

1. Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M , where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

Part A

Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \leq t \leq 10$. Show the work that leads to your answer, and indicate units of measure.

Part B

- i. Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t) dt$. Show the work that leads to your answer.
- ii. Interpret the meaning of $\int_0^{30} M(t) dt$ in the context of the problem.

Part C

The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

$$F(t) = \begin{cases} 0 & \text{for } 0 \leq t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \leq t \leq 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

Part D

On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

- A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.
- B. Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.
- C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) dt$. Show the work that leads to your answer.
- D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t) \, dt$. Interpret the meaning of $\frac{1}{12} \int_0^{12} C(t) \, dt$ in the context of the problem.

(c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$,

where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$.

Show the setup for your calculations.

(d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$,

determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate.

Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r , where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \leq t \leq 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \leq t \leq 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t , $0 \leq t \leq 3$, for which $r'(t) = -6$? Justify your answer.

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of

$$\int_0^{12} r'(t) \, dt.$$

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is

100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with

respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r

and height h is $V = \frac{1}{3} \pi r^2 h$.)

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

(a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$$2\pi \int_0^4 r f(r) dr.$$

Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four

subintervals indicated by the data in the table.

(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H , where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

(b) Explain why there must be at least one time t , for $2 < t < 10$, such that $H'(t) = 2$.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \leq t \leq 10$.

(d) The height of the tree, in meters, can also be modeled by the function G , given by $G(x) = \frac{100x}{1+x}$, where

x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the

base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of

change of the height of the tree with respect to time, in meters per year, at the time when the tree is

50 meters tall?

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.
- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

3. Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.
- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.
- Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.
- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.