightarrow \theta = \tan^{-1} m$

$$\frac{1}{2} \int_0^{\tan^{-1} m} (r(\theta))^2 d\theta = \frac{1}{2} \left(\frac{1}{2} \int_0^{\sqrt{\pi}} (r(\theta))^2 d\theta \right)$$

(d) As $k \rightarrow \infty$, the circle $r = k \cos \theta$ grows to enclose all points to the right of the y -axis.

$$\begin{aligned} \lim_{k \rightarrow \infty} A(k) &= \frac{1}{2} \int_0^{\pi/2} (r(\theta))^2 d\theta \\ &= \frac{1}{2} \int_0^{\pi/2} (3\sqrt{\theta} \sin(\theta^2))^2 d\theta = 3.324 \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{equates polar areas} \\ 1 : \text{inverse trigonometric function} \\ \text{applied to } m \text{ as limit of} \\ \text{integration} \\ 1 : \text{equation} \end{cases}$

2 : $\begin{cases} 1 : \text{limits of integration} \\ 1 : \text{answer with integral} \end{cases}$

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Question 3

(a) $\int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx$

$$\Rightarrow 7 = \int_{-6}^{-2} f(x) dx + 2 + \left(9 - \frac{9\pi}{4}\right)$$

$$\Rightarrow \int_{-6}^{-2} f(x) dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$$

(b) $\begin{aligned} \int_3^5 (2f'(x) + 4) dx &= 2 \int_3^5 f'(x) dx + \int_3^5 4 dx \\ &= 2(f(5) - f(3)) + 4(5 - 3) \\ &= 2(0 - (3 - \sqrt{5})) + 8 \\ &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5} \end{aligned}$

— OR —

$$\begin{aligned} \int_3^5 (2f'(x) + 4) dx &= [2f(x) + 4x]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

(c) $g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

(d) $\begin{aligned} \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\ &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}} \end{aligned}$

3 : $\begin{cases} 1 : \int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ 1 : \int_{-2}^5 f(x) dx \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$

1 : answer

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Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$

$$\left. \frac{dV}{dt} \right|_{h=4} = \pi \left. \frac{dh}{dt} \right|_{h=4} = \pi \left(-\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5} \text{ cubic feet per second}$$

2 : $\begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$

(b) $\frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10} \sqrt{h} \right) = \frac{1}{200}$

Because $\frac{d^2 h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

3 : $\begin{cases} 1 : \frac{d}{dh} \left(-\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$

$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10} dt$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$h(t) = \left(-\frac{1}{20}t + \sqrt{5} \right)^2$$

4 : $\begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

2019 SCORING GUIDELINES

Question 5

(a) $f'(x) = \frac{-(2x-2)}{(x^2-2x+k)^2}$

$$f'(0) = \frac{2}{k^2} = 6 \Rightarrow k^2 = \frac{1}{3} \Rightarrow k = \frac{1}{\sqrt{3}}$$

3 : $\begin{cases} 1 : \text{denominator of } f'(x) \\ 1 : f'(x) \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{x^2-2x-8} = \frac{1}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$

$$\Rightarrow 1 = A(x+2) + B(x-4)$$

$$\Rightarrow A = \frac{1}{6}, B = -\frac{1}{6}$$

3 : $\begin{cases} 1 : \text{partial fraction} \\ \text{decomposition} \\ 1 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \int_0^1 f(x) dx &= \int_0^1 \left(\frac{1}{6} \frac{1}{x-4} - \frac{1}{6} \frac{1}{x+2} \right) dx \\ &= \left[\frac{1}{6} \ln|x-4| - \frac{1}{6} \ln|x+2| \right]_{x=0}^{x=1} \\ &= \left(\frac{1}{6} \ln 3 - \frac{1}{6} \ln 3 \right) - \left(\frac{1}{6} \ln 4 - \frac{1}{6} \ln 2 \right) = -\frac{1}{6} \ln 2 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int_0^2 \frac{1}{x^2-2x+1} dx &= \int_0^2 \frac{1}{(x-1)^2} dx = \int_0^1 \frac{1}{(x-1)^2} dx + \int_1^2 \frac{1}{(x-1)^2} dx \\ &= \lim_{b \rightarrow 1^-} \int_0^b \frac{1}{(x-1)^2} dx + \lim_{b \rightarrow 1^+} \int_b^2 \frac{1}{(x-1)^2} dx \\ &= \lim_{b \rightarrow 1^-} \left(-\frac{1}{x-1} \Big|_{x=0}^{x=b} \right) + \lim_{b \rightarrow 1^+} \left(-\frac{1}{x-1} \Big|_{x=b}^{x=2} \right) \\ &= \lim_{b \rightarrow 1^-} \left(-\frac{1}{b-1} - 1 \right) + \lim_{b \rightarrow 1^+} \left(-1 + \frac{1}{b-1} \right) \end{aligned}$$

Because $\lim_{b \rightarrow 1^-} \left(-\frac{1}{b-1} \right)$ does not exist, the integral diverges.

3 : $\begin{cases} 1 : \text{improper integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer with reason} \end{cases}$

2019 SCORING GUIDELINES

Question 6

- (a)
- $f(0) = 3$
- and
- $f'(0) = -2$

The third-degree Taylor polynomial for f about $x = 0$ is

$$3 - 2x + \frac{3}{2!}x^2 + \frac{-2}{3!}x^3 = 3 - 2x + \frac{3}{2}x^2 - \frac{23}{12}x^3.$$

- (b) The first three nonzero terms of the Maclaurin series for
- e^x
- are

$$1 + x + \frac{1}{2!}x^2.$$

The second-degree Taylor polynomial for $e^x f(x)$ about $x = 0$ is

$$\begin{aligned} 3\left(1 + x + \frac{1}{2!}x^2\right) - 2x(1 + x) + \frac{3}{2}x^2(1) \\ = 3 + (3 - 2)x + \left(\frac{3}{2} - 2 + \frac{3}{2}\right)x^2 \\ = 3 + x + x^2. \end{aligned}$$

- (c)
- $h(1) = \int_0^1 f(t) dt$

$$\begin{aligned} &\approx \int_0^1 \left(3 - 2t + \frac{3}{2}t^2 - \frac{23}{12}t^3\right) dt \\ &= \left[3t - t^2 + \frac{1}{2}t^3 - \frac{23}{48}t^4\right]_{t=0}^{t=1} \\ &= 3 - 1 + \frac{1}{2} - \frac{23}{48} = \frac{97}{48} \end{aligned}$$

- (d) The alternating series error bound is the absolute value of the first omitted term of the series for
- $h(1)$
- .

$$\int_0^1 \left(\frac{54}{4!}t^4\right) dt = \left[\frac{9}{20}t^5\right]_{t=0}^{t=1} = \frac{9}{20}$$

$$\text{Error} \leq \left|\frac{9}{20}\right| = 0.45$$

$$2 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \end{cases}$$

$$2 : \begin{cases} 1 : \text{three terms for } e^x \\ 1 : \text{three terms for } e^x f(x) \end{cases}$$

$$2 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : \text{uses fourth-degree term} \\ \quad \text{of Maclaurin series for } f \\ 1 : \text{uses first omitted term} \\ \quad \text{of series for } h(1) \\ 1 : \text{error bound} \end{cases}$$

2017

AP Calculus BC

Scoring Guidelines

2017 SCORING GUIDELINES

Question 1

	1 : units in parts (a), (c), and (d)
(a) Volume = $\int_0^{10} A(h) \, dh$ $\approx (2 - 0) \cdot A(0) + (5 - 2) \cdot A(2) + (10 - 5) \cdot A(5)$ $= 2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5$ $= 176.3 \text{ cubic feet}$	2 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$
(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.	1 : overestimate with reason
(c) $\int_0^{10} f(h) \, dh = 101.325338$ The volume is 101.325 cubic feet.	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) Using the model, $V(h) = \int_0^h f(x) \, dx$. $\left. \frac{dV}{dt} \right _{h=5} = \left[\frac{dV}{dh} \cdot \frac{dh}{dt} \right]_{h=5}$ $= \left[f(h) \cdot \frac{dh}{dt} \right]_{h=5}$ $= f(5) \cdot 0.26 = 1.694419$ When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.	3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2017 SCORING GUIDELINES

Question 2

(a) $\frac{1}{2} \int_0^{\pi/2} (f(\theta))^2 \, d\theta = 0.648414$ The area of R is 0.648.	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $\int_0^k ((g(\theta))^2 - (f(\theta))^2) \, d\theta = \frac{1}{2} \int_0^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) \, d\theta$ — OR — $\int_0^k ((g(\theta))^2 - (f(\theta))^2) \, d\theta = \int_k^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) \, d\theta$	2 : $\begin{cases} 1 : \text{integral expression} \\ \text{for one region} \\ 1 : \text{equation} \end{cases}$
(c) $w(\theta) = g(\theta) - f(\theta)$ $w_A = \frac{\int_0^{\pi/2} w(\theta) \, d\theta}{\frac{\pi}{2} - 0} = 0.485446$ The average value of $w(\theta)$ on the interval $\left[0, \frac{\pi}{2}\right]$ is 0.485.	3 : $\begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$
(d) $w(\theta) = w_A$ for $0 \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = 0.517688$ $w(\theta) = w_A$ at $\theta = 0.518$ (or 0.517). $w'(0.518) < 0 \Rightarrow w(\theta)$ is decreasing at $\theta = 0.518$.	2 : $\begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$

2017 SCORING GUIDELINES

Question 3

$$(a) f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 = 3$$

$$f(5) = f(-2) + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 = 10 - 2\pi$$

$$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$$

$$(b) f'(x) > 0 \text{ on the intervals } [-6, -2) \text{ and } (2, 5).$$

Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

$$(c) \text{ The absolute minimum will occur at a critical point where } f'(x) = 0 \text{ or at an endpoint.}$$

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

$$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

$$(d) f''(-5) = \frac{2-0}{-6-(-2)} = -\frac{1}{2}$$

$$2 : \begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{cases}$$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

2017 SCORING GUIDELINES

Question 4

$$(a) H'(0) = -\frac{1}{4}(91 - 27) = -16$$

$$H(0) = 91$$

An equation for the tangent line is $y = 91 - 16t$.

$$3 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$$

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

$$(b) \frac{d^2H}{dt^2} = -\frac{1}{4} \frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$$

1 : underestimate with reason

$$H > 27 \text{ for } t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0 \text{ for } t > 0$$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

$$(c) \frac{dG}{(G - 27)^{2/3}} = -dt$$

$$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1) dt$$

$$3(G - 27)^{1/3} = -t + C$$

$$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$$

$$3(G - 27)^{1/3} = 12 - t$$

$$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3 \text{ for } 0 \leq t < 10$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \text{uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

The internal temperature of the potato at time $t = 3$ minutes is

$$27 + \left(\frac{12 - 3}{3}\right)^3 = 54 \text{ degrees Celsius.}$$

2017 SCORING GUIDELINES

Question 5

$$(a) f'(x) = \frac{-3(4x-7)}{(2x^2-7x+5)^2}$$

$$f'(3) = \frac{(-3)(5)}{(18-21+5)^2} = -\frac{15}{4}$$

$$(b) f'(x) = \frac{-3(4x-7)}{(2x^2-7x+5)^2} = 0 \Rightarrow x = \frac{7}{4}$$

The only critical point in the interval $1 < x < 2.5$ has x -coordinate $\frac{7}{4}$.

f' changes sign from positive to negative at $x = \frac{7}{4}$.

Therefore, f has a relative maximum at $x = \frac{7}{4}$.

$$(c) \int_5^\infty f(x) dx = \lim_{b \rightarrow \infty} \int_5^b \frac{3}{2x^2-7x+5} dx = \lim_{b \rightarrow \infty} \int_5^b \left(\frac{2}{2x-5} - \frac{1}{x-1} \right) dx$$

$$= \lim_{b \rightarrow \infty} \left[\ln(2x-5) - \ln(x-1) \right]_5^b = \lim_{b \rightarrow \infty} \left[\ln \left(\frac{2x-5}{x-1} \right) \right]_5^b$$

$$= \lim_{b \rightarrow \infty} \left[\ln \left(\frac{2b-5}{b-1} \right) - \ln \left(\frac{5}{4} \right) \right] = \ln 2 - \ln \left(\frac{5}{4} \right) = \ln \left(\frac{8}{5} \right)$$

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\int_5^\infty \frac{3}{2x^2-7x+5} dx$ converges.

— OR —

$$\frac{3}{2n^2-7n+5} > 0 \text{ and } \frac{1}{n^2} > 0 \text{ for } n \geq 5.$$

$$\text{Since } \lim_{n \rightarrow \infty} \frac{\frac{3}{2n^2-7n+5}}{\frac{1}{n^2}} = \frac{3}{2} \text{ and the series } \sum_{n=5}^\infty \frac{1}{n^2} \text{ converges,}$$

the series $\sum_{n=5}^\infty \frac{3}{2n^2-7n+5}$ converges by the limit comparison test.

2 : $f'(3)$

2 : $\begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \text{with justification} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$

2 : answer with conditions

2017 SCORING GUIDELINES

Question 6

$$(a) f(0) = 0$$

$$f'(0) = 1$$

$$f''(0) = -1(1) = -1$$

$$f'''(0) = -2(-1) = 2$$

$$f^{(4)}(0) = -3(2) = -6$$

The first four nonzero terms are

$$0 + 1x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}.$$

The general term is $\frac{(-1)^{n+1}x^n}{n}$.

$$(b) \text{ For } x = 1, \text{ the Maclaurin series becomes } \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n}.$$

The series does not converge absolutely because the harmonic series diverges.

The series alternates with terms that decrease in magnitude to 0, and therefore the series converges conditionally.

$$(c) \int_0^x f(t) dt = \int_0^x \left(t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots + \frac{(-1)^{n+1}t^n}{n} + \cdots \right) dt$$

$$= \left[\frac{t^2}{2} - \frac{t^3}{3 \cdot 2} + \frac{t^4}{4 \cdot 3} - \frac{t^5}{5 \cdot 4} + \cdots + \frac{(-1)^{n+1}t^{n+1}}{(n+1)n} + \cdots \right]_{t=0}^{t=x}$$

$$= \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \cdots + \frac{(-1)^{n+1}x^{n+1}}{(n+1)n} + \cdots$$

(d) The terms alternate in sign and decrease in magnitude to 0. By the alternating series error bound, the error $\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right|$ is bounded

by the magnitude of the first unused term, $\left| -\frac{(1/2)^5}{20} \right|$.

$$\text{Thus, } \left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| \leq \left| -\frac{(1/2)^5}{20} \right| = \frac{1}{32 \cdot 20} < \frac{1}{500}.$$

3 : $\begin{cases} 1 : f''(0), f'''(0), \text{ and } f^{(4)}(0) \\ 1 : \text{verify terms} \\ 1 : \text{general term} \end{cases}$

2 : converges conditionally with reason

3 : $\begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

1 : error bound